Quanto Credit Default Swaps

Theory, Pricing & Practice

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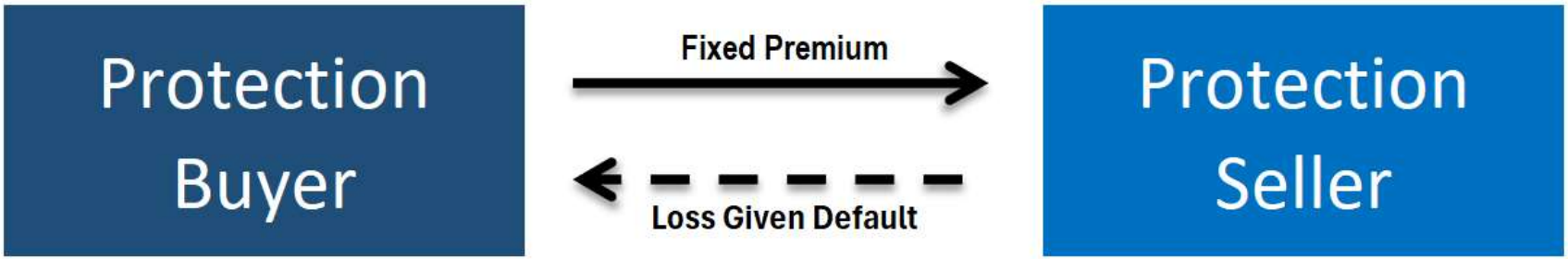
Quanto CDS Research & Support
https://github.com/nburgessx/QuantResearch/blob/main/Quanto%20CDS

PART ONE - THEORY

Quanto CDS – Introduction to Quanto CDS

Credit Default Swap

- An insurance contract to protect against losses should the reference bond default
- The protection buyer pays regular **Fixed Premiums** in exchange for the **Loss Given Default,** LGD = N(1 – RR)



Liquid vs Contractual Currency

- The **Liquid Currency** for a CDS contract is USD, regardless of bond domicile & issue currency
- When the CDS **Contractual Currency** is not the liquid currency, the CDS is considered a **Quanto CDS**

Credit Default Swaps – Pricing

Probability of Default at End of Period
$= \lambda\, Q(t_i)$

Probability of Survival
$= Q(t_i)$

Present Value is the sum of **Risky Discounted** cash flows

$$CDS\ PV^{1} = \underbrace{\sum_{i=1}^{n} N(1-\mathrm{R})\,\lambda\, Q(t_i)\, P(0,t_i)}_{Protection\ Leg} - \underbrace{\sum_{i=1}^{n} N\, s\, \tau_i\, Q(t_i)\, P(0,t_i)}_{Premium\ Leg} \quad (1)$$

[1] There are other ways to compute conditional default probabilities, here we use $\lambda\, Q(t_i)$ to simplify the par spread calculation.

Credit Default Swaps – Par Spread

Quotation

- CDS contracts typically quote as a **Par Spread, s**
- This is the spread that makes the premium and protection leg PVs equal.

Par Spread Formula

- Setting the PV in equation (1) to zero and rearranging gives (2),

$$s = \frac{\sum_{i=1}^{n} N(1-R)\,\lambda\, Q(t_i)\, P(0,t_i)}{\sum_{i=1}^{n} N\,\tau_i\, Q(t_i) P(0,t_i)} \quad (2)$$

Par Spread Rule of Thumb

- Terms in red cancel and assuming annual premiums i.e., $\tau = 1$ gives a simple rule of thumb,

$$s \approx (1-R)\,\lambda \quad (3)$$

Quanto CDS – The Quanto Market Data Problem

The Problem – Market Data in a Different Currency (USD)

- We want to price a BRL CDS on a BRL Sovereign Bond.
- However, only USD CDS market data quotes are available.

The Solution – Quanto Market Data Adjustments

- Brigo, Pede and Petrelli (2018) suggest we make a Quanto Adjustment to the USD market data.
- Naming conventions, they call BRL the Contract currency (or Quanto) and USD the Liquid currency.
- They derive a model to convert USD hazard rates into BRL equivalent rates.
- The quanto adjustment incorporates FX crash risk and FX covariance risk.

Quanto CDS – Quanto FX Risks

Quanto FX Risk

- We want to trade BRL CDS to buy/sell protection on a BRL Bond
- Typically, the BRL Quanto CDS will be illiquid, so we might use liquid USD CDS market data as a proxy.
- When using USD market data to price Quanto CDS we need to adjust for Quanto FX risk

Quanto FX Risk = Crash Risk + Covariance Risk

FX Crash Risk

- FX devaluation or crash risk on default
- Quanto CDS Spreads cheapen relative to USD

FX Covariance Risk

- FX covariance with credit spreads
- As default intensity increases, Quanto currency weakens
- Quanto protection becomes cheap relative to USD

Quanto CDS – Quanto Market Data Adjustments

Quanto Market Data Adjustment

- **Li, Mercurio & Resnick (2018)** present a quanto market data adjustment formula in [1], see equation (25)

$$\underbrace{\lambda^{BRL}}_{Conract\ Currency} = (1+\gamma)\left(1+\underbrace{\frac{1}{2}\,\rho_{(C,FX)}\,\sigma_C\,\sigma_{FX}\,t}_{Covariance\ Term}\right)\underbrace{\lambda^{USD}}_{Liquid\ Currency} \quad (4)$$

- The FX jump parameter, γ captures a single FX jump at default in %
- The FX jump dominates and covariance is a second order effect
- Setting the covariance term to zero we get a useful approximation or 'rule of thumb'.

$$\lambda^{BRL} = (1+\gamma)\lambda^{USD} \quad (5)$$

- Equivalently, using (3) gives an expression in terms of CDS par spreads, s

$$s^{BRL} = (1+\gamma)s^{USD} \quad (6)$$

Credit Default Swaps – FX Jump on Default Explained

Sovereign CDS - Brazil Case Study

- Consider Brazil Sovereign CDS in USD (Liquid Currency)
- and Brazil CDS in EUR (Quanto Contractual Currency)
- Let's define an FX process as Z = Quanto/USD
- The FX process Z = BRL/USD i.e., how many USD needed to buy 1 BRL?
- We can measure the jump $\gamma\%$ in the FX rate if there is a sovereign default

Credit Default and Impact of FX Devaluation

- Should Brazil default we expect BRL will weaken versus USD
- That is the amount of USD to buy 1 BRL to jump down i.e., Negative Jump, $\gamma\%$
- Therefore, Quanto protection payments in BRL are worth less relative to USD
- Par spreads measure protection payment value.
- Consequently, we expect Lower Quanto Par Spreads in BRL relative to USD

Key Points

- Negative FX Jump on Default
  i.e. BRL Devaluation
- Quanto Protection Cheapens
  Relative to USD
- Lower Quanto CDS Spreads
  USD CDS > BRL CDS

Credit Default Swaps – Quanto Hazard Rates

Consider Brazil CDS in BRL (Quanto) and USD (Liquid)

- Z = BRL/USD (Quanto/USD)
- On Default Expect Negative FX Jump γ
- Quanto Protection Cheaper Relative to USD

Quanto Rules of Thumb

- $s^{Quanto} = (1+\gamma)s^{Liquid}$
- $\lambda^{Quanto} = (1+\gamma)\lambda^{Liquid}$

Cheaper Quanto Protection

- BRL CDS Spread < USD CDS Spread
- BRL Hazard Rates < USD Hazard Rates, since $s \approx (1-R)\,\lambda$

Quanto Hazard Rates & Market Data

- Intuitively, we can deduce that $S^{Quanto} = (1+\gamma)S^{Liquid}$
- Furthermore, $s \approx (1-R)\,\lambda$ implies $\lambda^{Quanto} = (1+\gamma)\lambda^{Liquid}$
- Therefore, we should not use raw USD hazard Rates (Liquid) to price BRL CDS (Quanto)
- We must scale hazard rates for Quanto CDS pricing and instead use λ^{Quanto}

Credit Default Swaps – Mathematics of Quanto Hazard Rates I

- Consider a default process D(t) = 1{τ<t} for default time τ
- Now D(t) is not a martingale, so we apply a compensator to create a Martingale process, M(t)
- The compensator removes the predictable drift from the D(t) process
- Here the compensator term represents expected defaults over the time-period (0, t)

$$M(t) = D(t) - \int_0^t \big(1 - D(s)\big)\lambda(s)ds$$

or

$$dM(t) = dD(t) - \big(1 - D(t)\big)\lambda(t)dt(7)$$

Credit Default Swaps – Mathematics of Quanto Hazard Rates II

- The Quanto Martingale process, $M^Q(t)$ can be computed using the Girsanov theorem to change to Quanto probability measure. Given a Radon-Nikodym derivative R(t) we have,

$$dM^Q(t) = dM(t) - \frac{dD(t), R(t))}{R(t)}$$

$$= dM(t) - d(D(t), \gamma D(t))$$

$$= dM(t) - \big(1 - D(t)\big)\gamma\lambda(t)$$

- Substituting (7) for dM(t) gives,

$$dM^Q(t) \; = dD(t) - \big(1 - D(t)\big) \underbrace{(1+\gamma)\lambda(t)}_{=\lambda^Q(t)} dt \; (8)$$

- Comparing $dM^Q(t)$ with dM(t) we can see the Quanto hazard rate process is given by,

$$\lambda^Q = \; (1+\gamma)\lambda \; (9)$$

PART TWO – PRICING & PRACTICE

Eurozone Case Study – Quanto Effect on CDS Spreads

Eurozone FX Crash Risk

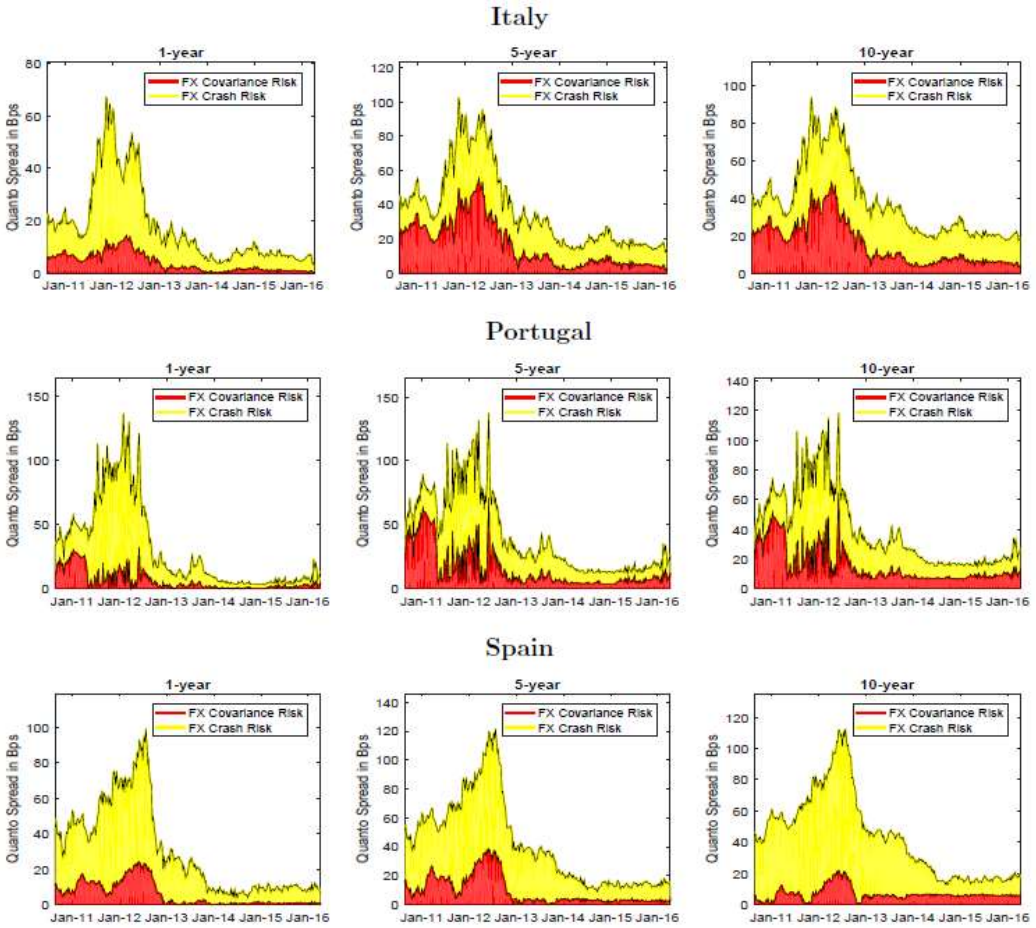
- Varies depending on credit regime & market distress
- On average accounts for ca. 10 bps of the Quanto Basis

Eurozone FX Covariance

- Similar to FX crash risk, covariance risk of ca. 10 bps
- Covariance risk grows with CDS maturity

Eurozone Quanto Basis

- Combining FX Crash and Covariance Risk
- Total Quanto Basis ≈ 20 bps
- i.e. USD CDS Spread > EUR CDS Spread



Source: Lando & Nielson (2018)

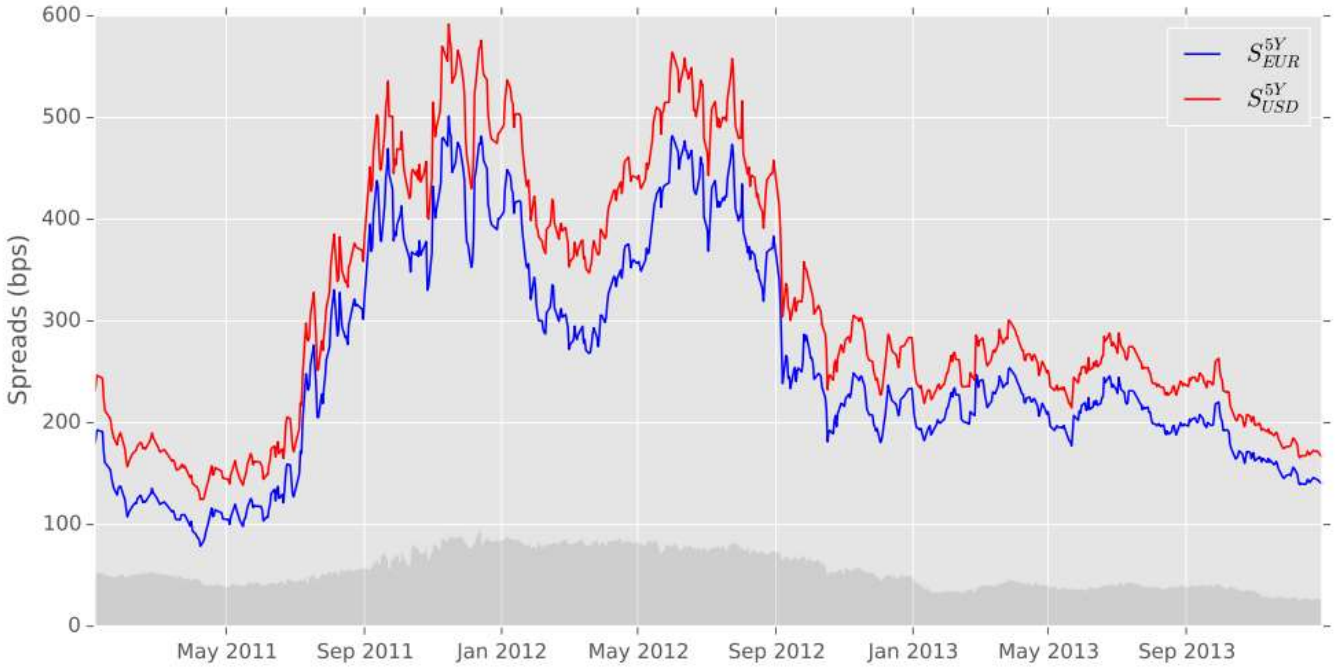
Italy Case Study – CDS Par Spreads EUR vs USD

Quanto Par Spreads

- Typically, Quanto CDS spreads are lower than the liquid currency CDS in USD

Italy CDS Spreads: EUR vs USD

- Italy CDS spreads in EUR (Quanto) are lower than those in USD by ca. 20 bps



Italy CDS Spreads, Source: Brigo (2016) – Quant Summit Europe

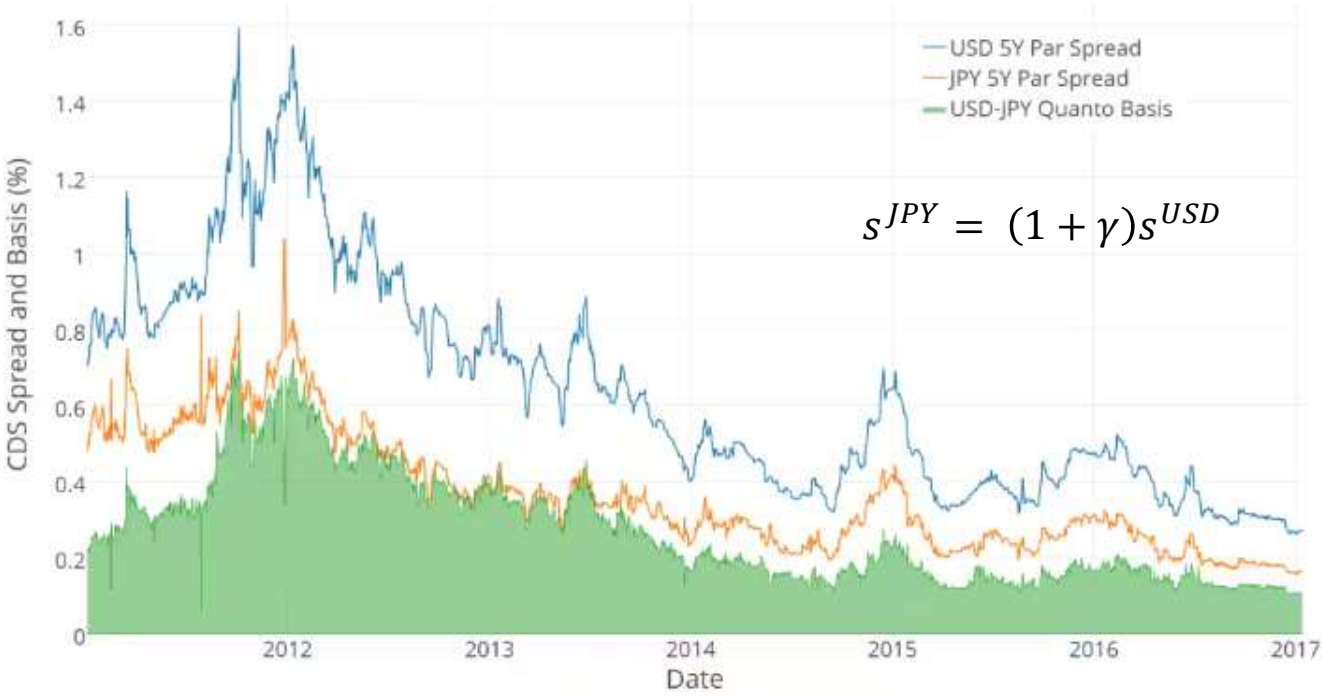
Japan Case Study – Sovereign Quanto Basis Spread

Quanto Basis

- The difference between USD and Quanto CDS Spreads is called the Quanto Basis

Japan Sovereign CDS Spreads

- Japan CDS spreads in JPY (Quanto) are lower than those in USD by ca. 20 bps

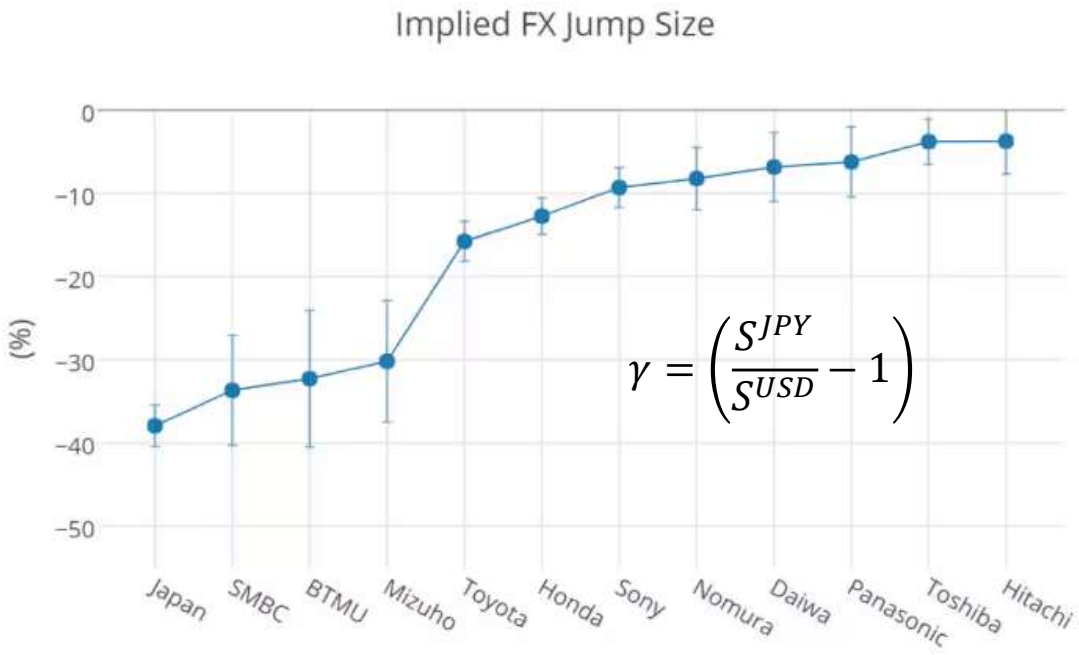


Japan Sovereign CDS Spreads, Source: S&P Global (2018)

Japan Case Study – Corporate Quanto Basis

How to Imply FX Jump Sizes and Quanto Basis for Corporates?

- In Japan corporate CDS market, significant negative FX Jumps (JPY Devaluation) priced in.
- FX jump sizes are strongly correlated with systemic importance of corporate
- There is evidence of similar FX jump sizes across industry groups. Suggests possibility of building sector basis curves



Japan Corporate Implied FX Jump Sizes, Source: S&P Global (2018)

Japan Case Study – Industry Sector Implied FX Jump Sizes

Sector Basis Curves

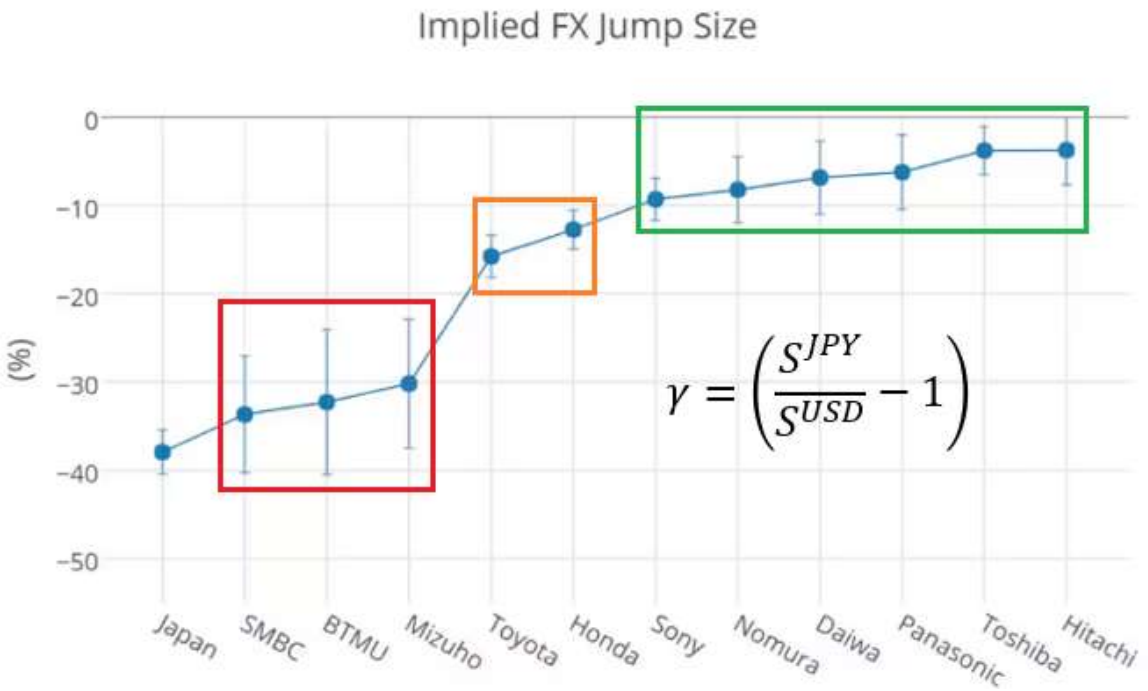
There is evidence of similar FX jump sizes across industry groups. Jump sizes correlated to systemic importance.

JPY Implied Jump Sizes by Sector

- JPY Sovereign -40%
- JPY Banking Sector -35%
- JPY Automotive Sector -15%

(Automotive sector constitutes 90% of total GDP!)

- JPY Large Corporates -10%
- JPY Mid-Small Corporates -5%



JPY Implied FX Jump Sizes by Sector, Source: S&P Global (2018)

Brazil Case Study – Quanto CDS & Quanto Basis

Brazil Sovereign CDS Spreads (BRL)

- Consider an FX jump of -40%
- CDS spreads in BRL (Quanto) would be lower than those in USD by ca. 71 bps

Present Value (PV)	Without Adj	With Adj	+/-
CreditDefaultSwap	242,273	5,648	-236,626
PremiumLeg	-335,060	-342,377	-7,316
AccruedInterest	-964	-581	383
ProtectionLeg	578,297	348,605	-229,692

Par CDS Rate	Without Adj	With Adj	+/-	Quanto Basis
Par CDS Rate, p	1.7231%	1.0165%	-0.7066%	70.66
ProtectionPV - AccruedInterest	-577,334	-348,024	229,309	
RiskyFixedAnnuity	-33,506,024	-34,237,674	-731,650	

CDS Trade Parameters & Price

BuyOrSellProtection	BUY	
BuySellIndicator, φ	1	
Notional, N	10,000,000	
HazardRate, $\lambda^{\text{Domestic CCY}}$	2.2947%	
RecoveryRate, R	25.00%	Spread (bps)
CreditSpread, s	1.0000%	100.00
PremiumFrequency	QUARTERLY	
PremiumYearFraction, Δt	0.2500	
Maturity, years	5.0000	
ZeroRate, z	14.00%	

Quanto Parameters

FX Jump on Default, J_{FX}	-40.00%	
Corrleation, $\rho_{(C,FX)}$	-0.25	
Credit Volatility, σ_C	25.00%	
FX Volatility, σ_{FX}	10.00%	Lambda Adj
HazardRate, $\lambda^{\text{Quanto CCY}}$	1.3553%	-0.9394%

$$\lambda^{Quanto}(t) = (1 + J_{FX})\left(1 + \tfrac{1}{2}\,\varrho_{C,FX}\,\sigma_C\,\sigma_{FX}\,t\right)\lambda^{Domestic}(t)$$

What-If Analysis

Correlation \ Quanto Basis: 70.66	FX Jump on Default: 0.00%	-5.00%	-10.00%	-20.00%	-30.00%	-40.00%	-50.00%
1.00	-10.80	-1.62	7.56	25.91	44.25	62.58	80.89
0.75	-8.10	0.94	9.99	28.07	46.14	64.19	82.24
0.50	-5.40	3.51	12.42	30.23	48.02	65.81	83.58
0.25	-2.70	6.07	14.85	32.38	49.91	67.43	84.93
0.00	0.00	8.64	17.28	34.54	51.80	69.04	86.28
-0.25	2.70	11.20	19.70	36.70	53.68	70.66	87.62
-0.50	5.40	13.77	22.13	38.86	55.57	72.27	88.97
-0.75	8.10	16.33	24.56	41.01	57.46	73.89	90.32
-1.00	10.80	18.89	26.99	43.17	59.34	75.51	91.66

Brazil Case Study – Corporate Implied FX Jumps

Brazil Implied FX Jump (Estimates)

- Sovereign -40%
- Banking Sector -35%
- Large Corporates -10%
- Mid-Small Corporates -5%

Can be implied from Direct Market Quotes

or Sector Quanto Basis Curves

$$\gamma = \left(\frac{S^{JPY}}{S^{USD}} - 1 \right)$$

What-If Analysis

		FX Jump on Default			
Correlation	Quanto Basis	MID CORP	LARGE CORP	BANKS	SOVEREIGN
	70.66	-5.00%	-10.00%	-35.00%	-40.00%
	1.00	-1.62	7.56	44.25	62.58
	0.75	0.94	9.99	46.14	64.19
	0.50	3.51	12.42	48.02	65.81
	0.25	6.07	14.85	49.91	67.43
	0.00	8.64	17.28	51.80	69.04
	-0.25	11.20	19.70	53.68	70.66
	-0.50	13.77	22.13	55.57	72.27
	-0.75	16.33	24.56	57.46	73.89
	-1.00	18.89	26.99	59.34	75.51

Brazil Case Study – Quanto CDS & Quick Rule of Thumb

Brazil Implied FX Jump (Estimates)

- Sovereign -40%
- Banking Sector -35%
- Large Corporates -10%
- Mid-Small Corporates -5%

BRL Par Spreads: As a Multiple of USD CDS

- Sovereign 60%
- Banking Sector 55%
- Large Corporates 90%
- Mid-Small Corporates 95%

Rule of Thumb: $S^{BRL} = (1+\gamma)S^{USD}$

Example: BRL CDS(Sovereign) ≈ (1 - 40%) USD CDS

= 60% x USD CDS

= 60% x 172.10bps = 103.26 bps

References

- Bloomberg (2016), Pricing Default-Contingent Contracts with Deterministic Credit
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- Itkin A., Shcherbakov V. and Veygman A. (2019), A New Model for Pricing Quant Credit Default Swaps
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Have questions or want further info?

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