Quanto CDS Spreads

David Lando and Andreas Bang Nielsen*

April 20, 2018

Abstract

Quanto CDS spreads are differences in CDS premiums of the same reference entity but in different currency denominations. Such spreads can arise in arbitrage-free models and depend on the risk of a jump in the exchange rate upon default of the underlying and the covariance between the exchange rate and default risk. We develop a model that separates the contribution of these two effects to quanto spreads and apply it to four eurozone sovereigns. Furthermore, using our model estimates, we provide evidence that quanto effects can explain a significant part of the yield spread between eurozone sovereign bonds issued in Euro and U.S. dollar. Our findings suggest that comparing bond yields across currency denominations using standard FX forward hedges misses an important quanto effect component.

Keywords: Sovereign credit risk, CDS premiums, currency risk, systemic risk

JEL Codes: H63, G13, F31, G01

*Both authors are at Department of Finance and Center for Financial Frictions (FRIC). Copenhagen Business School, Solbjerg Plads 3, DK-2000 Frederiksberg, Denmark. E-mail: abn.fi@cbs.dk and dl.fi@cbs.dk.
We acknowledge support from the Danish Social Science Research Council and through our affiliation with the Center for Financial Frictions (FRIC), grant no. DNRF102 from the Danish National Research Foundation.

1 Introduction

During the European debt crisis, the European sovereign credit market experienced tremendous distress with sovereign credit spreads widening to unprecedented levels. But not only did the levels of CDS premiums for sovereigns spike; the difference between CDS premiums on European sovereigns denominated in EUR and USD, the so called quanto spread, also increased significantly. The 5-year quanto spread reached 95 bps for Italy, 105 bps for Spain and 145 bps for Portugal and it has continued to be substantial after the crisis. Since the EUR and USD-denominated CDS contracts are issued under the same standardized ISDA terms—including same recovery rate and trigger events—the quanto spread is not due to contractual differences.

It is well known that quanto spreads can arise without any frictions. If there is a risk of a crash in the exchange rate coinciding with default of the reference name of the CDS, then this leads to a quanto spread. It is less obvious, and seemingly less recognized, that correlation between FX-rate fluctuations and the default intensity of the reference name also leads to a quanto spread, and that this contribution to the spread can arise even if there is no depreciation of one currency in the event of default. An accurate assessment of currency crash risk in the event of default from quanto spread requires a correction for this correlation effect.

We propose here a simple two-factor discrete-time model in which the effects can be understood simply and rigorously. The first factor, the FX crash risk factor, captures the market's (risk-neutral) anticipation of a jump in foreign currency (EUR) against domestic currency (USD) in the event of a sovereign default. If crash risk is present, it implies a smaller expected recovery on a EUR contract relative to a similar USD contract and thus causes protection in USD to be more expensive. The second factor, the currency/default risk covariance factor, captures the propensity for the EUR to depreciate (appreciate) against the U.S. dollar when eurozone sovereign credit risk rises (falls). If there is a positive (negative) shock to credit risk, CDS premiums in both EUR and USD increase (decrease). However, if the EUR simultaneously decreases (increases) relative to the USD, the gain (loss) is larger (smaller) on the USD CDS compared to the similar EUR CDS. Therefore, the expected gains are smaller, and the expected losses are greater on the EUR CDS compared to the USD CDS, implying a positive quanto CDS spread.

The model offers a number of important insights on how these two channels affect quanto spreads and how we can distinguish between them. Importantly, we show that short-term quanto spreads are primarily driven by crash risk, as the maturity goes to zero, this is the only factor that drives quanto spreads. Quanto spreads at longer maturities, on the other hand, are impacted by both crash risk and covariance factor—with the latter gaining more significance as time to maturity increases. A key implication of the model is therefore that the term structure of quanto spreads can help to differentiate between crash and covariance risk.

Based on the insights of the discrete-time model, we propose an affine term structure model that captures both time-varying default risk, covariance between the FX-rate and the default intensity and currency jump risk associated with sovereign default. We estimate the model using USD-denominated CDS, quanto CDS spreads, and EURUSD currency options. Currency options are included in the estimation to identify the dynamics of exchange rate risk which is an important contributor to quanto spreads through the covariance risk channel.

We find that the covariance component is highly time-varying and tends to spike in times of crisis, while the crash risk component is persistent over the sample period, and, on average, accounts for the largest fraction of quanto CDS spreads. In essence, the covariance component reflects the distress-related part of quanto spreads; it shoots up in times when volatilities of credit risk and exchange rates are high and when they covary strongly. On the other hand, the crash risk component is of more static nature, because it captures the expected depreciation conditional on default. For example, in a model with no uncertainty surrounding credit risk (e.g., constant default risk) the covariance component is clearly zero, while crash risk causes a quanto spread if the market anticipates a jump in the exchange rate in reaction to a default.

Furthermore, we document that the relative contribution of covariance risk and crash risk to quanto spreads depends on the maturity. The short end of the quanto CDS term structure is almost exclusively driven by crash risk, while the covariance component increases in time to maturity. Intuitively, this is because the crash risk component causes a parallel shift in the term structure of quanto CDS spreads, while the covariance component affects the slope of the quanto CDS term structure. As a consequence, we find that covariance risk is particularly important for the relative pricing across currency denominations for longer-dated credit risky securities.

More specifically, we use our model to decompose the quanto CDS spreads, at maturities from 1-10 years, into a crash risk and a covariance risk component for Italy, Spain, Portugal, and Ireland over the period from August 2010 to April 2016. For Spain and Italy, we estimate the impact of a sudden sovereign default on the EURUSD to 15.6% and 9.6%, respectively. While for Portugal and Ireland, we estimate the currency crash to be significantly smaller at 5.3% and 5.0%, respectively.

Based on our model, we find that for Portugal and Ireland the average covariance components are 15.2 bps and 23.5 bps for the 5-year quanto spreads, corresponding to shares of 35% and 75% of their average quanto spreads. Consistent with our intuition that the covariance component is particularly important in times of distress, we indeed find that covariance risk is largest at the peak of the European debt crisis. For Ireland and Portugal, the covariance components during this period reach up to 60-70 bps which, in fact, exceed the contribution of crash risk to their quanto spreads. Without taking into account covariance risk, we would erroneously interpret the large quanto spreads for Portugal and Ireland as a sign of risk of a large downward jump in the Euro upon the default of these sovereigns.

The covariance components are not only substantial for the peripheral sovereigns, they also account for a large proportion of the quanto spreads for Spain and Italy. We find that the average of the covariance components at the 5-year maturity are 9.42 bps and 16.35 bps, which corresponds to 20% and 35% of their total quanto spreads. However, as is the case for the peripheral sovereigns, their covariance components exhibit strong time-variation and reach 38.51 bps and 55.25 bps at the peak of the European debt crisis, corresponding to 40%

and 65% of their total spreads.

Quanto effects also apply to yield spreads of bonds issued by the same entity in different currencies. The advantage of studying quanto spreads from the perspective of CDS contracts is that recovery rates are the same for CDS contracts denominated in different currencies. This eliminates uncertainty related to differences in recovery rates, for example due to legal risk, between local currency and foreign currency denominated bonds, as addressed for example in Du and Schreger (2016).

On this basis, we use the model estimated from CDS data to construct model-implied quanto bond yield spreads, and we investigate if they can explain the observed yield spreads on bonds denominated in EUR and USD issued by Italy, Spain, and Portugal. We find that a significant part of the contemporaneous variation in quanto yield spreads can be explained by our model-implied quanto yield spreads, especially during the peak of the European debt crisis. An implication of our findings is thus that the previous literature that compares bonds across currency denominations using FX forward hedges, without accounting for quanto effects, may potentially miss an important component of yield spreads caused by quanto effects.

2 Literature

The unpublished work of Ehlers and Schönbucher (2006) is, to our knowledge, the first to recognize the joint effects of crash risk and covariance risk on CDS premiums in different currencies. While they focus on developing a theoretical framework that can be used to construct models for credit risky securities in different currencies, we focus on understanding and quantifying, both theoretically and empirically, the driving factors of quanto CDS spreads.

There are two closely related papers that study quanto CDS spreads in the eurozone which both focus on using quanto CDS spreads to imply out expected depreciations in the Euro versus the U.S. dollar at different horizons. Mano (2013) uses quanto CDS spreads for eurozone sovereigns to imply out risk-neutral expected depreciations upon default, without distinguishing between crash risk and covariance risk. In more recent and contemporaneous research, Augustin, Chernov, and Song (2018) propose an affine term structure model for eurozone quanto CDS spreads, which they use to estimate objective expected depreciations in the EURUSD conditional on sovereign defaults at different horizons. Our work differs from these papers in its main objective, we focus on what causes quanto CDS spreads and differences in bond yields across currency denominations. We identify two risk factors, covariance risk and currency crash risk, and we estimate their contribution to quanto CDS spreads and their time-series variation. Furthermore, we also use our model to explain what causes yield spread differences for eurozone sovereign bonds issued in Euro and U.S. dollar. Besides this, there are two other relevant papers that study eurozone quanto CDS spreads, De Santis (2015) and Brigo et al. (2016). The former uses quanto CDS spreads for eurozone sovereigns to estimate redenomination risk, that is, compensation for risk that EUR-denominated securities are redenominated into a new devalued currency. The latter focuses on developing a pricing model for quanto CDS spreads and calibrate it to Italian quanto CDS spreads.

Carr and Wu (2007b) provide evidence that sovereign credit risk is priced in the currency option markets for Brazil and Mexico. They obtain inference on the (risk-neutral) jump size in local currency upon sovereign default by estimating a joint model for options and sovereign CDS. Since option prices are driven by numerous factors apart from sovereign credit risk, e.g., macroeconomic news (Chernov et al., 2016), this approach makes it difficult to quantify the effect of sovereign default on local currency. Since the payoff on a quanto CDS is directly linked to currency jump risk at default, we contribute by providing a clean method for estimating the crash risk upon default.

Our paper is related to the vast literature that studies sovereign credit risk through the lens of CDS premiums, e.g., Longstaff, Pan, Pedersen, and Singleton (2011), Aït-Sahalia, Laeven, and Pelizzon (2014), Pan and Singleton (2008), Benzoni, Collin-Dufresne, Goldstein, and Helwege (2015), and Della Corte, Sarno, Schmeling, and Wagner (2016). The latter is, perhaps, the closest related to this paper. They document empirically a significant relationship between sovereign credit risk and returns on currencies and currency option strategies. While their paper is purely empirical, our objective is to develop models that allow us to quantify and understand the interconnection between credit and currency risk.

We contribute to the literature that studies pricing of similar credit risky securities across currency denominations, in particular bonds. There is a growing literature that analyzes deviations in yields for sovereign bonds across currency denominations (Buraschi et al., 2014; Corradin and Rodriguez-Moreno, 2016; Du and Schreger, 2016).

In these papers, the objective is to use the so-called "yield basis", defined as the difference between yields on a domestic and a synthetic domestic bond (which is constructed from foreign currency denominated bonds using FX forwards), to measure violations of the law of one price. Corradin and Rodriguez-Moreno (2016) show that the yield basis for eurozone sovereigns is large and volatile, and they attribute it to differences in collateral value and ECB purchases of EUR-denominated bonds. Buraschi, Menguturk, and Sener (2014) find a substantial yield basis for emerging market bonds during the 2007-2008 crisis and explain it by frictions in banking capital structure and non-conventional policy interventions. However, our theory shows that a yield basis may arise because of crash risk and covariance risk. Our empirical results suggest that this not only a theoretical concern. We provide evidence that indicates that the yield spread between EUR and USD-denominated bonds for eurozone sovereigns reflects compensation for risk related to covariance and crash risk.

3 Default and Recovery in Different Currencies

CDS contracts on the same reference entity but denominated in different currencies share a number of characteristics that are important to understand before setting up a model.

A Credit Default Swap (CDS) is an insurance against default on debt of an underlying reference entity. The contract involves two parties: a protection buyer and a protection seller. Every period, if no credit event has occurred of the reference entity, the buyer pays a percent-wise premium (often quarterly) of an agreed notional amount to the seller. If a credit event occurs, the buyer receives a recovery of the notional protected. Credit events are defined by the International Swaps and Derivatives Association (ISDA) and involves different scenarios, including outright bankruptcy, restructuring of debt, or deferred interest payments.

If a credit event occurs, an auction is held to determine the recovery rate based on a pool of bonds delivered into the auction. Importantly, the recovery rate is the same for all CDS contracts, independently of the currency denomination (see below for more details).

The auction is typically conducted between 30-35 days following the event determination date. Once an event has occurred, protection buyers are entitled to settle by physically delivering any of the specified deliverable obligations to settle the contract.

According to the standardized ISDA terms, the deliverable bonds are subject to a number of requirements. The payments of the obligation must be made in one of the *specified currencies* which for reference entities of Western Sovereigns are CAD, CHF, EUR, GBP, JPY, or USD. This means, for example, that a holder of a CDS contract denominated in EUR on Germany can choose to deliver German sovereign bonds denominated in USD. The relevant exchange rates for delivering obligations in a different currency to the CDS contract are fixed the day before the auction at 4pm at the WM/Reuters 4pm London mid-point rate.

4 The Quanto Spread in a Discrete Model

The option to choose in which currency to deliver bonds of the defaulted issuer means that the currency denomination becomes important. This can be seen through a very simple example: Consider two CDS contracts on Germany: One EUR-denominated with a notional amount of 1 EUR and one USD-denominated with a notional amount of 1 USD. Imagine for simplicity that the exchange rate is 1 at the initiation of the contract. If a default occurs before maturity, and at the same time the EUR drops to, say, a value of 0.5 USD, then the scale of protection offered by the two contracts differs. The holder of the EUR-denominated CDS can deliver 1 EUR notional and receive 1 EUR, whereas the holder of the USD protection can deliver a notional amount of 2 EUR, since the USD equivalent notional of 2 EUR is now only 1 USD because of the 'crash' of the EUR. Hence the amount of notional protected becomes effectively larger for the USD contract.

A similar mechanism is at play when currency depreciation has a positive correlation with a decrease in credit quality. Again, a simple example can provide the intuition. Imagine, as above, that the time 0 exchange rate is 1, and that the value of 1 USD can become 1.2 Euro or 0.8 Euro with equal probabilities 0.5 (under the USD risk-neutral measure) in the period 1, and that the exchange rate stays put in the second period until the CDS matures at time 2. Assume also for simplicity that the default probability of the reference entity is perfectly correlated with the exchange rate and becomes 3 percent in the state where the exchange rate is 1.2 and 1 percent in the other state. Assume zero interest rate in both currencies, and zero recovery in default. In this case, the USD value of protection of the CDS contract in two states is summarized in the following table:

State/denomination	USD	EUR
1.2/3%	0.03	$\frac{0.03}{1.2} = 0.025$
0.8/1%	0.01	$\frac{0.01}{0.8} = 0.0125$

Since $0.5 \cdot 0.025 + 0.5 \cdot 0.0125 = 0.0187 < 0.5 \cdot 0.01 + 0.5 \cdot 0.03 = 0.02$, we see that the value of the protection leg at time 1 is smaller for the EUR-denominated contract. If we assume (again for simplicity) that default risk is 0 between time 0 and time 1, then we have shown that the effect also applies for correlated default probability and FX-rate.

4.1 Model Assumptions and Definitions

We now build a simple discrete-time model that makes these observations rigorous. The model allows us to derive comparative statics and to analyze term structure effects. For the remainder of the paper, we define the exchange rate at time t, X_t, as units of domestic currency per unit of foreign currency, i.e., an increase in X_t implies that the foreign currency has appreciated against the domestic currency. Furthermore, we assume the existence of fixed riskless interest rates in both foreign and domestic currency, which we denote r_d and r_f, and we let $P_i(t, T) = e^{-r_i(T-t)}$ denote the price at time t of a zero-coupon bond paying one unit of currency $i = d, f$ at time T. In a no-arbitrage setting, we can then express the time t forward exchange rate with maturity T, $F(t,T)$, in terms of the foreign and domestic bond prices and the spot exchange rate as

$$F(t,T) = X_t \frac{P_f(t,T)}{P_d(t,T)}$$

Our model has a time horizon of $\bar{t}$ and we subdivide the time horizon into N equidistant time points which we label $t_0 = 0, t_1 = 1, \ldots, t_N = \bar{t}$. In each time period t there is a probability λ_t that the reference entity will default between time t and time $t+1$. We model FX crash risk upon default of the reference entity by assuming that the exchange rate drops by a fixed fraction of δ of the (risk-neutral) unconditional expectation of the exchange rate. Specifically, conditional on default between t and $t+1$, the exchange rate takes two possible values at $t+1$: $\delta \cdot uX_t$ and $\delta \cdot u^{-1}X_t$ with probabilities q and $1-q$, respectively. Conditional on no default, the exchange rate takes the values $C(\lambda_t) \cdot u$ and $C(\lambda_t) \cdot u^{-1}$ with respective probabilities q and $1-q$, where $C(\lambda_t)$ is a compensating factor $C(\lambda_t)$ defined as

$$C(\lambda_t) = \frac{1 - \delta\lambda_t}{1 - \lambda_t}$$

and it is needed to ensure no-arbitrage by compensating the exchange rate movement for crash risk. Had there been no crash risk, the exchange rate would either move up by a factor of u or down by a factor of u^{-1}. We show formally in Appendix 10.1 that this model is consistent with no-arbitrage. For tractability, we choose to do the compensation of crash risk through the jump size rather than through the martingale probabilities, which is an alternative option. We assume that the default probability can assume two values (λ^U, λ^D) in each period, and for simplicity we assume that the respective probabilities q^λ and $(1-q^\lambda)$ do not depend on the current state. To capture the joint dynamics of default risk and exchange rates, we introduce correlation between the movements in the exchange rate and the default probability. Let Q_{ij} denote the one-step probability of the exchange rate to reach state i and the default probability to reach state j (conditional on survival), where

$i = 1/j = 1$ correspond to an up move, and $i = 0/j = 0$ to a down move. At any point in time, we specify the joint distribution of the exchange rate and default probability as

$$Q_{11} = q(q^{\lambda} + A_1), \;\; Q_{10} = q(1 - q^{\lambda} - A_1) \tag{1}$$

$$Q_{01} = (1 - q)(q^{\lambda} - A_0), \;\; Q_{00} = (1 - q)(1 - q^{\lambda} + A_0) \tag{2}$$

where, $A_1 = \rho\sqrt{\frac{q^{\lambda}}{q}(1-q)(1-q^{\lambda})}$ and $A_0 = \rho\sqrt{\frac{q^{\lambda}}{1-q}q(1-q^{\lambda})}$. The important parameter here is ρ, which is the correlation between the Bernoulli variables controlling the up and down moves of the exchange rate and default probability. Clearly, if $\rho < 0$, then $A_1 < 0$ and $A_0 < 0$, which implies that the exchange rate and the default probability tend to move in the opposite direction compared to the uncorrelated case ($\rho = 0$). Note that it only takes a specification of the unconditional probabilities q and q^{λ} and the correlation parameter to specify all the relevant quantities. q^{λ} and ρ can be chosen freely in $(0, 1)$ and $(-1, 1)$, respectively, but q is endogenously determined through the no-arbitrage condition for the currency movement which can be expressed simply in terms of the one-period forward rate $F = F(t, t + 1)$ as

$$q = \frac{F/X_t - u^{-1}}{u - u^{-1}} \tag{3}$$

See Appendix 10.1 for the derivation. Figure 1 illustrates the joint dynamics of the exchange rate and the default probability over two periods. The multi-period dynamics are obtained by repeating this tree from each individual node. After default of the reference entity, the tree terminates.

4.2 Pricing the Domestic and Foreign CDS

We model a Credit Default Swap (CDS) contract focusing on the 'fair running premium' that the buyer of protection should pay to obtain credit protection. For a contract with maturity T, we assume that no payment is exchanged at time 0 and that at every period $t_i \leq t_N \equiv T$, the buyer of the CDS contract pays a premium if the reference issuer has not defaulted at this time. If default occurs in the time interval $(t_{i-1}, t_i]$, the seller of insurance pays $1 - R$ per unit face value—which we without loss of generality assume to be 1.

In this setting, the CDS premium in domestic currency with maturity T, $S^d(0,T)$, is given by

$$S^d(0,T) = (1-R)\frac{\sum_{i=1}^{N} P_d(0,t_i)Q\left(\tau = t_i\right)}{\sum_{i=1}^{N} P_d(0,t_i)Q\left(\tau > t_i\right)} \tag{4}$$

According to the standardized rules of ISDA, the foreign CDS contract is subject to the exact same contractual terms as the domestic contract, apart from currency denomination (CDS premiums are paid in foreign currency, and in the event of default, the recovery is received in foreign currency). The rules imply that the recovery rate is the same regardless of currency denomination of the contract.

Recall, that Q is the risk-neutral pricing measure when using the domestic bank account as numeraire. Defining Q^f as the risk-neutral measure corresponding to having the foreign account as numeraire, we can now express the premium of the same CDS contract denominated in the foreign currency as

$$S^f(0,T) = (1-R)\frac{\sum_{i=1}^{N} P_f(0,t_i)Q^f\left(\tau = t_i\right)}{\sum_{i=1}^{N} P_f(0,t_i)Q^f\left(\tau > t_i\right)} \tag{5}$$

where $P_f(0,t)$ denotes the discount factor corresponding to the foreign interest rate. To compare the two expressions we will need to understand the relationship between Q and Q^f.

Let M_t^i denote the pricing kernel for currency denomination $i = d, f$. Starting with the objective measure, P, we can price any foreign-denominated security with a price, Z_t^f, using the foreign pricing kernel:

$$1 = E_t^P\left(\frac{M_T^f}{M_t^f}\frac{Z_T^f}{Z_t^f}\right) = E_t^{Q^f}\left(P_f(t,T)\frac{Z_T^f}{Z_t^f}\right) \tag{6}$$

As in, e.g., Backus, Foresi, and Telmer (2001), we construct a domestic security from the foreign security using the exchange rate: $X_t Z_t^f$. Since this claim is denominated in domestic currency, we can price it using the domestic pricing kernel:

$$1 = E_t^P \left(\frac{M_T^d}{M_t^d} \frac{X_T Z_T^f}{X_t Z_t^f} \right) = E_t^Q \left(P_d(t,T) \frac{X_T Z_T^f}{X_t Z_t^f} \right) \tag{7}$$

Equations (6) and (7) hold for any security which implies that there is the following relationship between the domestic and foreign pricing kernels, the exchange rate, and the foreign and domestic risk-neutral measures:

$$\frac{M_T^f}{M_T^d} \frac{M_t^d}{M_t^f} = \frac{X_T}{X_t}, \quad M_T = \frac{X_T}{X_t} \frac{P_d(t,T)}{P_f(t,T)} \tag{8}$$

where M_T changes measure from the foreign to the domestic risk-neutral measure (i.e., $M_T = \frac{dQ^f}{dQ}(T)$). We refer to Appendix 10.2.1 and 10.2.2 for the closed-form model expressions of the domestic and foreign CDS premiums as well as their derivation.

4.3 Quanto CDS Spreads Comparative Statics

We now discuss how each parameter of the model impacts the quanto spread. First, we show that the quanto spread widens in the expected severity of the crash in foreign currency upon default.

Proposition 1. *The quanto spread, $QS(0,T)$, is decreasing in δ for all T*

Proof. See Appendix 10.2.3 □

To gain some intuition on Proposition 1, we propose a stylized example with a fixed default probability (implying independence between the default probability and the exchange rate), and a crash risk premium of δ. In Appendix 10.2.4, we show that in this case, the

CDS premiums in domestic and foreign currency, of any maturity, are given by

$$S^d = (1-R)\frac{\lambda}{(1-\lambda)} \tag{9}$$

$$S^f = (1-R)\frac{\lambda\delta}{(1-\lambda\delta)} \tag{10}$$

In the case of a fixed default probability, the riskless interest rates do not affect CDS premiums, i.e., the expressions for the CDS premiums in (9) and (10) hold for any choice of foreign and domestic interest rates. Assume $\delta < 1$, which implies that foreign currency depreciates upon default. Under this assumption, the recovery payment on the foreign CDS, $(1-R)\delta$, is strictly smaller compared to the domestic CDS. The net present value of the premium leg payments, on the other hand, is larger than on the domestic CDS, because the foreign currency is expected to appreciate vs. domestic currency conditional on survival. Therefore, when $\delta < 1$, the value of the premium leg is greater and the value of the protection leg is smaller than for the domestic CDS, implying a positive quanto spread.

Figure 2 shows the CDS premiums denominated in foreign and domestic currency plotted against the expected depreciation upon default. The foreign CDS premium decreases as the risk-neutral expected crash in the currency increases, while the domestic CDS premium is fixed for a given level of the default probability, implying that the quanto spread increases in the severity of the crash.

Proposition 2. *The quanto CDS spread, $QS(0,T)$, is decreasing in ρ for all $T \geq 2$. Furthermore, if $\rho < 0$ ($\rho > 0$) then $QS(0,T)$ is increasing (decreasing) in u and $\lambda^U - \lambda^D$.*

Proof. See Appendix 10.2.3 □

The intuition behind Proposition 2 is that if there is negative correlation between the exchange rate and default risk, it is more likely that default occurs in states in which foreign currency has depreciated relative to its unconditional expectation. This effectively causes the foreign contract (converted into domestic currency) to deliver a smaller expected recovery payment, in the event of a default, compared to the domestic contract. The value of the premium leg, on the other hand, is largest on the foreign contract. This is because the risk-neutral expectation of the exchange rate conditional on survival must be larger than its unconditional expectation, otherwise, the currency forward is not priced consistently with no-arbitrage. The exchange rate thus tends to move unfavourably in both default and non-default states for the buyer of foreign CDS, implying that the fair foreign CDS premium must be smaller than the domestic CDS premium, i.e., a positive quanto spread.

An increase in the volatility of the exchange rate or the default probability, measured by the spread between up and down states (i.e., u and $\lambda^U - \lambda^D$), causes the quanto CDS spread to widen. An intuitive explanation for this is as follows. When credit risk goes up (down), then there are gains (losses) on both the foreign and the domestic CDS in the respective currencies. However, if the exchange rate tends to simultaneously decrease (increase), then the gain (loss) is smaller (larger) on the foreign CDS compared to the domestic CDS. Thus, the larger the moves in the credit risk and the exchange rate, the smaller (greater) the expected gains (losses) on the foreign CDS versus the domestic CDS, causing the quanto CDS spread to widen.

Finally, an important aspect of Proposition 2 is that the one-period quanto CDS spread is exclusively driven by crash risk, while the quanto CDS spread of two periods or more are impacted by both crash risk and covariance risk. Crash risk and covariance risk thus affect the term structure of quanto CDS spreads differently which allows us to distinguish between them by using data for quanto CDS spreads at different horizons.

4.4 Calibrating the Quanto CDS Term Structure

In the following, we use the discrete-time model to get a grasp of the magnitude of the crash and covariance risk embedded in quanto CDS spreads. The purpose is to gain intuition on how crash and covariance risk affect quanto spreads and to get an approximate estimate of their effect on observed quanto CDS spreads. Although the model is static, the central intuition gained from the model carries over to a richer dynamic term structure model, which we will analyze further in section 7.

We calibrate the model using CDS premiums for Spain, Italy, Portugal, and Ireland over the period August 2010-August 2012, i.e., at the height of the European debt crisis where CDS and quanto CDS spreads peaked. More specifically, the model parameters are calibrated such that they match the average observed 5-year quanto CDS spread, the 5-year CDS spread volatility, the EURUSD FX volatility, and the realized correlation between FX spot and 5-year USD CDS spread changes. We proxy ρ, the default probability/exchange rate correlation, with the correlation between daily percent-wise changes in the 5-year USD-denominated CDS premium and the EURUSD exchange rate. The parameter u is chosen such that the model's FX volatility matches the average 1-year risk-neutral volatility[1]. We compute the risk-neutral volatility from EURUSD currency options using the "model-free" methodology of Bakshi, Kapadia, and Madan (2003) (see section 6 for further details on the data). The empirical moments used for the calibration are reported in Table 1.

Fixing ρ and u as described above, we calibrate the default probability parameters, $(\lambda^D, \lambda^U, q^\lambda)$, and the currency crash risk parameter, δ, such that the model exactly matches the average 5-year CDS premiums denominated in USD and EUR. The calibration shows that the risk-neutral expected crash in the EURUSD in the event of a default is substantially larger for Spain and Italy relative to Portugal and Ireland. In particular, in the event of default of Spain and Italy, we estimate the risk-neutral expected depreciation in the EURUSD to 16% and 15%, respectively, while for Portugal and Ireland we estimate it to 5% and 7%, respectively. The results seem reasonable; the Euro is expected to take a much larger hit in the event of a Spanish or Italian default as these countries are more important economies for the eurozone.

If we were to ignore covariance risk ($\rho = 0$), the impact of a sovereign default on the EURUSD exchange rate would have been overestimated. In this case, for Spain and Italy, we estimate the crash risk to 21% and 19%, and 7% and 9% for Portugal and Ireland, underlining the importance of including covariance risk in the model to get an accurate assessment of the implied effect of a sovereign default on the exchange rate.

[1]Since the one-year FX volatility, σ_{FX}, and the size of the up step, u, in a Cox, Ross, and Rubinstein (1979) tree are related as $u = e^{\sigma_{FX}}$.

In Figure 3, we show the calibrated term structure of quanto spreads for Portugal, Ireland, Italy, and Spain. We see that the quanto spread increases in time to maturity. In the model—as shown explicitly in equations (9) and (10)—the term structures of foreign and domestic CDS premiums are flat when there is no covariance risk. Hence, the upward sloping quanto CDS curve is caused by covariance risk. The orange graph shows the quanto CDS spread in the case of no crash risk, i.e., the case where the entire quanto spread stems from covariance risk. We see that the curve is upward sloping in maturity, implying that covariance risk accounts for larger share of the quanto spread at longer maturities. Therefore, consistent with the intuition discussed previously, we can infer the magnitude of covariance risk from the slope of the quanto CDS term structure.

4.5 Bond Pricing in Different Currencies

A growing empirical literature studies the pricing of bonds issued by the same issuer denominated in different currencies, e.g., Buraschi, Menguturk, and Sener (2014); Corradin and Rodriguez-Moreno (2016); and Liao (2016). In these papers, they compare yields of domestic bonds with yields on synthetic domestic bonds that are constructed from foreign-denominated bonds using FX forward hedges. However, as we will show below, the yield of a synthetic bond constructed in this manner only has the same yield as the domestic bond if there is no crash or covariance risk.

Consider two coupon bonds, on the same issuer, in foreign and domestic currency with prices $P_C^f(0,T)$ and $P_C^d(0,T)$, with respective coupons C_t^f and C_t^d. To focus on quanto effects, we assume the same coupons on the domestic and the foreign bond, but in different currencies, the exchange rate is 1 at time 0, no recovery payment at default, and that risk-free rates are 0. The price of the domestic and foreign risky bonds are:

$$P_C^d(0,T) = \underbrace{E_t^Q\left(\sum_{i=1}^{N} C_{t_i}^d 1_{(\tau>t_i)}\right)}_{\text{in domestic currency}} \tag{11}$$

$$P_C^f(0,T) = \underbrace{E_t^{Qf}\left(\sum_{i=1}^{N} C_{t_i}^f 1_{(\tau>t_i)}\right)}_{\text{in foreign currency}} \tag{12}$$

We construct a synthetic domestic bond, which consists of the foreign bond and a portfolio of currency forward contracts entered at time 0 which converts each foreign-denominated coupon payment into domestic currency. The time 0 price of this synthetic bond in terms of domestic currency is:

$$\begin{aligned} P_C^{d,synth}(0,T) = & \underbrace{\sum_{i=1}^{N} C_{t_i}^f E_0(X_{t_i}|\tau>t_i) Q(\tau>t_i)}_{\text{Value of foreign bond in domestic currency}} \\ & + \underbrace{\sum_{i=1}^{N} C_{t_i}^f \left(F(0,t_i) - E_0(X_{t_i}|\tau>t_i)\right) Q(\tau>t_i)}_{\text{Value of forwards conditional on survival}} \\ & + \underbrace{\sum_{i=1}^{N} C_{t_i}^f \left(F(0,t_i) - E_0^Q\left(X_{t_i}|\tau=t_i, \tau>t_{i-1}\right)\right) Q(\tau=t_i|\tau>t_{i-1})}_{\text{Value of forwards conditional on default}} \end{aligned} \tag{13}$$

It is natural to believe that the price of $P_C^{d,synth}(0,T)$ is the same as $P_C^d(0,T)$, since the forward contracts hedge the exchange rate risk inherent in the foreign coupon payments. However, this is only correct if we assume that the last expression is 0, that is, default risk and exchange rate risk are independent. Under this assumption, we get the expression of the synthetic bond price that Buraschi, Menguturk, and Sener (2014); Corradin and Rodriguez-Moreno (2016); and Liao (2016) use to measure deviations from the law of one price, that is, $\sum_{i=1}^{N} C_{t_i}^f F(0,t_i)Q(\tau>t_i)$. In general, however, this price of synthetic domestic bond does not equal the price of the domestic bond, because the value of the forward contracts conditional on default deviates from the (unconditional) value of the forward contracts. Rather, in order for the synthetic bond to have the same value as the domestic bond, the foreign bond payments must be hedged using forward contracts that cancel at default such that the last expression is 0 by construction of the hedge, and not by assumption.

We illustrate this point in Table 2 by comparing the payoffs of two risky zero coupon bonds issued in EUR and USD in a one-period model. We assume that the EUR falls by 50% versus the USD at default, risk-free rates are 0, the forward price is 1, and no recovery on the bonds. We see from the table that a strategy that buys the USD bond and sells the synthetic USD bond has zero payoff in survival states since the forward contract hedges any exchange rate risk. However, it has a negative payoff of 0.5 USD in the default state, because the seller of the synthetic bond is obliged to pay 1 USD per 1 EUR from the forward contract, which is now worth only $\frac{1}{2}$ USD, and neither the EUR bond nor the USD bond pay anything. Important to note is that the EUR is expected to appreciate versus the USD in survival states to compensate for the EUR crash, but this gain has been hedged out by the forward contract. As a consequence, the synthetic USD bond must trade at a premium to the "real" USD bond to compensate for the crash in the EUR in default states. This simple example illustrates that at least a part of the observed yield spreads between synthetic and "real" bonds may be caused by currency crash risk, unrelated to any market frictions or imperfections in the international bond markets.

Likewise, covariance risk affects bond yields across currency denominations. We illustrate this in a multi-period model using the discrete-time model with parameters calibrated to 5-year Spanish CDS data (the parameters are reported in Table 1). The coupon bonds are assumed to be 1 and the principal is set to 100 (in respective currencies). For simplicity to convey the main idea, we assume 0 recovery rate and interest rates. Table 3 shows the results. The first row is the yield of a synthetic coupon bond, including crash risk. The second row shows yields on a long synthetic bond assuming no crash risk, and the third row is the yield on the domestic bond. The synthetic bond is long a foreign coupon bond, which pays coupons of one unit foreign currency and 100 at maturity, and short a portfolio of FX forward contracts that match the bond's payments (conditional on no default). The yield of the synthetic bond is 127 bps lower than the yield of the domestic bond, where 36 bps stems from covariance risk and 91 bps from crash risk. Raising the volatility of the exchange rate to 20.5% (the maximum EURUSD volatility over 2010-2012), the covariance component increases to 51 bps, while the crash risk component is unaltered. Overall, the results show that the synthetic bond trade at a substantially lower yield using realistic parameters to derive the covariance and crash risk components. Furthermore, the model suggests that the difference between the domestic and the synthetic yield is expected to increase in FX volatility. However, this implication must be interpreted with some caution since the model is static. In what follows, we explore more rigorously the driving factors causing the time-series variation in quanto spreads by using a dynamic term structure model.

5 A Term Structure Model of Quanto CDS Spreads

The discrete-time model is useful for obtaining the main intuition on how quanto spreads are driven by crash risk and default/currency covariance risk, but the static nature of the model makes it unable to capture time variation in credit and exchange rate risk. To this end, we propose an affine term structure model that captures the salient features of quanto CDS spreads discussed in the discrete-time model.

5.1 The Risk-Neutral Dynamics of the Model

In the model, the default risk of a sovereign i is driven by a compound Poisson process with a stochastic arrival rate, $\lambda_{i,t}$. Sovereign i's default intensity consists of two components: a systematic factor, $l_{i,t}$, which is correlated with the exchange rate, and a country-specific idiosyncratic component, $z_{i,t}$, which is orthogonal to the systematic factor

$$\lambda_{i,t} = l_{i,t} + z_{i,t} \tag{14}$$

Under the domestic risk-neutral measure, we let the exchange rate follow a Heston (1993) type dynamics with stochastic volatility, v_t, and a jump component driven by the sovereign default risk intensities:

$$dX_t = X_{t-}\left(r_{d,t} - r_{f,t}\right)dt + \sqrt{v_t}X_{t-}\left(\rho dW_{sys,t} + \sqrt{1-\rho^2}dW_{x,t}\right) + X_{t-}\sum_{i=1}^{K}\left(\zeta_i dN_{i,t} + \zeta_i \lambda_{i,t} dt\right) \tag{15}$$

The drift of the exchange rate, that is, the difference between domestic and foreign risk-free interest rates, insures that forward contracts are priced consistently with no-arbitrage. The jump component captures jumps in the exchange rate induced by sovereign default: conditional on country i defaulting at time t, the exchange rate depreciates instantly by a percent-wise fraction: $\frac{X_t - X_{t-}}{X_{t-}} = 1 + \zeta_i$, where ζ_i is a fixed country-specific jump size parameter. We then add up all jump components to get the aggregate crash risk component in the exchange rate, i.e., K represents the number of sovereigns included in the model. We specify the domestic risk-neutral dynamics of the state variables for sovereign i as follows:

$$\begin{bmatrix} dv_t \\ dl_{i,t} \\ dz_{i,t} \\ dm_{i,t} \end{bmatrix} = \left(\begin{bmatrix} \kappa_v \theta_v \\ \kappa_{l,i}\theta_l \\ \kappa_{z,i} m_{i,t} \\ \kappa_{m,i}\theta_{m,i} \end{bmatrix} - \begin{bmatrix} \kappa_v v_t \\ \kappa_{l,i} l_{i,t} \\ \kappa_{z,i} z_{i,t} \\ \kappa_{m,i} m_{i,t} \end{bmatrix} \right) dt + \begin{bmatrix} \sigma_v \sqrt{v_t} & 0 & 0 \\ \sigma_{l,i}\sqrt{l_{i,t}} & 0 & 0 \\ 0 & \sigma_{z,i}\sqrt{z_{i,t}} & 0 \\ 0 & 0 & \sigma_{m,i}\sqrt{m_{i,t}} \end{bmatrix} \begin{bmatrix} dW_{sys,t} \\ dW_{zi,t} \\ dW_{mi,t} \end{bmatrix} \tag{16}$$

where $W_{sys,t}, W_{zi,t}$, and $W_{mi,t}$ are independent. The systematic Brownian shock, $W_{sys,t}$, causes correlation between the exchange rate and the instantaneous volatility/systematic default risk component, which is assumed fixed and denoted ρ (as in, e.g., Bates (1996) and Carr and Wu (2007b)). The state variable, $m_{i,t}$, induces a central tendency in the idiosyncratic factor, i.e., our model has two state variables capturing the shape (level and slope) of the term-structure of domestic CDS premiums (Balduzzi, Das, and Foresi, 1998). This allows for the systematic component of the default intensity to freely capture the default/currency correlation risk, which is an important feature of our model in order for it to appropriately fit the term structure of quanto CDS spreads.

5.2 Specification of Pricing Kernels

We use a change of numeraire technique to price the foreign-denominated CDS contract which is no different than the techniques used to price derivatives by changing from the objective measure to the risk-neutral measure. Specifically, $M_t = X_t \frac{P_d(0,t)}{P_f(0,t)}$, is used to change numeraire from the domestic bond to the foreign bond, or put differently, M_t relates the (risk-neutral) parameters that are used to price domestic and foreign CDS contracts. Since the exchange rate in (15) jumps in the event of a sovereign default, we must be capable of handling jumps in the process governing the change of measure. Thus, we formulate Lemma 1 in Appendix 11 which slightly extends the extended affine risk premium specification of Cheridito, Filipovic, and Kimmel (2007) to jump diffusions. Roughly, Lemma 1 states that diffusions are drift-adjusted under the foreign measure according to their covariance with the exchange rate, i.e., there is no drift-adjustment in the uncorrelated case. Furthermore, the ratio between the default intensity under the foreign and domestic measure equals the jump size in the exchange rate upon sovereign default.

Besides this, we also use Lemma 1 to specify risk premia by relating the objective measure P and the risk-neutral domestic measure Q, which thus completes a triangle that allows us to switch between the domestic, foreign, and objective measure. Equivalent to Cheridito, Filipovic, and Kimmel (2007), Lemma 1 shows that if the square root processes under both P and Q, as characterized by parameters Θ_P and Θ_Q, fulfil the Feller condition [2], then the dynamics governed by Θ_P and Θ_Q are consistent with no-arbitrage. Therefore, to preclude arbitrage opportunities, we assume that the P and Q-dynamics of each state variable follow square root processes that fulfil the Feller condition, but with different parameters.

[2] The boundary non-attainment condition is important for square root processes. Let $X_t = (b + \beta X_t)dt + \sigma\sqrt{X_t}dW_t^P$ and consider a risk premium, $\phi(t)$, that preserves the affine structure under Q, i.e., $\phi(t) = \frac{c+dX_t}{\sigma X_t}$. Then it is in general not the case that the Radon-Nikodym, $L_t \equiv \frac{dQ}{dP}$, is a true martingale and the probability measure Q need not exist. However, if we impose the zero boundary non-attainment conditions (the Feller condition) $b^P \geq \frac{\sigma^2}{2}$ and $b^Q \geq \frac{\sigma^2}{2}$ then L_t is indeed a true martingale.

We do not model a jump to default risk premium between P and Q, as studied extensively in Benzoni, Collin-Dufresne, Goldstein, and Helwege (2015) in the context of eurozone sovereign CDS. They measure the jump to default risk premium as the ratio between the objective and risk-neutral default intensity, which is parallel to our setup where the currency jump size upon default equals the ratio between the foreign and domestic default intensities. An important distinction between the jump to default risk premium and the currency crash risk premium is that CDS premiums in both foreign and domestic currency are observable, which helps us pin down currency crash risk, whereas the jump to default risk premium is not tied to any observable quantity.

5.3 CDS Premiums in Domestic Currency

The derivation of the domestic CDS premiums follows the same procedure as in Pan and Singleton (2008) and Longstaff, Pan, Pedersen, and Singleton (2011). Here we briefly go through the main steps that are specific for our case. First, let $S_d(t,T)$ denote the domestic CDS premium at time t at maturity T, $P_d(t,T)$ the domestic discount factor, and R a fixed recovery rate. The state variable vector for country i, $x_{i,t} \equiv \left[l_{i,t}\ z_{i,t}\ m_{i,t}\right]^T$, is affine which entails that we can compute the following transforms as

$$\psi\left(x_{i,t},t,T\right) \equiv E_t^Q\left(e^{-\int_t^T \lambda_{i,s}ds}\right) = e^{\alpha_i(t,T)+\beta_i(t,T)\cdot x_{i,t}} \tag{17}$$

$$\phi(x_{i,t},t,T) \equiv E_t^Q\left(\lambda_{i,T}e^{-\int_t^T \lambda_{i,s}ds}\right) = \psi(x_{i,t},t,T)\big(A_i(t,T)+B_i(t,T)\cdot x_{i,t}\big) \tag{18}$$

where $\alpha_i(t,T), \beta_i(t,T), A_i(t,T)$, and $B_i(t,T)$ solve a set of ordinary differential equations (see, e.g., Duffie, Pan, and Singleton (2000)). The exact specification of the ODEs are reported in Appendix 11.2. Given a quarterly payment scheme for the premium leg and a fixed recovery rate on the protection leg, we have that their present values are given by

$$\Pi^{prem}(t,T) = S_d(t,T)\frac{1}{4}\sum_{j=1}^{4T} P_d\left(t, t+\frac{j}{4}\right)\psi\left(x_{i,t}, t, t+\frac{j}{4}\right) \tag{19}$$

$$\Pi^{prot}(t,T) = (1-R)\int_t^{t+T} P_d\left(t, t+u\right)\phi(x_{i,t}, t, u)du \tag{20}$$

The domestic CDS premium, which is consistent with no arbitrage, is then determined such that the present values of the premium leg and the protection leg are equal:

$$S_d(t,T) = \frac{\Pi^{prot}(t,T)}{\Pi^{prem}(t,T)} \tag{21}$$

5.4 CDS premiums in Foreign Currency

In the discrete-time model, we derive the foreign CDS premium directly by using $M_t = \frac{X_t}{X_0}\frac{P_d(0,t)}{P_f(0,t)}$ to convert each foreign-denominated payment into a domestic payment. In the affine model, this is rather cumbersome. We take a more convenient approach and price the foreign-denominated CDS contract using a change of numeraire technique. Formally, $M_t = \frac{dQ_f}{dQ}$, is the Radon-Nikodym derivative that changes measure from the domestic to the foreign risk-neutral measure. To apply the change of numeraire technique, we need the dynamics of the Radon-Nikodym derivative between Q and Q_f, which is given by:

$$dM_t = M_t\sqrt{v_t}\left(\rho W_{sys,t} + \sqrt{1-\rho^2}dW_{x,t}\right) + M_t\sum_{i=1}^{K}\left(\zeta_i dN_{i,t} + \zeta_i\lambda_{i,t}dt\right) \tag{22}$$

By using Lemma 1 with M_t as the pricing kernel, the default intensity under the foreign risk-neutral measure is given by:

$$\lambda_{i,t}^f = \lambda_{i,t}(1+\zeta_i) \tag{23}$$

$$dv_t = \kappa_v^f(\theta_v^f - v_t)dt + \sigma_v\sqrt{v_t}dW_{sys,t}^f \tag{24}$$

$$dl_{i,t} = \left(\kappa_{l,i}(\theta_{l,i} - l_{i,t}) + \sigma_{l,i}\rho\sqrt{l_{i,t}v_t}\right)dt + \sigma_{l,i}\sqrt{l_{i,t}}dW_{sys,t}^f \tag{25}$$

where $\kappa_v^f = (\kappa_v - \sigma_v \rho)$, $\theta_v^f = \frac{\kappa_v \theta_v}{\kappa_v - \sigma_v \rho}$, and $\lambda_{i,t}$ is the domestic default intensity.

Lemma 1 states that the ratio between the default intensity under the foreign measure and domestic measure equals the jump size conditional on sovereign default: $\lambda_t^f = \lambda_t(1+\zeta)$. For this reason, very short-term quanto CDS spreads are exclusively driven by crash risk because $S^d(t,T) \approx (1-R)\lambda_t$ and $S^f(t,T) \approx (1-R)(1+\zeta)\lambda_t$, when t approaches T. Even in the case of a purely idiosyncratic default intensity (i.e., no covariance risk), a quanto CDS spread emerges solely through the crash risk channel. This is consistent with our intuition from the discrete-time model, where we showed that a quanto CDS spread arises in the case of a constant default probability through crash risk.

Under the foreign measure, each process that is exposed to $W_{sys,t}$ is drift-adjusted via the pricing kernel (22). For l_t, the drift adjustment is $\sigma_l \rho \sqrt{l_t v_t}$, i.e., it depends on the instantaneous volatility of the exchange rate, the systematic default component, and their correlation. If there is negative correlation between the exchange rate and the default intensity, then the drift correction is negative which causes the expected default risk to be smaller under the foreign measure than under the domestic measure, implying a positive quanto CDS spread.

The covariance adjustment has less impact at shorter horizons, because the drift adjustment does not affect the instantaneous default risk. An implication of the model is therefore that quanto CDS spreads tend to widen in maturity if there is negative covariance between default and exchange rate risk. This is consistent with the results of our calibration exercise based on the discrete-time model, where we showed that the quanto CDS spread widens in maturity because of covariance risk. To summarize, crash and covariance risk affect the foreign default intensity through different channels; crash risk scales and covariance risk drift-adjusts the default intensity, and this distinction is what allows us to separate the two effects using the term structure of quanto CDS spreads.

In order to fit the model into the affine framework, we approximate the term, $\sqrt{l_t v_t}$, in the systematic default risk's drift with a first-order Taylor expansion around the respective processes' mean reversion levels [3]. The foreign transforms are then computed as in the domestic setting

$$\psi_f\left(x_{i,t}, t, T\right) = e^{\alpha_{f,i}(t,T)+\beta_{f,i}(t,T)\cdot x_{i,t}} \tag{26}$$

$$\phi_f(x_{i,t}, t, T) = \psi_f(x_{i,t}, t, T)\big(A_{f,i}(t, T) + B_{f,i}(t, T) \cdot x_{i,t}\big) \tag{27}$$

and the foreign premium and protection legs are given by

$$\Pi_f^{prem}(t, T) = S_f(t, T)\frac{1}{4}\sum_{j=1}^{4T} P_f\left(t, t + \frac{j}{4}\right)\psi_f\left(x_{i,t}, t, t + \frac{j}{4}\right) \tag{28}$$

$$\Pi_f^{prot}(t, T) = (1 - R)\int_t^{t+T} P_f\left(t, t + u\right)\phi_f(x_{i,t}, t, u)du \tag{29}$$

From the dynamics of the foreign state variables, i.e., equation (25), we see that the currency/default covariance risk introduces v_t as an additional state variable compared to the domestic case, that is, $x_{i,t} \equiv \left[l_{i,t}\ z_{i,t}\ m_{i,t}\ v_t\right]^T$. The exact specification of the ODEs which $\alpha_{f,i}, \beta_{f,i}, A_{f,i}$, and $B_{f,i}$ solve are provided in Appendix 11.2.

6 Data and Descriptive Analysis

6.1 Credit Default Swap Data

We collect CDS premiums from Markit on eurozone sovereign bonds issued by Austria, Belgium, Germany, Finland, Ireland, France, Italy, Netherlands, Portugal, and Spain denominated in EUR and USD. Markit provides us with daily quotes at maturities of 1, 3, 5, 7, and 10 years. We use the complete restructuring clause on the CDS contracts which allows the protection buyer to deliver bonds of any maturity (and currency denomination) into the CDS auction. Markit performs a number of data cleaning procedures on the CDS data that they receive from their contributors, e.g., to avoid stale quotes and outliers, and

[3]The exact form of the Taylor approximation is given by: $\sqrt{l_t v_t} = 1/2\left(v_t\left(\frac{\theta_l}{\theta_v}\right)^{1/2} + l_t\left(\frac{\theta_v}{\theta_l}\right)^{1/2}\right)$.

they only report quotes if there are at least three quotes from different contributors. Before August 2010, Markit aggregated quotes across currency denominations into one quote. As our focus is on the impact of currency denomination on the pricing of CDS contracts, we initiate our analysis in August 2010, and our sample ends in April 2016.

6.2 Currency Options Data

One of our main objectives is to estimate the contribution of covariance risk to quanto spreads which essentially depends on three factors: risk-neutral exchange rate volatility, volatility of systematic default risk, and the correlation between credit risk and the exchange rate. The latter two factors can be identified from USD-denominated CDS premiums and quanto CDS spreads, but CDS data are not particularly informative about the first factor. Therefore, in order to pin down the risk-neutral distribution of exchange rate volatility, we include currency options data in our estimation, as in, e.g., Bates (1996); Carr and Wu (2007a,b).

We collect EURUSD currency options data from Bloomberg from August 2010 to April 2016. The data consist of Garman and Kohlhagen (1983) implied volatilities of delta-neutral straddles, 10, 25-delta risk reversals, and 10, 25-delta butterfly spreads which are the common quoting conventions in currency option markets. The maturities are fixed and are 1, 2, 3, 6, 9, and 12 months.

A straddle is a portfolio which is long a call and a put option with the same strike and maturity. The payoff of a straddle is directionless and the buyer of the straddle is long at-the-money volatility.

A risk reversal consists of a long position in an out-of-the money (OTM) put option and a short position in an OTM money call option with symmetric deltas[4]. The long position in the OTM put protects against large depreciations in foreign currency (EUR), and in contrast, the short OTM call loses money when large depreciations in the USD occur. Risk reversals therefore measure the slope of the implied volatility curve against moneyness, also called the skew of the implied volatility curve.

[4] Sometimes the risk reversal is quoted conversely as a long position in a call option and a short position in a put.

A butterfly spread is the difference between the average IV of and OTM call and an OTM put and the IV of the delta-neutral straddle. If the butterfly spread is positive, it reflects that the market price of hedging large FX movements (in either direction) is more expensive compared to the case in which returns are log-normal, i.e., the risk-neutral distribution of exchange rate changes is fat tailed.

Using the Garman and Kohlhagen (1983) formula for the IVs derived from the straddles, risk reversals, and butterflies, we recover five different strikes, spanning from the strike of a put with a delta of -10 percent to the strike of a call option with a delta of 10 percent. We skip the details on how this procedure works and refer to Della Corte, Sarno, Schmeling, and Wagner (2016) and Jurek (2014) for an elaborate explanation.

6.3 Interest Rate Data

For the pricing of CDS denominated in Euro and U.S. dollar, we need to compute discount curves in both currencies. We take the most common approach and build discount curves from overnight index swap rates, OIS for U.S. dollar, and EONIA for Euro. We use overnight index swap rates rather than LIBOR swap rates because it is well-documented that they contain a default risk component. Since 2010, maturities of up to 10 years of overnight index swaps have been traded. We therefore exclusively use overnight index swap rates as proxies for riskless interest rates, since the longest maturity in our CDS data is 10 years. Based on the overnight index swap interest rates, we construct zero-coupon curves in Euro and U.S. dollar using a standard bootstrapping procedure. We collect the data on overnight index swap rates from Bloomberg, and the maturities are 3, 6, 9 months, and 1-10 years, and the data start in August 2010 and end in April 2016.

6.4 Descriptive Data Analysis

Table 4 reports the averages and standard deviations of eurozone sovereign CDS premiums denominated in EUR and USD, spanning maturities from 1-10 years, over the period August 2010 to April 2016. First, we note that the USD CDS premium is, on average, unambiguously higher than the corresponding EUR CDS premium for all sovereigns. In absolute terms, the average quanto CDS spreads, e.g., at the 5-year maturity, are largest for Ireland, Italy, Portugal, and Spain, ranging from 36-48 bps, while they are the smallest for Finland, Germany, Netherlands, and Austria, ranging from 8-22 bps. In general, the non-GIIPS countries have much smaller average CDS premiums, indicating that the market deemed it unlikely that sovereign defaults would occur for these sovereigns. As an example, the average 5-year USD CDS premium for Portugal is more than ten times larger than for Germany.

In Figures 4-6, we show the time series of quanto CDS spreads and USD-denominated CDS premiums for all sovereigns at maturities ranging from 1-10 years. The quanto CDS spreads are positive in the entire sample period for all sovereigns. As is the case for the USD CDS premiums, the quanto CDS spreads peak for all sovereigns between the last quarter of 2011 and the Summer of 2012. During this period, the 5-year quanto CDS spreads exceed 100 bps for Spain and Portugal, and almost reach 100 bps for Italy and Ireland as well. From July 2012, in the wake of Mario Draghi's speech in which he insured that the ECB would do whatever it takes to preserve the Euro, the quanto CDS spreads gradually decline, but they stay positive throughout the sample period.

Table 5 reports the averages and standard deviations for implied volatilities of straddles, risk reversals, and butterflies for each maturity. The implied volatility for both the 10 and 25-delta risk reversals are, on average, negative, in fact, they are negative throughout our sample period at all maturities. This shows that large downside risk in the Euro has historically been more expensive to insure relative to symmetric downside risk in the U.S. dollar.

The focus of our analysis is the relation between currency risk and credit risk. As a first step in exploring this relation, we proxy aggregate eurozone credit risk by the first principal component of eurozone 5-year USD CDS premiums and investigate its relation to EURUSD implied volatility and spot changes. The principal component analysis shows that there is a strong commonality in CDS premiums for eurozone sovereigns. The first principal component of weekly changes in 5-year USD CDS premiums explains 77% of the common variation of the changes in 5-year USD CDS premiums[5], consistent with Longstaff, Pan, Pedersen, and Singleton (2011), who document strong commonality in global CDS premiums.

Table 6 shows results from regressions of weekly innovations in the EURUSD spot exchange rate and the delta-neutral straddle implied volatility on the first principal component of the eurozone CDS premiums. Over the entire sample period, there is a significantly negative relation between changes EURUSD spot rate and eurozone credit risk, with a t-statistic of -3.69 and an R^2 of 8.1%. This result suggest that the Euro tends to depreciate when eurozone credit risk rises. Most of the significance, however, stems from the European debt crisis period, i.e., from August 2010 to December 2012. In the post-crisis period (January 2013 to April 2016), there is a negative, but insignificant, relation (t-statistic of -1.34), and a miniscule part of the variation in spot exchange rates is explained by exposure to sovereign credit risk.

The at-the-money implied volatility and eurozone credit risk are significantly positively related over the entire sample period (t-statistic of 3.84), with an R^2 of 12.1%, i.e., increasing forward-looking EURUSD volatility tends to be associated with increasing eurozone credit risk. Our results are consistent with those of Della Corte, Sarno, Schmeling, and Wagner (2016), who document, for a large sample of countries, that exchange rate spot movements and implied volatilities of options are tightly related to sovereign credit risk. The positive relation between EURUSD implied volatility and eurozone credit risk is highly significant in the crisis period, with a t-statistic of 7.70 and an $R^2 = 27.2\%$, but their relation is barely significant in the post-crisis period (t-statistic of 2.17, $R^2 = 3.2\%$). Consequently, the results of our regression analysis indicate that eurozone sovereign credit risk and the currency spot rate and implied volatility primarily co-vary in times of distress.

According to our discrete-time model, the significant covariance between exchange rate risk and sovereign credit risk implies a positive quanto CDS spread for eurozone sovereigns, even without any exchange rate crash risk at default. Moreover, the results of the regressions suggest that the covariance risk components embedded in quanto CDS spreads are most pronounced during the crisis period from 2010-2012. In the next section, we analyze these

[5]Similar results are obtained when using EUR-denominated CDS.

conjectures using the proposed affine term structure model to decompose quanto CDS spreads into a covariance risk component and a crash risk component.

7 Model Results and Estimation

7.1 Estimation Approach

We focus on estimating the model for the GIIPS countries: Portugal, Ireland, Italy, and Spain, excluding Greece. We exclude Greece from the analysis because Breuer and Sauter (2012) document that there was virtually no trading activity in the Greek CDS from early 2011, as the market anticipated a Greek default, which, in fact, occurred on March 9, 2012. CDS markets also reflected that a Greek default was anticipated, with elevated CDS premiums on Greek government bonds reaching several thousand bps by the last of quarter of 2011.

We focus on the GIIPS countries (excluding Greece) because they are the least creditworthy in our sample and, arguably, the focal point of the European debt crisis. For example, the 5-year CDS premiums (in USD) for the GIIPS all reached levels exceeding 600 bps, with Portugal and Ireland being the most extreme cases with CDS premiums exceeding 1000 bps. In comparison, the German 5-year CDS barely touched 100 bps, and the French 5-year CDS spiked at about 200 bps.

In the estimation, we use weekly data (each Wednesday) of quanto CDS spreads, USD-denominated CDS premiums, and currency option implied volatilities. Each week, we have 30 option prices (five strikes at six maturities), five CDS premiums denominated in USD, and five quanto CDS spreads at maturities of 1, 3, 5, 7, and 10 years.

If we were to estimate the model in one joint estimation, we would have an unmanageably large set of parameters and a high dimensional state variable vector. For instance, in the case of four sovereigns, the model has 12 state variables and a very large parameter vector containing systematic, country-specific, and measurement error parameters. One approach to reduce the dimension of the state vector is to introduce common factors or to use just one state variable to capture country-specific default risk. However, since we are interested in making accurate assessments of the magnitude of the quanto spreads driven by crash and covariance risk, we need precise estimations. Our estimations suggest that at least two country-specific factors are necessary for the model to accurately fit the cross-section and time-series dynamics of USD CDS and quanto CDS premiums simultaneously.

For this reason, we estimate the model stepwise. In the first step, we estimate a time series of the instantaneous currency volatility, v_t, and its objective and risk-neutral parameters from currency option implied volatilities. We estimate the model using maximum likelihood estimation in conjunction with the unscented Kalman filter. In the next step, now treating v_t as observable and its parameters as fixed, we estimate the parameters for the idiosyncratic and systematic default intensity components, i.e., l_t, z_t, and m_t, using data for USD CDS and quanto CDS spreads for one country at the time. The estimation procedure is described in detail in Appendix 12.

7.2 Estimation Results

Table 7 presents the maximum likelihood estimates of the model, and Figure 7 illustrates the estimated state variables l_t, z_t, and m_t for each sovereign. For all sovereigns, the idiosyncratic component of the default intensity, z_t, spikes between the last quarter of 2011 and the Summer of 2012. In the wake of Mario Draghi's (president of the ECB) famous speech in July 2012, in which it was announced that the ECB would do whatever it takes to preserve the Euro within its mandate, the EURUSD exchange rate and the eurozone sovereign credit markets stabilized, which caused both z_t and l_t to decrease rapidly, for all sovereigns.

The systematic component, which captures the part of the default intensity correlated with the foreign exchange rate, l_t, exhibits two peaks (with the exception of Portugal), in early 2011 and by mid-2012. The systematic default component has a more stable path over the sample period compared to the idiosyncratic components that have stronger mean reversion and seem to capture transient credit risk shocks. Clearly, for all the sovereigns, m_t, is highly time-varying, indicating that it is an important feature of our model to allow the mean-reversion level of z_t to be stochastic. Consistent with this, we find considerable improvements in model fits when using a three-factor model instead of a two-factor model. For example, we find that a model in which z_t has a constant mean-reversion level is not sufficiently rich to provide reasonable fits of the USD CDS term structure and the quanto CDS term structure.

Using the estimated parameters and the filtered state variables, we compute model-implied USD CDS premiums and quanto CDS spreads and compare them to their observed counterparts. We show in table 8 the summary statistics for the model pricing errors, both in terms of root mean squared errors (RMSEs) and mean absolute pricing errors (APEs) in bps. The time-series fits are illustrated in Figures 8-9 at maturities of 1, 5, and 10 years.

The average RMSE across the 1-10 years maturities for the USD CDS range from 23.21-26.68 bps for Italy, Spain, and Ireland. The average RMSEs for Portugal, however, are significantly larger at 37.92 bps, especially the 1-year RMSE is comparatively large. Using the APE metric, the Portuguese fit is better, which indicates that large outliers are important contributors to its RMSEs. For all sovereigns, the general pattern is that the pricing errors decline in maturity, i.e., the shorter maturities are the most difficult to capture for the model. A likely explanation for this is that the short end is more volatile/noisy than the long end of the term structure, as shown in Table 4.

The model seems to fit the quanto CDS premiums reasonably well, as seen from Figures 8-9. This is also reflected by relatively small average RMSEs for all sovereigns, with the lowest being 0.98 bps for Ireland and the largest being 4.90 bps for Spain. The RMSEs tend to increase in the maturity of the quanto CDS spread, most notably for Spain. From Figure 9, we see that for Spain, the model tends to underestimate the 10-year quanto CDS premium and overestimate the 10-year USD CDS premium. Such a bias, however, is not present for the other sovereigns and does not seem to be a general issue with the model. Overall, considering the large fluctuations in the CDS premiums over a relatively short sample period, we believe that the model performs well in capturing both the USD CDS and the quanto CDS dynamics across all tenors. As an example, to underline the strong time-variation of the CDS premiums over our sample period, the 1-year USD CDS premium for Portugal and Ireland range between 0.23%-23% and 0.07%-14.5%, respectively.

Next, we use the model estimates to decompose quanto CDS spreads for Italy, Spain, Ireland, and Portugal into a currency/default covariance component and a crash risk component. We compute the covariance and crash risk component of the quanto spread as:

$$\text{FX/default covariance risk component} = S^d_{\zeta=1}(t,T) - S^f_{\zeta=1}(t,T) \tag{30}$$

$$\text{FX crash risk component} = S^d(t,T) - S^f(t,T) - \Big(S^d_{\zeta=1}(t,T) - S^f_{\zeta=1}(t,T)\Big) \tag{31}$$

where $S^d_{\zeta=1}(t,T) - S^f_{\zeta=1}(t,T)$ denotes the model-implied quanto spread assuming no currency crash at default. Hence, if crash risk accounts for the entire quanto spread, the covariance component is zero. The crash risk component is the residual part of the quanto spread after correcting for covariance risk, i.e., the difference between the total quanto spread and the FX/default covariance component.

Figure 10 illustrates the time series of the decompositions at maturities of 1, 5, and 10 years for Spain, Italy, Portugal, and Ireland. Table 9 shows descriptive statistics for the decompositions. First, we discuss the estimates of ζ, i.e., the risk-neutral expected percent-wise jump in the EURUSD immediately after sovereign default is announced. For all sovereigns in our estimations, we find that ζ is negative and highly significantly different from zero. This indicates that the Euro is expected to take an immediate hit conditional on the announcement of a sovereign default. The general pattern we find is that the Euro is expected to take a larger downward jump at default of sovereigns that are fundamentally more important for the eurozone economy. Specifically, we estimate ζ for Spain, Italy, Portugal, and Ireland to be -15.6%, -9.6%, -5.3%, and -5.0%, respectively.

Turning to the decompositions of the quanto spreads, we find that the covariance component is economically large and accounts for a large proportion of the quanto spreads for all sovereigns in our estimations. Over the entire sample period, the covariance component of the 5-year quanto spread ranges, on average, from $9.2-16.4$ bps (20-38% of total spread) for Spain, Italy, and Portugal, and it is 23.5 bps (75% of total spread) for Ireland. Importantly, covariance risk is strongly time-varying and is especially pronounced during the European debt crisis, where credit and exchange rate risk are strongly co-varying and volatile. From August 2010 to December 2012, the average covariance component at the 5-year maturity is 18.4 bps (25% of total spread) for Spain and ranges between 27-36 bps for Portugal, Italy, and Ireland (35%-58% of total spread). During this period, covariance risk reaches as much as 38.5-65.8 bps and accounts for 40-76% of the total 5-year quanto spreads for Spain, Portugal, and Italy and virtually for the entire 5-year quanto spread for Ireland.

We expect that a larger part of quanto spreads at shorter maturities is due to crash risk and that the contribution of covariance risk increases in maturity. The intuition for this is that when the maturity approaches zero, the domestic (USD) CDS premium is well-approximated by $S^d(t,T) \approx (1-R)\lambda_t$, and according to Lemma 1, the foreign (EUR) CDS premium is well-approximated by $S^f(t,T) \approx (1-R)(1+\zeta)\lambda_t$. The longer-term quanto spreads are more exposed to covariance risk, because the covariance between credit risk and exchange rate risk reduces the drift of the Euro default intensity and hence has a larger impact over longer horizons (see section 5.4 for an elaborate discussion). Consistent with this reasoning, we indeed find that crash risk accounts, on average, for the largest part of quanto spreads at the 1-year maturity and gradually decreases in maturity. The average term structure of crash risk is particularly steep between the 1-year and 5-year maturity, but almost flat from the 5-year maturity and beyond. Specifically, the crash risk component accounts, on average, for 46% (25%) for Ireland, 80% (65%) for Portugal, 81% (62%) for Italy, and 87% (80%) for Spain of the 1-year (5-year) quanto spreads.

The average quanto spread is steeply upward sloping up to the 5-year maturity and virtually flat at maturities beyond that (see Table 4), our estimations suggest that this shape of the quanto spread is because of covariance risk. If only crash risk were present, we would expect a flat quanto spread term structure because crash risk scales the default intensity, i.e., causes parallel-shifts of the quanto spread term structure.

Overall, our findings indicate that covariance between sovereign credit risk and currency risk accounts for a significant share of quanto spreads, especially in times of financial distress.

Anecdotal evidence confirms the importance of covariance risk in eurozone credit markets during the European debt crisis. Between 2010-2011, several research notes were released by major investment banks discussing the practicalities of hedging currency/credit risk for eurozone sovereigns and banks (e.g., Barclays Research Note (2011) and J.P. Morgan Research Note (2010)), indicating a large hedging/speculative demand for FX/default covariance risk.

Based on our decompositions, we shed some light on redenomination risk, that is, the risk that a sovereign redenominates its EUR-denominated debt into a new (devalued) domestic currency. According to the standardized ISDA terms, if Spanish (or Portuguese/Irish) sovereign bonds are redenominated into a new currency, i.e., a new "Pesetas", it triggers the Spanish CDS contracts, whereas redenomination is not considered a credit event for Italy. The Euro CDSs for Italy are therefore not protected against a redenomination event, while they are for Spain. Our estimations suggest that redenomination risk is not priced in quanto spreads as a sudden event, because a larger part of the quanto spreads for Spain is caused by crash risk compared to Italy. However, this does not imply that redenomination risk is not a contributing factor to quanto spreads, but rather that it is not priced as a jump event. In support of this finding, articles written by major market participants (e.g., Credit Suisse Research Note (2010)) seemed to share the view that redenomination is legally and practically very difficult to implement "overnight".

Our estimations provide us with the parameters under both the objective and the risk-neutral measure which we can use to calculate the time series of credit risk and quanto credit risk premiums. Longstaff, Pan, Pedersen, and Singleton (2011) argue that a reasonable measure for the credit risk premium—the risk premium associated with holding unpredictable variation in the default arrival rate—is the difference between the CDS premiums based on the risk-neutral parameters (Q-parameters) and the objective parameters (P-parameters). Presumably, since providing credit insurance on eurozone sovereigns is associated with large losses at times of high marginal utility, we expect that credit risk premiums are positive, on average.

In the same spirit, we define a quanto risk premium as the risk premium associated with taking exposure to crash and covariance risk, as defined in equations (30)-(31). We measure the quanto risk premium as the difference in quanto CDS spreads calculated based on the Q-parameters and the P-parameters. That is, the credit risk premium and the quanto risk premium are defined as:

$$CRP(t,T) = S_d^Q(t,T) - S_d^P(t,T) \tag{32}$$

$$QRP(t,T) = S_d^Q(t,T) - S_f^Q(t,T) - (S_d^P(t,T) - S_f^P(t,T)) \tag{33}$$

where $S_i^M(t,T)$ is the CDS premium based on parameters under measure $M = Q, P$ in currency i at maturity T. Figure 11 illustrates the time series of the quanto and credit risk premiums for each sovereign, and Table 10 reports the mean risk premiums in basis points, and the fraction of the risk premiums to total spreads. We find substantial positive risk premiums associated with taking exposure to eurozone sovereign credit risk and quanto risk, especially at the peak of the European debt crisis in 2011-2012. For Spain, Italy, and Portugal, the average 5-year credit risk premiums range from $114-211$ bps, which in relative terms correspond to 59-66% of the total average USD CDS premiums. The large credit risk premiums suggest that investors demand high compensation for providing credit insurance compared to premiums based on objective default risk. In general, the credit risk premiums for Ireland are quite small compared to the other countries and account, on average, for less than 6% of the total USD CDS 5-year spread. For Italy and Spain, the credit risk premiums are positive throughout the sample period, with peaks in 2012, while for Ireland and Portugal, the risk premiums are briefly negative for a period in 2011, but positive for the rest of the sample. At shorter maturities, the risk premium accounts for a smaller part of CDS spreads for all sovereigns, since the unpredictable variation in default risk is smaller.

Finally, we document sizeable and highly time-varying quanto risk premiums for the eurozone sovereigns. The quanto risk premiums are positive at all horizons and account for a significant share of the quanto CDS spreads. The quanto risk premiums are of greatest magnitude for Spain and Italy, both in relative and nominal terms, consistent with the notion that investors demand a larger risk premium for holding quanto risk for more systematically important sovereigns. For example, at the 5-year maturity, the quanto risk premium accounts, on average, for 61% and 73% of the total quanto CDS spreads for Spain and Italy, and for 40% and 15% of the quanto CDS spreads for Portugal and Ireland. The 5-year quanto risk premium is largest in the last part of 2012, where it reaches 28 bps for Spain and 35 bps for Italy. Even though the quanto CDS spreads are of similar order of magnitude for Ireland and Portugal, they have much smaller maximum quanto risk premiums of 6 bps and 18 bps, respectively.

7.3 Quanto Effects on Bond Yields

Quanto spreads are not only present in eurozone sovereign CDS, it has also been documented in previous research that the difference in yields on a USD-denominated bond and a EUR-denominated bond tends to be positive, that is, a positive quanto yield spread (Corradin and Rodriguez-Moreno, 2016). In this section, we investigate if the quanto yield spread is attributed to compensation for exposure to crash and covariance risk (quanto effects). To this end, we use the estimated parameters and state variables to compute model-implied yields for EUR and USD-denominated bonds, and we then investigate if they explain those observed in data. There are only a few eurozone sovereigns that have bonds issued USD. Our analysis focuses on Italian, Spanish, and Portuguese government bonds issued in EUR and USD (Ireland has no government bonds issued in USD).

In the presence of no frictions, the model-implied quanto yield spreads should explain all the variation in the observed quanto yield spreads. However, there are many factors unrelated to quanto effects that may cause the observed quanto yield spreads to deviate from zero. First, quanto yield spreads may simply be caused by differences in terms of the bonds, because it is typically not possible to pair EUR and a USD-denominated bonds that have the same maturity, coupon payments, recovery rates, etc. Second, there is evidence for a specialness premium attached to holding EUR-denominated bonds in the eurozone due to favourable regulatory treatment of debt issued in local currency over foreign currency debt. For instance, EUR-denominated bonds tend to have relatively smaller haircuts in repo transactions and carry lower capital weights on banks' balance sheets compared to USD-denominated bonds (Corradin and Rodriguez-Moreno, 2016). Since none of the above-mentioned factors impact quanto CDS spreads, we can use our model to derive cross-currency bond yield spreads caused only by quanto effects.

7.3.1 Constructing the Quanto Yield Spread

We now discuss how to construct quanto yield spreads from observed bond yields, and we then examine how they relate to model-implied quanto yield spreads estimated from CDS data. There are very few bonds issued in USD by eurozone sovereigns which makes it difficult to find matching EUR and USD bonds. We circumvent this issue by constructing a synthetic USD bond from EUR-denominated bonds, which matches the maturity, coupon rate, and coupon frequency of the traded USD bond.

To this end, we calculate the full term structure of riskless zero-coupons in EUR and USD, as well as the zero-coupons in EUR of the risky sovereign. We express the price of the risky zero-coupon in EUR, $P^E(t,s)$, as

$$P^E(t,s) = \frac{1}{\left(1 + r^E(t,s) + s^E(t,s)\right)^{s-t}} \tag{34}$$

where $r^E(t,s)$ are the riskless EUR interest rates, and $s^E(t,s)$ are the credit spreads for the risky EUR bonds. From the zero-coupon term structure of risky EUR bonds, we use (34) to calculate $s^E(t,s)$ at any maturity. Using $s^E(t,s)$ and the riskless USD interest rates, $r^U(t,s)$, we construct a synthetic USD bond, $PC^U_{synth}(t,T)$, with matching coupons, notional, and maturity of the observed USD bond, $PC^U_{obs}(t,T)$. The prices of the synthetic and the traded USD bonds are given by

$$\begin{aligned} PC^U_{synth}(t,T) &= \sum_s C_s \frac{1}{(1 + s^E(t,s) + r^U(t,s))^{s-t}} + N \frac{1}{(1 + s^E(t,T) + r^U(t,T))^{T-t}} \\ PC^U_{obs}(t,T) &= \sum_s C_s \frac{1}{(1 + s^U(t,s) + r^U(t,s))^{s-t}} + N \frac{1}{(1 + s^U(t,T) + r^U(t,T))^{T-t}} \end{aligned} \tag{35}$$

We then define the observed synthetic quanto yield spread as the yield differential between the USD bond and its synthetic counterpart:

$$QY_{synth}(t,T) \equiv y^U_{obs}(t,T) - y^U_{synth}(t,T) \tag{36}$$

If there are no quanto effects, and no other frictions, the observed quanto yield spread should be zero, since the credit spreads in this case are the same. However, if quanto effects are present, it causes a positive quanto yield spread (i.e., $s^U(t,s) > s^E(t,s)$).

Using the estimated model parameters, we compute model-implied quanto yield spreads. We choose the time to maturity, coupons, and notional amount such that they exactly match those of the traded USD bond. We assume fixed recovery of par value at default and calculate the risky zero-coupon price in currency $i = EUR, USD$ as:

$$P^i(t,T) = E^{Q^i}_t\left(e^{-\int_t^T (r_{i,u}+\lambda_{i,u})du}\right) + R\int_t^T E^{Q^i}_t\left(\lambda_{i,s}e^{-\int_t^s (r_{i,v}+\lambda_{i,v})dv}\right)ds \tag{37}$$

In order to emulate the methodology used for constructing the observed quanto yields spreads, we derive a EUR credit spread curve from the risky and riskless EUR zero-coupon prices. We then use this EUR credit spread curve in conjunction with the USD riskless zero-coupon prices to construct a synthetic model-implied USD bond price, exactly as in (35). We then compute the model-implied quanto yield spread as the difference in yields between the USD bond and the synthetic USD bond.

7.3.2 Empirical Results

For each sovereign, we obtain the full term structure of risky EUR zero-coupon prices using the benchmark government yield curve provided by Reuters at maturities ranging from six months to 10 years. The riskless zero-coupon prices in EUR and USD are bootstrapped from their respective overnight index swap rates.

For Italy, we study a USD-denominated bond that matures in February 2017, and for Spain we study two USD-denominated bonds with maturities in June 2013 and March 2018, respectively, i.e., the entire sample period from 2010-2016 is covered by a USD-denominated bond for both countries. Portugal, however, only has one USD-denominated bond traded in our sample period with maturity in March 2015. We calculate the observed quanto yield spread as in (35), which we refer to as the "synthetic" quanto yield spread. Besides this, we compute a bond quanto yield spread, defined as the yield spread between a USD bond and a EUR bond with similar maturities corrected for the riskless interest rate differential:

$$QY_{bond}(t,T) \equiv y^U_{obs}(t,T) - y^E_{obs}(t,T) - \left(\bar{r}^U(t,T) - \bar{r}^E(t,T)\right) \tag{38}$$

The bonds that we use are specified in the footnote [6]. One advantage with the measure specified in (38) is that it does not involve the extraction of a full term structure of zero-coupon prices and credit spreads. This spread, however, is a cruder measure than the synthetic quanto yield spread, since it does not take into account the term structure of the risky zero coupon prices, differences in coupon schemes, or maturity mismatch.

The justification for this measure is that if there were no quanto effects, or other frictions, only the riskless interest rate differential drives the yield spreads across currency denominations. We would thus expect (38) to be close to zero if there are no quanto effects. In the presence of no frictions, the bond quanto yield spread is exactly zero for zero-coupon bonds [7], but it is not necessarily zero for coupon bonds.

[6] The Italian EUR-denominated government bond matures on 1st of February 2017, ISIN: IT0004164775. 4% coupon semi-annual. The USD-denominated Italian government bond has maturity on 12th of June 2017. ISIN: US465410BS63, 5.375% coupon semi-annual. First Spanish bond couple: EUR-denominated government bond matures on January 31 th 2014, ISIN: ES00000121H0, 4.25% coupon semi-annual, and the USD-denominated June 17th 2013. ISIN: XS0363874081, 3.625% coupon semi-annual. Spain bond couple for latter period: EUR bond: 30th of July 2018 4.1% semi-annual coupon rate, and USD bond: maturity 6th of March 2018, 4% semi-annual coupon rate. Portugal bond couple: maturity EUR bond 15th oct 2014 PTOTEOOE0017 and 3.6% coupon rate semi-annual, USD bond maturity 25th march 2015 XS0497536598 and 3.5% coupon rate semi-annual.

[7] To see this, consider two risky zero-coupon bonds in EUR and USD: $P^E(t,T)$ and $P^U(t,T)$ and assume independence between the exchange rate and the default event $(1_{\tau>T})$:

$$\begin{aligned} P^E(t,T) &= E_t^{Q_E}\left(exp\left(-\int_t^T r^E(s)ds\right)1_{\tau>T}\right) = E_t^{Q^U}\left(\frac{X_T}{X_t}exp\left(-\int_t^T r^U(s)ds\right)1_{\tau>T}\right) \\ &= E_t^{Q^U}\left(\frac{X_T}{X_t}\right)E_t^{Q^U}\left(exp\left(-\int_t^T r^U(s)ds\right)1_{\tau>T}\right) \Leftrightarrow \frac{P^E(t,T)}{P^U(t,T)} = \frac{exp\left(\int_t^T r^U(s)ds\right)}{exp\left(\int_t^T r^E(s)ds\right)} \\ &\Leftrightarrow y^U(t,T) - y^E(t,T) - \bar{r}^U(t,T) - \bar{r}^E(t,T) = 0 \end{aligned}$$

However, if the duration of the bond is short, the yield spread between a coupon bond and zero-coupon bond is close to zero, which is the case in our sample, where we consider only bond maturities of less than seven years [8].

In Table 11, we report summary statistics of the observed quanto yield spreads. We divide the sample into a crisis period, from August 2010 to March 2013, and a post-crisis period March 2013 to April 2016.

For Italy and Spain, the crisis period is characterized by positive and highly significant quanto yield spreads (i.e., t-statistics exceeding > 5.42), with respective averages of 40.8 bps (59.7 bps) and 62.7 bps (99.0 bps) of the synthetic (bond) quanto yield spread. For these countries, the corresponding average model-implied quanto yield spreads are in the same order of magnitude of 61 bps and 59 bps. The Portuguese observed quanto yield spreads based on the synthetic and the bond method have respective means of 4.3 bps and 28.6 bps, which are both insignificantly different from zero. However, if restrict the sample period to August 2010 to July 2012, i.e., we consider the sample period prior to Draghi's speech, then the synthetic quanto yield spread is significant for Portugal as well. In general, the Portuguese quanto yield spread is more noisy than for Spain and Italy and exhibits larger positive and negative swings.

In the post-crisis period, we only study bonds issued by Italy and Spain, since there are no USD-denominated bond data for Portugal. In this period, the quanto yield spreads are much smaller (albeit still positive) and less significant compared to the crisis period. For Italy, the synthetic bond yield spread has en insignificant average of just 14.0 bps, and the spread is contained within a more narrow range compared to the crisis period, with a 95% percentile of 56 bps relative to 123 bps in the crisis period. Likewise is the average Spanish synthetic quanto yield spread smaller (33.3 bps) in the post-crisis period compared to the crisis period. The corresponding means of the model-implied quanto yield spreads are about 25 bps and 41 bps for Italy and Spain, our model thus captures the falling trend in the

[8] For example, for Italy, the 7-year EUR spread between the coupon bond and zero-coupon is always negative, with a minimum of -15 bps (3% in relative terms), and since the USD-bond is subject to the same bias, we presume that the bias' affect in the quanto yield spread is small.

quanto yield spreads.

Next, we test if the observed quanto yield spreads are explained by their model-implied counterparts. We find a significant and positive relation between observed quanto yield spreads and their model-implied counterparts during the peak of the European debt crisis, while they are insignificantly related in the post-crisis period. Our results indicate that a significant portion of the observed yield deviations between EUR and USD-denominated eurozone sovereign bonds is attributable to quanto risk and that quanto yield spreads do not necessarily reflect mispricings. Positive quanto yield spreads persist post-crisis, although much smaller compared to the crisis period, but they are seemingly caused by other factors, such as differences in liquidity and specialness associated with currency denomination (Corradin and Rodriguez-Moreno, 2016).

Table 12 shows results from regressions of the observed quanto yield spreads, using both the synthetic (36) and the bond method (38), on the model-implied counterparts. We also include the 5-year quanto CDS spread in the regressions as an alternative measure for quanto effects. In the crisis period, for Spain and Italy, there is in general a significant positive relationship between the observed quanto yield spreads and the model-implied quanto yield spreads and the quanto CDS spreads.

In particular for Spain, the slope coefficients of the model-implied quanto yield spread and the 5-year quanto CDS spread range between 1.24-1.93, with t-statistics between 2.99 and 6.60, and R^2s ranging from 20.75%-30.54%. Likewise for Italy, there is a positive relation between the synthetic quanto yield spread and its model-implied counterpart and the 5-year quanto CDS spread both with slope coefficients close to unity, with respective t-statistics and R^2s of: 1.43, $R^2 = 8.66\%$ and 2.07, $R^2 = 17.06\%$. Using the bond method to derive the observed quanto yield spreads, we also find significant positive slope coefficients near unity. In this case, a substantial part of the variation in the observed quanto yield spreads are explained by quanto effects, with R^2s between 15.90% and 29.19%. In the post-crisis period, we find an insignificant relation between observed and model-implied quanto yield spreads and quanto CDS spreads for both Spain and Italy.

To conclude, our findings suggest that joint modeling of credit risk and currency risk is a key ingredient in understanding bond yields across currency denominations and that it becomes increasingly important when sovereign bond markets are under distress. In accordance with our above findings for the eurozone, Du and Schreger (2016) construct a quanto yield spread for emerging market sovereign bonds and find that the covariance between currency and credit risk explains a significant part of the quanto yield spread. These findings suggest that our model could be useful for understanding the variation in yield spreads across currency denominations in emerging bond markets, we leave this topic for future research.

8 Conclusion

In this paper we analyze quanto spreads in the context of eurozone sovereign CDS contracts. We develop a discrete-time no-arbitrage model, which illustrates how, even in a frictionless setting, quanto CDS spreads arise as a compensation for exposure to two risk factors. The first risk factor is an FX crash risk factor, which captures the market's (risk-neutral) anticipation of a large adverse jump in foreign currency (EUR) against domestic currency (USD) in the event of a sovereign default. The second factor, the currency/default risk covariance factor, captures the propensity for the EUR to depreciate (appreciate) against the U.S. dollar when eurozone sovereign credit risk rises (declines). Our simple model allows for simple comparative statics.

To estimate the relative importance of these factors, we propose an affine term structure model that allows us to distinguish between the two effects and capture their time-variation. We use our model to decompose the quanto spreads for Spain, Italy, Portugal, and Ireland, and find that both covariance and currency crash risk contribute substantially to quanto CDS spreads. The covariance risk factor is highly time-varying and increases in times of distress, when the currency and credit markets are volatile and co-move. However, the implied currency crash risk from sovereign defaults differ greatly in the four cases.

We estimate the (risk-neutral) expected jump in the EURUSD conditional on sovereign default for Spain and Italy to 15.6% and 9.6% which, consistent with our intuition, is significantly larger than the estimated currency jump size of about 5% in the event of a Portuguese or Irish default. We document a significant risk premium associated with currency/default covariance risk and currency crash risk, i.e., a risk premium associated with selling protection in the 'expensive' currency (USD) and buying protection in the 'cheap' currency (EUR). This risk premium is especially large for Spain and Italy, where it accounts for most of the quanto CDS spread.

Finally, we provide evidence that quanto yield spreads, which are differences in yields on USD and EUR-denominated bonds, are significantly related to quanto effects estimated based on our model. This highlights the importance of taking into account currency crash risk and covariance risk when assessing the relative pricing of bonds across currency denominations.

References

Aït-Sahalia, Y., R. J. Laeven, and L. Pelizzon (2014). Mutual excitation in Eurozone sovereign CDS. *Journal of Econometrics 183*(2), 151–167.

Augustin, P., M. Chernov, and D. Song (2018). Sovereign credit risk and exchange rates: Evidence from CDS quanto spreads. Working paper.

Backus, D. K., S. Foresi, and C. I. Telmer (2001). Affine term structure models and the forward premium anomaly. *The Journal of Finance 56*(1), 279–304.

Bakshi, G., N. Kapadia, and D. Madan (2003). Stock return characteristics, skew laws, and the differential pricing of individual equity options. *Review of Financial Studies 16*(1), 101–143.

Balduzzi, P., S. R. Das, and S. Foresi (1998). The central tendency: A second factor in bond yields. *Review of Economics and Statistics 80*(1), 62–72.

Barclays Research Note (2011, June). Quanto of solace. Technical report.

Bates, D. S. (1996). Jumps and stochastic volatility: Exchange rate processes implicit in deutsche mark options. *Review of Financial Studies 9*, 69–107.

Benzoni, L., P. Collin-Dufresne, R. Goldstein, and J. Helwege (2015). Modeling credit contagion via the updating of fragile beliefs. *Review of Financial Studies 28*(7), 1960–2008.

Breuer, A. and O. Sauter (2012). The impact of a sovereign default within the euro-zone on the exchange rate. *Applied Economics Quarterly 58*(1), 1–18.

Brigo, D., N. Pede, and A. Petrelli (2016). Multi currency credit default swaps: Quanto effects and FX devaluation jumps.

Buraschi, A., M. Menguturk, and E. Sener (2014). The geography of funding markets and limits to arbitrage. *Review of Financial Studies 28*(4), 1103–1152.

Carr, P. and D. Madan (1999). Option pricing and the fast fourier transform. *Journal of Computational Finance 2*(4), 61–73.

Carr, P. and L. Wu (2007a). Stochastic skew in currency options. *Journal of Financial Economics 86*(1), 213–247.

Carr, P. and L. Wu (2007b). Theory and evidence on the dynamic interactions between sovereign credit default swaps and currency options. *Journal of Banking and Finance 31*(8), 2383–2403.

Cheridito, P., D. Filipovic, and R. L. Kimmel (2007). Market price of risk specifications for affine models: Theory and evidence. *Journal of Financial Economics 83*(1), 123–170.

Chernov, M., J. J. Graveline, and I. Zviadadze (2016). Crash risk in currency returns. *Journal of Financial and Quantitative Analysis, forthcoming*.

Corradin, S. and M. Rodriguez-Moreno (2016). Limits to arbitrage: Empirical evidence from euro area sovereign bond markets. Working paper.

Cox, J. C., S. A. Ross, and M. Rubinstein (1979). Option pricing: A simplified approach. *Journal of Financial Economics 7*(3), 229–263.

Credit Suisse Research Note (2010, March). 'Leaving EMU' is just an expensive way to default. Technical report.

De Santis, R. A. (2015). A measure of redenomination risk.

Della Corte, P., L. Sarno, M. Schmeling, and C. Wagner (2016). Exchange rates and sovereign risk. *Working Paper*.

Du, W. and J. Schreger (2016). Local currency sovereign risk. *Journal of Finance, forthcoming*.

Duffie, D. and R. Kan (1996). A yield-factor model of interest rates. *Mathematical Finance 6*(4), 379–406.

Duffie, D., J. Pan, and K. Singleton (2000). Transform analysis and asset pricing for affine jump-diffusions. *Econometrica 68*(6), 1343–1376.

Ehlers, P. and P. Schönbucher (2006). The influence of FX risk on credit spreads. ETH Zurich working paper.

Garman, M. B. and S. W. Kohlhagen (1983). Foreign currency option values. *Journal of International Money and Finance 2*(3), 231–237.

Heston, S. L. (1993). A closed-form solution for options with stochastic volatility with applications to bond and currency options. *Review of Financial Studies 6*(2), 327–343.

J.P. Morgan Research Note (2010, October). Trading credit in different currencies via quanto CDS. Technical report.

Jurek, J. W. (2014). Crash-neutral currency carry trades. *Journal of Financial Economics 113*(3), 325–347.

Liao, G. Y. (2016). Credit migration and covered interest rate parity. *Working Paper*.

Longstaff, F. A., J. Pan, L. H. Pedersen, and K. J. Singleton (2011). How sovereign is sovereign credit risk? *American Economic Journal: Macroeconomics 3*(2), 75–103.

Mano, R. (2013). Exchange rates upon sovereign default. *Working Paper*.

Newey, W. K. and K. D. West (1987). A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica*, 703–708.

Pan, J. and K. J. Singleton (2008). Default and recovery implicit in the term structure of sovereign CDS spreads. *The Journal of Finance 63*(5), 2345–2384.

Schwartz, E. S. and A. B. Trolle (2009). Unspanned stochastic volatility and the pricing of commodity derivatives. *Review of Financial Studies 22*(11), 4423–4461.

Table 1: **Model parameters calibrated to moments for the EURUSD and CDS premiums.** This table shows parameter values for the model calibrated to average 5-year quanto CDS spreads for Portugal, Ireland, Italy, and Spain. First column reports the calibrated values for ρ, which is estimated for each sovereign as the correlation between percent-wise changes in the 5-year USD-denominated CDS premium and the EURUSD exchange rate. The second column reports the value for e^u which equals the average risk-neutral volatility derived from EURUSD options maturing in one year. The third column shows annualized standard deviations of daily percent-wise changes in the USD-denominated CDS premiums, and the fourth and fifth columns report the average 5-year CDS premiums of the USD and EUR-denominated contracts, respectively. All moments are estimated over the period August 2010 to August 2012.

	$\rho = \text{Corr}\big(\Delta S^U(t,5y), \Delta X_t\big)$	$e^u = \sigma_{FX}$	$\text{Std}\big(\Delta S^U(t,5y)\big)$	$\text{mean}\big(S^U(t,5y)\big)$	$\text{mean}\big(S^E(t,5y)\big)$
Portugal	−36%	14.6%	57%	8.51%	7.81%
Ireland	−38%	——	51%	6.48%	5.84%
Italy	−56%	——	68%	3.30%	2.67%
Spain	−57%	——	73%	3.44%	2.68%

Table 2: **One-period example with crash risk in synthetic bond price.** This table shows the payoffs for a long position in a USD zero-coupon bond and a short position in a synthetic USD zero-coupon bond—which is short a EUR zero-coupon bond and long a forward contract. There are no recovery payments on the bonds. All contracts are initiated at time 0 and expire at time 1. The riskless interest rates are 0, the exchange rate is 1 at time 0, and the forward exchange rate is 1. The default states are assumed to be associated with a 50% depreciation in the EUR against the USD.

	$t = 0$	No default at $t = 1$	Default at $t = 1$
Long USD Bond	$-P^{USD}$	1 USD	0 USD
Short Synthetic USD Bond	$P^{USD,synth}$	-1 EUR + 1 EUR - 1 USD	1 EUR -1 USD
Cash Flow L/S in USD	0	0	-0.5 USD

Table 3: **Crash risk and currency/default covariance risk in bond yields.** This table compares yields on domestic and synthetic domestic coupon bonds derived via the discrete-time model. The synthetic domestic bond consists of a long position in a foreign bond that pays 1 at $t = 1, \ldots 5$ and 100 at maturity in foreign currency, and a short position in currency forward contracts that match those payments. The yield of the synthetic bond is reported in the first row with crash risk, and in the second row under the assumption of no crash risk. The third row shows the yield on a domestic coupon bond which pays 1 at $t = 1, \ldots 5$ and 100 at maturity. Rows 4-6 show the corresponding prices of the coupon bonds and the prices for each of the coupon payments. All bond payments are conditional on no default, and there are no recovery payments. The parameters used in the model are calibrated to 5-year EUR and USD CDS data for Spain and EURUSD moments (as reported in Table 1).

	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$
Yield Synthetic Coupon Bond, ($\delta = 0.84$)					4.46 %
Yield Synthetic Coupon Bond, ($\delta = 1$)					5.37 %
Yield Domestic Coupon Bond					5.73 %
Price Synthetic Domestic Coupon, ($\delta = 0.84$)	0.95	0.91	0.88	0.84	80.41
Price Synthetic Domestic Coupon, ($\delta = 1$)	0.95	0.90	0.85	0.81	76.99
Price Domestic Coupon	0.95	0.89	0.85	0.80	75.67

Table 4: **Summary statistics for USD CDS premiums and quanto CDS premiums.** This table reports sample estimates of the means and standard deviations of the USD-denominated CDS premiums and quanto CDS premiums for Austria, Belgium, Germany, Finland, France, Ireland, Italy, Netherlands, Portugal, and Spain. For each sovereign, the quanto CDS premium is defined as the difference in premiums on a USD and a EUR-denominated CDS contract at the same maturity. Panel A reports the time-series means of the premiums of the USD-denominated CDS contracts and the quanto CDS contracts in basis points at maturities of 1-10 years. Panel B reports the standard deviations of the premiums on the USD-denominated CDS contracts and the quanto CDS contracts in percentages at maturities of 1-10 years. The sample consists of daily quotes obtained from Markit from August 2010 to April 2016 (1402 observations for each series).

Panel A: Mean in bps

	USD CDS					Quanto CDS				
	1 yr	3 yrs	5 yrs	7 yrs	10 yrs	1 yr	3 yrs	5 yrs	7 yrs	10 yrs
AUS	26.74	43.96	65.09	78.38	89.88	8.75	14.64	22.20	27.07	30.79
BEL	53.78	82.84	108.98	124.08	136.27	12.12	21.96	30.55	35.24	39.25
GER	11.26	21.72	39.26	51.72	63.08	3.70	9.07	16.90	21.48	25.51
FIN	12.87	21.05	34.27	43.98	52.93	2.54	5.00	8.49	11.10	12.96
FRA	29.01	54.02	82.49	100.45	115.90	8.39	18.60	28.90	34.17	38.42
IRE	271.14	303.88	297.09	298.27	291.82	22.59	31.35	36.46	38.64	40.55
ITA	132.99	193.37	224.06	240.09	250.72	21.84	32.45	38.39	41.01	43.11
NET	17.50	29.83	48.56	61.00	72.15	5.06	10.24	17.51	22.31	25.97
POR	437.00	489.10	478.21	471.72	456.50	34.43	37.71	41.04	42.72	44.56
SPA	142.89	198.90	225.70	239.26	247.96	30.16	41.60	47.68	50.36	52.84

Panel B: Standard Deviation in %

	USD CDS					Quanto CDS				
	1 yr	3 yrs	5 yrs	7 yrs	10 yrs	1 yr	3 yrs	5 yrs	7 yrs	10 yrs
AUS	0.35	0.45	0.53	0.50	0.49	0.12	0.15	0.17	0.17	0.17
BEL	0.71	0.85	0.85	0.78	0.70	0.17	0.21	0.23	0.22	0.21
GER	0.13	0.19	0.27	0.27	0.27	0.05	0.08	0.13	0.14	0.15
FIN	0.13	0.16	0.19	0.18	0.17	0.03	0.03	0.03	0.04	0.05
FRA	0.35	0.47	0.55	0.52	0.50	0.12	0.18	0.23	0.24	0.24
IRE	3.48	3.37	2.82	2.52	2.20	0.25	0.27	0.26	0.25	0.24
ITA	1.34	1.39	1.33	1.22	1.11	0.20	0.24	0.25	0.25	0.24
NET	0.20	0.25	0.31	0.31	0.30	0.07	0.09	0.11	0.13	0.14
POR	5.03	4.55	3.59	3.08	2.61	0.39	0.29	0.25	0.23	0.22
SPA	1.27	1.45	1.41	1.30	1.17	0.28	0.33	0.34	0.33	0.32

Table 5: **Summary statistics for currency options data.** This table reports the means and standard deviations of implied volatilities for EURUSD delta-neutral straddles (STR), EURUSD 10 and 25-delta risk reversals (RR10 and RR25, respectively), and EURUSD 10 and 25-delta butterfly spreads (BF10 and BF25, respectively). All quantities are reported in percentages. The data are obtained from Bloomberg and the sample consists of daily quotes from August 2010 to April 2016 (1402 observations for each series).

	Mean (%)						Std (%)					
	1 mo	2 mo	3 mo	6 mo	9 mo	1 yr	1 mo	2 mo	3 mo	6 mo	9 mo	1 yr
STR	9.83	9.93	10.01	10.25	10.42	10.56	2.74	2.68	2.64	2.57	2.52	2.47
RR10	-1.70	-2.26	-2.70	-3.20	-3.47	-3.61	1.41	1.52	1.62	1.60	1.62	1.61
RR25	-1.02	-1.30	-1.51	-1.78	-1.90	-1.97	0.82	0.85	0.87	0.85	0.84	0.83
BF10	11.08	11.54	11.93	12.66	13.10	13.42	3.19	3.26	3.33	3.41	3.42	3.41
BF25	10.27	10.47	10.62	10.97	11.20	11.36	2.85	2.82	2.80	2.77	2.72	2.68

Table 6: **Regressions of FX spot and implied volatility changes on eurozone sovereign credit risk.** This table presents estimates from regressions of contemporaneous weekly changes in the EURUSD spot exchange rate and the EURUSD implied volatility on eurozone sovereign credit risk:

$$\Delta X_t = \alpha + \beta \Delta PC1_t^{CDS} + \varepsilon_t, \quad \Delta IV_t = \alpha + \beta \Delta PC1_t^{CDS} + \varepsilon_t$$

EURUSD volatility is proxied by the 1-month implied volatility of a delta-neutral straddle, and eurozone credit risk is measured as the first principal component of weekly 5-year CDS premiums for 10 eurozone sovereigns. Columns 1-2 show the results of the regressions using the full sample, columns 3-4 show the results from the crisis period (August 2010 to December 2012), and columns 5-6 show the results for the post-crisis period (January 2013 to April 2016). Newey and West (1987) t-statistics are reported in brackets, and the superscripts *, **, and *** indicate statistical significance at 10 %, 5 %, and 1 %, respectively. The currency spot and implied volatility data are from Bloomberg, and the CDS data are from Markit. The sample period is from August 2010 to April 2016 (281 weekly observations).

	2010-2016		2010-2012		2013-2016	
	ΔX	ΔIV	ΔX	ΔIV	ΔX	ΔIV
α	-0.0001	-0.0000	0.0005	0.0007	-0.0003	-0.0003
	[-0.27]	[-0.06]	[0.64]	[0.97]	[-1.38]	[-0.80]
β	-0.0988^{***}	0.2330^{***}	-0.1720^{***}	0.5046^{***}	-0.0191	0.0672^{**}
	[-3.69]	[3.84]	[-4.01]	[7.70]	[-1.34]	[2.17]
R^2	0.081	0.121	0.162	0.272	0.002	0.032

Table 7: **Parameter estimates for the proposed affine model.** This table reports parameter estimates of the affine model specified in equation (16). The numbers in parentheses are standard errors of the estimates. The parameters are estimated using maximum likelihood estimation in conjunction with the unscented Kalman filter, using premiums on USD-denominated CDS and quanto CDS contracts with maturities of 1-10 years, and EURUSD option-implied volatilities at five strikes and six maturities spanning 1-12 months. Each time series consists of 281 weekly observations (each Wednesday) from August 2010 to April 2016.

	Intensity Parameters				**FX Parameters**	
	Ireland	Italy	Portugal	Spain		
κ_l	0.0326	0.2533	0.0308	0.0907	κ_v	1.2129
	(0.0027)	(0.0367)	(0.0133)	(0.0597)		$(0.5890 \cdot 10^{-3})$
θ_l	0.1760	0.0018	0.0608	0.0188	θ_v	0.0183
	(0.0051)	(0.0011)	(0.0019)	(0.0069)		$(0.9021 \cdot 10^{-4})$
σ_l	0.4392	0.2892	0.3484	0.4525	σ_v	0.1452
	(0.0099)	(0.0154)	(0.0081)	$(2.56 \cdot 10^{-5})$		$(0.3740 \cdot 10^{-3})$
κ_l^P	0.0469	0.0005	0.0007	0.0001	κ_v^P	1.5935
	(0.0136)	(0.8737)	(0.0101)	(0.0119)		$(0.1305 \cdot 10^{-3})$
θ_l^P	0.1295	0.0092	(0.0059)	0.0336	θ_v^P	0.0174
	(0.0086)	(0.0068)	(0.0007)	(0.0267)		$(0.9532 \cdot 10^{-2})$
κ_z	0.2620	0.2460	0.2450	0.1283	ρ	-0.6817
	(0.0073)	(0.0298)	(0.0053)	(0.0208)		$(0.9032 \cdot 10^{-3})$
σ_z	0.0000	0.0013	0.3660	0.0445	σ_O	$0.8512 \cdot 10^{-4}$
	(0.0093)	0.0025	0.0031	(0.0617)		$(0.5815 \cdot 10^{-3})$
κ_z^P	0.0037	0.0041	0.0000	0.0010		
	(0.0159)	(0.1480)	(0.0060)	(0.1539)		
κ_m	0.0035	0.0012	0.0241	0.0023		
	(0.0164)	(0.0027)	(0.0057)	(0.0035)		
θ_m	0.0000	0.0000	0.0043	0.0116		
	(0.0071)	(0.0092)	(0.0139)	(0.0052)		
σ_m	0.2200	0.1099	0.2059	0.1201		
	(0.0051)	(0.0100)	(0.0015)	(0.0063)		
κ_m^P	0.0010	0.1641	0.0009	0.0639		
	(0.1690)	(0.0967)	(0.0037)	(0.4256)		
θ_m^P	0.0000	0.0000	0.2615	0.1002		
	(0.0027)	(0.0300)	(0.0123)	(0.6966)		
ζ	-0.0502	-0.0960	-0.0543	-0.1559		
	(0.0041)	(0.0050)	(0.0018)	(0.0012)		
l_0	(0.0098)	(0.0105)	(0.0042)	(0.0033)		
	(0.0003)	(0.0035)	(0.0008)	(0.0032)		
z_0	0.0015	0.0029	0.0272	0.0000		
	(0.002)	(0.0107)	(0.0040)	(0.1965)		
m_0	0.0435	0.0459	(0.0022)	(0.0793)		
	(0.0023)	(0.0093)	(0.0040)	(0.0120)		
σ_U	$3.24 \cdot 10^{-6}$	$1.06 \cdot 10^{-6}$	$1.91 \cdot 10^{-6}$	$5.16 \cdot 10^{-6}$		
	$(4.40 \cdot 10^{-6})$	$(3.53 \cdot 10^{-6})$	$(2.91 \cdot 10^{-6})$	$(2.51 \cdot 10^{-6})$		
σ_{UE}	$4.12 \cdot 10^{-5}$	$2.47 \cdot 10^{-5}$	$0.74 \cdot 10^{-5}$	$0.45 \cdot 10^{-5}$		
	$(4.97 \cdot 10^{-6})$	$(1.549 \cdot 10^{-6})$	$(2.22 \cdot 10^{-6})$	$(6.22 \cdot 10^{-6})$		

Table 8: **Summary statistics of model pricing errors for USD CDS and quanto CDS premiums.** This table reports the root mean squared errors and mean absolute pricing errors for model-implied USD-denominated CDS premiums and quanto CDS premiums at maturities from 1-10 years. Both are reported in basis points (bps). The pricing error is defined as the difference between the observed CDS premium/quanto CDS premium and the model-implied CDS premium/quanto CDS premium (using the updated state variable). The model is estimated using maximum likelihood estimation in conjunction with the unscented Kalman filter using USD CDS data, quanto CDS data (both from Markit), and currency options data from Bloomberg. The sample consists of 281 weekly observations from August 2010 to April 2016.

Panel A: Root Mean Squared Errors (bps)												
	USD CDS						Quanto CDS					
	1 yr	3 yrs	5 yrs	7 yrs	10 yrs	Mean	1 yr	3 yrs	5 yrs	7 yrs	10 yrs	Mean
IRE	37.61	36.93	24.55	18.93	15.35	26.68	0.65	0.67	0.44	1.04	2.09	0.98
ITA	26.65	25.23	25.18	21.58	17.41	23.21	3.89	2.16	1.48	2.41	5.36	3.06
POR	50.50	45.82	36.81	28.35	28.12	37.92	3.48	1.77	0.43	1.42	4.70	2.36
SPA	26.27	23.61	20.42	18.70	34.04	24.61	0.15	1.92	5.79	7.66	8.99	4.90
Panel B: Mean Absolute Pricing Errors (bps)												
	USD CDS						Quanto CDS					
	1 yr	3 yrs	5 yrs	7 yrs	10 yrs	Mean	1 yr	3 yrs	5 yrs	7 yrs	10 yrs	Mean
IRE	28.44	32.48	18.92	13.52	9.07	20.49	7.24	8.06	9.86	11.01	11.27	9.49
ITA	19.44	18.72	18.76	15.58	13.97	17.29	6.82	6.31	6.90	7.31	8.80	7.23
POR	30.93	36.70	25.75	15.71	18.12	25.44	11.02	7.22	6.74	7.61	9.43	8.40
SPA	19.81	17.18	15.11	13.58	26.70	18.48	3.81	5.13	7.20	8.29	9.13	6.71

Table 9: **Summary statistics for decompositions of quanto CDS spreads.** This table reports summary statistics for model decompositions of quanto CDS spreads into a covariance risk component and a crash risk component. Panel A reports the mean and the maximum of the covariance component in basis points (bps) over the full sample period. Panel B reports the mean and maximum share for the covariance component of the total quanto CDS spread over the full sample. Panel C and D report the same quantities but for the debt crisis period (August 2010 to December 2012). The model is estimated using maximum likelihood estimation in cojunction with the unscented Kalman filter based on USD CDS data, quanto CDS data (both from Markit), and currency options data from Bloomberg. The sample consists of 281 weekly observations from August 2010 to April 2016.

Panel A: Full sample (August 2010 - April 2016)										
	Mean covariance component (bps)					Max covariance component (bps)				
	1 yr	3 yrs	5 yrs	7 yrs	10 yrs	1 yr	3 yrs	5 yrs	7 yrs	10 yrs
IRE	8.09	18.95	23.49	25.55	27.25	31.26	59.04	60.60	57.23	54.62
ITA	3.98	12.08	16.35	16.84	15.40	14.37	42.19	55.23	55.15	48.39
POR	5.98	13.02	15.16	15.54	15.51	32.24	64.36	70.97	69.32	64.56
SPA	5.66	9.86	9.24	8.03	6.77	24.06	43.61	38.51	30.23	21.89
	Share of spread from covariance risk					Max share of spread from covariance risk				
	1 yr	3 yrs	5 yrs	7 yrs	10 yrs	1 yr	3 yrs	5 yrs	7 yrs	10 yrs
IRE	0.54	0.72	0.75	0.77	0.79	0.82	0.96	0.98	0.98	0.99
ITA	0.20	0.34	0.38	0.38	0.37	0.39	0.60	0.65	0.65	0.62
POR	0.21	0.31	0.35	0.37	0.40	0.56	0.72	0.76	0.78	0.78
SPA	0.13	0.17	0.20	0.21	0.21	0.35	0.46	0.40	0.31	0.21
Panel B: Debt Crisis Period (August 2010 - December 2012)										
	Mean covariance component (bps)					Max covariance component (bps)				
	1 yr	3 yrs	5 yrs	7 yrs	10 yrs	1 yr	3 yrs	5 yrs	7 yrs	10 yrs
IRE	15.78	32.27	35.81	36.04	36.02	31.26	59.04	60.60	57.23	54.62
ITA	7.77	23.14	30.70	30.92	27.34	14.37	42.19	55.23	55.15	48.39
POR	11.95	24.62	27.18	26.57	25.11	32.24	64.36	70.97	69.32	64.56
SPA	12.49	22.00	18.35	13.23	8.18	24.06	43.61	38.51	30.23	21.89
	Share of spread from covariance risk					Max share of spread from covariance risk				
	1 yr	3 yrs	5 yrs	7 yrs	10 yrs	1 yr	3 yrs	5 yrs	7 yrs	10 yrs
IRE	0.37	0.54	0.58	0.60	0.62	0.61	0.74	0.76	0.77	0.78
ITA	0.27	0.48	0.53	0.53	0.50	0.39	0.60	0.65	0.65	0.62
POR	0.23	0.35	0.38	0.39	0.40	0.55	0.72	0.76	0.78	0.78
SPA	0.22	0.30	0.25	0.18	0.11	0.35	0.46	0.40	0.31	0.21

Table 10: **Summary statistics for risk premiums of USD CDS and quanto CDS.** This table shows risk premiums associated with holding USD CDS and quanto CDS for Ireland, Italy, Portugal, and Spain. Panel A reports the mean risk premiums for holding USD CDS and quanto CDS in basis points at maturities of 1-10 years. Panel B reports the average risk premiums for USD CDS and quanto CDS as a fraction of total spreads. The model is estimated using maximum likelihood estimation in conjunction with the unscented Kalman filter based on USD CDS data, quanto CDS data (both from Markit), and currency options data from Bloomberg. The sample consists of 281 weekly observations from August 2010 to April 2016.

Panel A: Mean risk premium in bps

	USD CDS					Quanto CDS				
	1 yr	3 yrs	5 yrs	7 yrs	10 yrs	1 yr	3 yrs	5 yrs	7 yrs	10 yrs
IRE	0.66	1.90	2.34	1.68	-3.18	0.92	2.36	3.14	3.44	3.33
ITA	36.60	96.01	134.81	156.92	171.41	3.76	11.46	15.94	16.88	15.81
POR	55.96	146.97	211.29	251.03	278.33	3.00	7.96	11.18	12.83	13.76
SPA	28.84	76.59	114.31	144.67	177.44	2.85	6.64	8.48	9.25	8.56

Panel B: Mean risk premium as a fraction of spread

	USD CDS					Quanto CDS				
	1 yr	3 yrs	5 yrs	7 yrs	10 yrs	1 yr	3 yrs	5 yrs	7 yrs	10 yrs
IRE	0.03	0.05	0.06	0.06	0.05	0.25	0.17	0.15	0.14	0.12
ITA	0.38	0.58	0.66	0.69	0.72	0.63	0.69	0.73	0.75	0.77
POR	0.42	0.57	0.64	0.69	0.75	0.25	0.42	0.40	0.39	0.38
SPA	0.28	0.48	0.59	0.65	0.71	0.49	0.51	0.61	0.45	0.67

Table 11: **Summary statistics for observed quanto yield spreads.** This table reports summary statistics for observed quanto yield spreads. The synthetic quanto yield spread is the difference in yields between a USD bond and a synthetic USD bond, which is constructed based on EUR bond credit spreads. The synthetic USD bond is constructed such that it matches the coupon scheme, notional value, and time to maturity of the USD bond. The quanto bond yield spread is computed as the difference in yields on coupon bonds denominated in USD and EUR with similar maturities corrected for the riskless interest rate differential. Newey-West t-statistics of the means are reported in square brackets. The superscripts *, **, and *** indicate statistical significance at 10%, 5%, and 1%, respectively. The sample period is from August 2010 to April 2016 (281 observations).

Panel A: Debt Crisis (August 2010 – March 2013)						
	IT (synth)	IT (bond)	ES (synth)	ES (bond)	PT (synth)	PT (bond)
Mean (bps)	40.82***	59.67***	62.65***	98.98***	4.31	28.61
	[5.42]	[9.45]	[6.31]	[7.85]	[0.35]	[1.02]
Std (%)	0.39	0.39	0.62	0.81	0.67	1.91
Skew	0.44	0.46	0.65	0.61	0.25	1.00
Q5 (bps)	-16.35	1.76	-26.61	-3.44	-91.89	-198.73
Q95 (bps)	123.06	140.06	178.21	240.36	112.53	360.88
Fraction > 0	0.86	0.95	0.89	0.91	0.50	0.40
Panel B: Post Debt Crisis (March 2013 – April 2016)						
	IT (synth)	IT (bond)	ES (synth)	ES (bond)		
Mean (bps)	14.04	25.81***	33.31***	22.15***		
	[0.84]	[3.21]	[4.47]	[3.47]		
Std (%)	0.27	0.22	0.19	0.20		
Skew	-0.34	-0.17	-0.41	0.09		
Q5 (bps)	-34.66	-8.84	-1.12	-8.30		
Q95 (bps)	55.74	60.70	60.35	53.22		
Fraction > 0	0.74	0.88	0.94	0.85		

Table 12: **Regressions of observed quanto yield spreads on model-implied quanto yield spreads.** This table shows the results from regressing observed quanto yield spreads on model-implied quanto yield spreads (Model QY) and observed 5-year quanto CDS spreads (5Y QCDS). The observed synthetic quanto yield spread is the difference in yields on a USD bond and a synthetic USD bond, which is constructed from EUR credit spreads. The synthetic USD bond is constructed such that it matches the coupon scheme, notional value, and time to maturity of the USD bond. The quanto bond yield spread is computed as the difference in yields on comparable coupon bonds denominated in USD and EUR corrected for the riskless interest rate differential. Newey-West t-statistics are reported in square brackets. The superscripts *, **, and *** indicate statistical significance at 10%, 5%, and 1%, respectively. The sample period is from August 2010 to April 2016 (281 observations).

Panel A: Debt Crisis (August 2010 – March 2013)						
	IT (synth)	IT (bond)	ES (synth)	ES (bond)	PT (synth)	PT (bond)
Model QY	0.99	1.03**	1.37***	1.60***	1.16*	2.85
	[1.43]	[2.28]	[2.99]	[6.60]	[1.70]	[1.03]
Intercept	0.00	-0.00	0.00	0.01	-0.01	−0.02
	[-0.29]	[-0.03]	[0.32]	[-0.99]	[-1.57]	[-0.60]
R^2 (%)	8.66	15.90	20.75	35.54	5.83	3.29
5Y QCDS	0.92**	1.19***	1.24***	1.93***	0.15	0.16
	[2.07]	[3.22]	[4.19]	[4.30]	[0.21]	[0.06]
Intercept	0.00	0.00	0.00	-0.01	0.00	0.01
	[-0.68]	[-0.71]	[-1.69]	[-1.30]	[-0.19]	[0.27]
R^2 (%)	17.06	29.19	25.28	35.44	0.19	0.00
Panel B: Post Debt Crisis (March 2013 – April 2016)						
	IT (synth)	IT (bond)	ES (synth)	ES (bond)		
Model QY	-1.37	-0.54	0.27	0.38*		
	[-1.60]	[-0.51]	[-0.93]	[1.66]		
Intercept	0.00***	0.00**	0.00*	0.00		
	[2.66]	[2.85]	[1.88]	[0.54]		
R^2 (%)	17.60	5.71	3.38	6.81		
5Y QCDS	-0.49	-0.08	0.24	0.32*		
	[-1.13]	[-0.21]	[1.44]	[1.86]		
Intercept	0.00***	0.00***	0.00***	0.00***		
	[2.66]	[3.17]	[4.23]	[2.35]		
R^2 (%)	7.72	0.26	3.86	6.50		

9 Figures

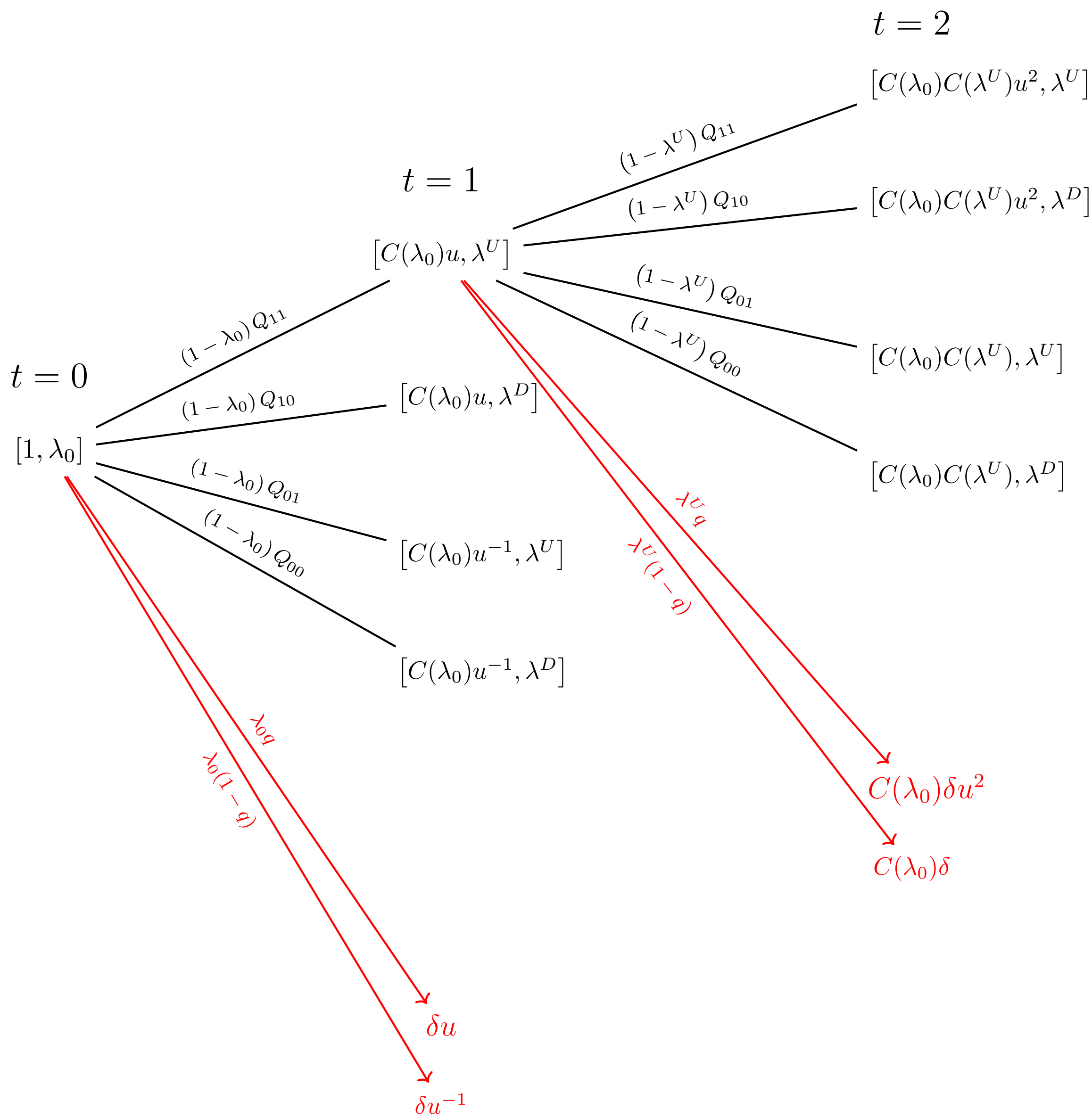


Figure 1: **Two-period model of the default probability and the exchange rate.** This figure illustrates the joint dynamics of the default probability and the exchange rate over two periods. At time 0, the exchange rate is 1 and default occurs with a probability of λ_0. If default occurs, the exchange rate is adjusted by δ relative to the state of the exchange rate if there were no crash risk. Conditional on survival, which occurs with probability $1 - \tilde{\lambda}$, the exchange rate is adjusted by the compensating factor $C(\tilde{\lambda})$, where $\tilde{\lambda} = \lambda^U$ or $\tilde{\lambda} = \lambda^D$. Simultaneously, if survival occurs, a new one-period default probability is drawn which takes either a high value λ^U or a low value λ^D, and a relative one-period change of the exchange rate is realized taking two possible values (u, u^{-1}). That is, in total there are four possible outcomes for the default probability and the exchange rate change at each node. The joint probability distribution for reaching each of those four possible states are specified in equations (1)-(2). There are the same possible states in each survival node. Due to space constraints, we only show the possible states at time 2 starting from the survival node in which the default probability and the exchange rate went up $((\lambda^U, u))$.

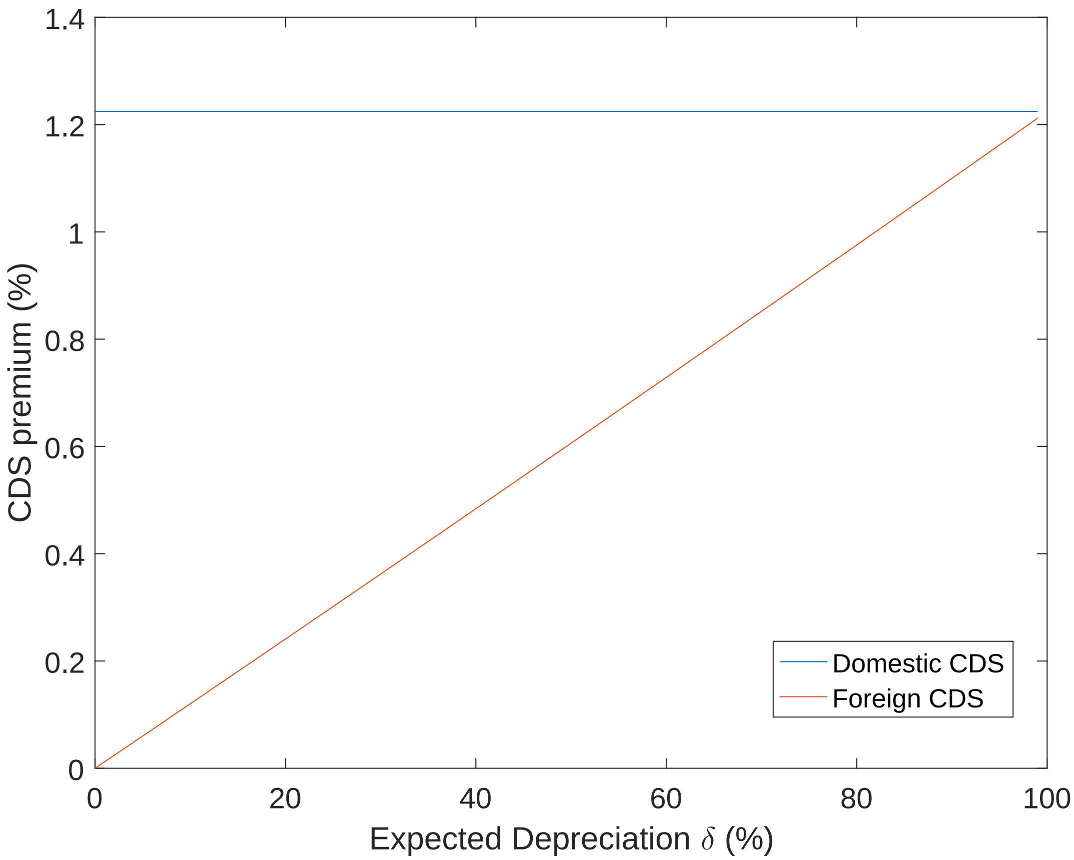


Figure 2: **Currency crash risk induced quanto CDS spreads.** This figure illustrates the impact of an expected depreciation upon default, δ, on the premiums of CDS contracts denominated in foreign and domestic currency. The blue graph is the CDS premium in domestic currency, and the red graph is the CDS premium in foreign currency on the same underlying reference entity. The CDS premiums are computed based on a model with fixed default probability and a fixed risk-neutral expected depreciation upon default. Interest rates do not affect CDS premiums in the model when the default probability is constant.

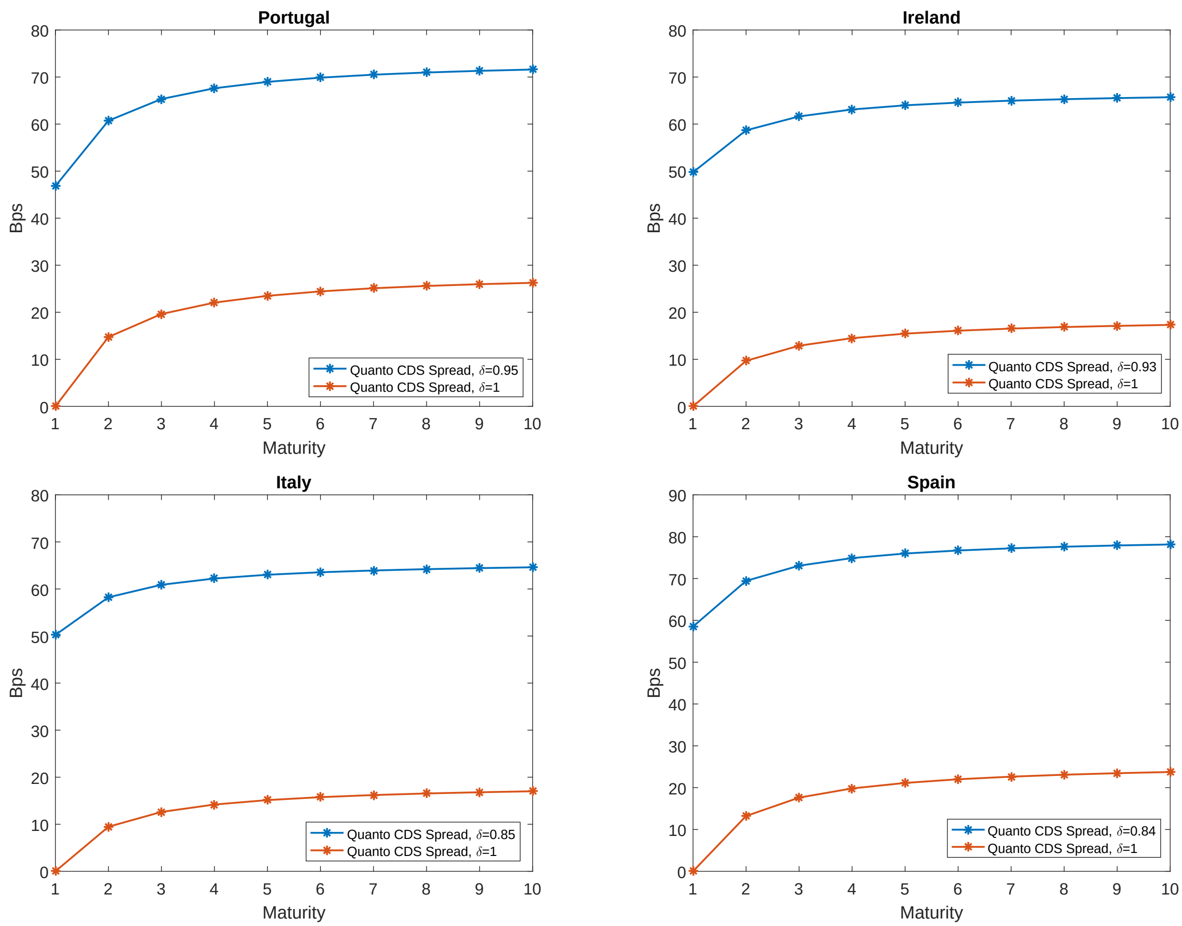


Figure 3: **Term structures of calibrated quanto CDS spreads.** This figure illustrates the term structure of model-generated quanto CDS spreads at maturities of one to ten years. The quanto spread is the difference between the CDS premiums on the same reference entity denominated in USD and EUR. The parameters are calibrated to match the empirical average 5-year EUR and USD CDS premiums, the 1-year EURUSD risk-neutral volatility, and the correlation between the 5-year USD CDS premium and the EURUSD spot exchange rate. All model parameters are assumed fixed, and the calibration period is August 2010 to August 2012. The blue graph illustrates the quanto spread at different maturities. The orange graph is the share of the quanto spread stemming from default/currency covariance risk, i.e., the case of $\delta = 1$. The recovery rate is assumed to be 40%, and the choice of foreign and domestic interest rates has no impact on the quanto spread in the model.

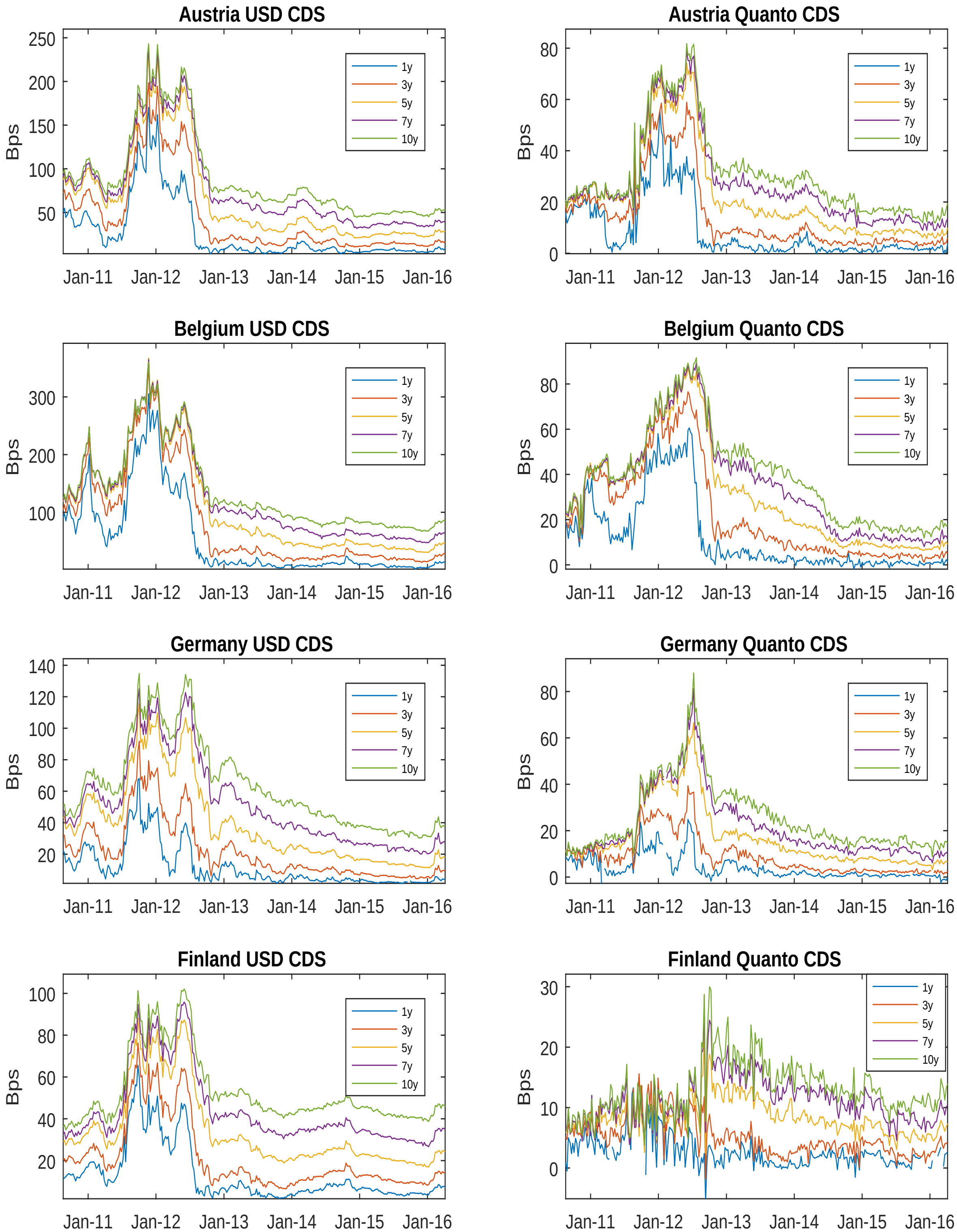


Figure 4: **USD CDS and quanto CDS spreads for Austria, Belgium, Germany, and Finland.** This figure shows USD CDS premiums and quanto CDS spreads–defined as the difference between USD and EUR-denominated CDS premiums of the same underlying reference entity–for Austria, Belgium, Germany, and Finland. The sample period is August 2010 to April 2016 and comprises 1402 daily observations obtained from Markit.

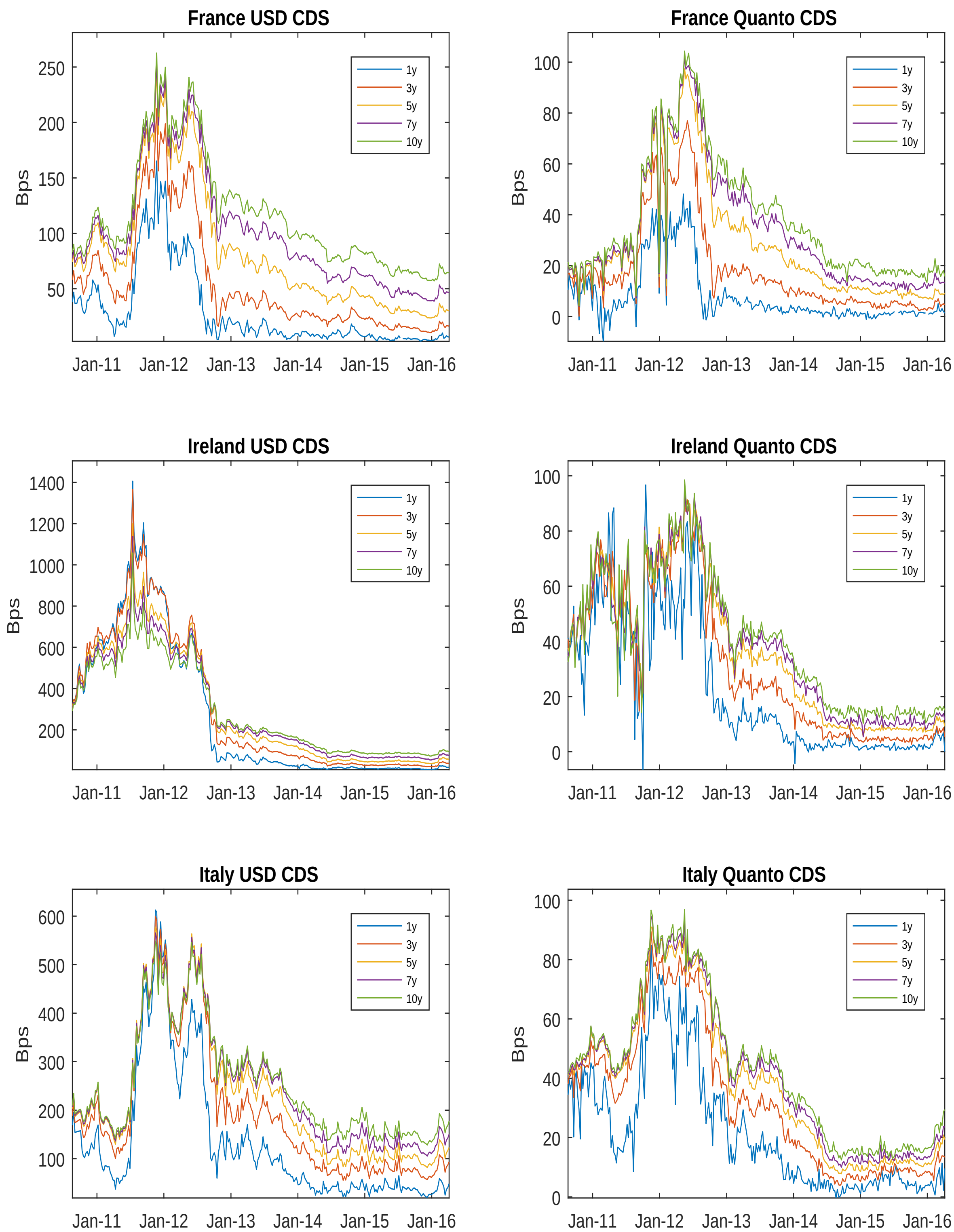


Figure 5: **USD CDS and quanto CDS spreads for France, Ireland, and Italy.** This figure shows USD CDS premiums and quanto CDS spreads–defined as the difference between USD and EUR-denominated CDS premiums of the same underlying reference entity–for France, Ireland, and Italy. The sample period is August 2010 to April 2016 and comprises 1402 daily observations obtained from Markit.

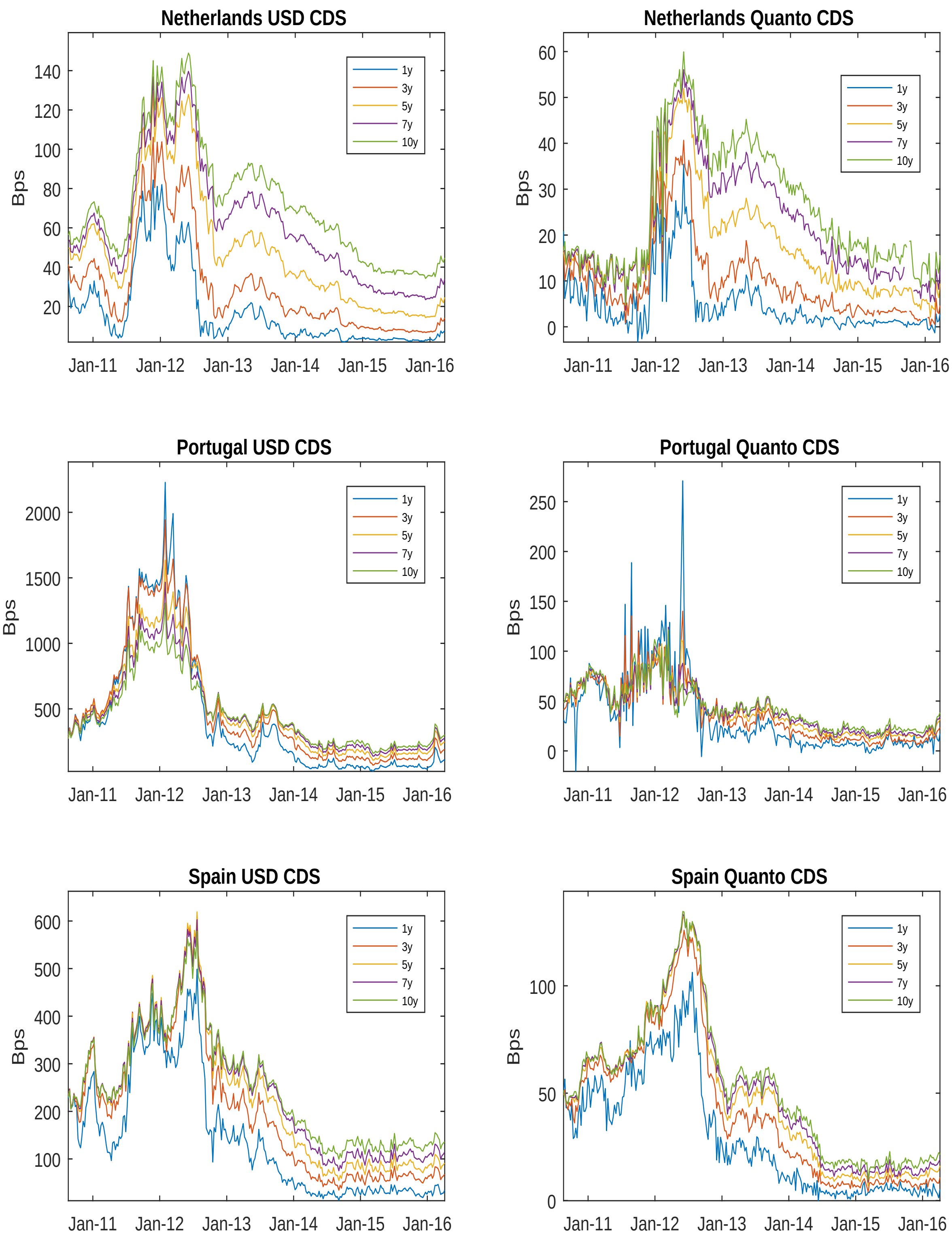


Figure 6: **USD CDS and quanto CDS spreads for Netherlands, Portugal, and Spain.** This figure shows USD CDS premiums and quanto CDS spreads–defined as the difference between USD and EUR-denominated CDS premiums of the same underlying reference entity–for Netherlands, Portugal, and Spain. The sample period is August 2010 to April 2016 and comprises 1402 daily observations obtained from Markit.

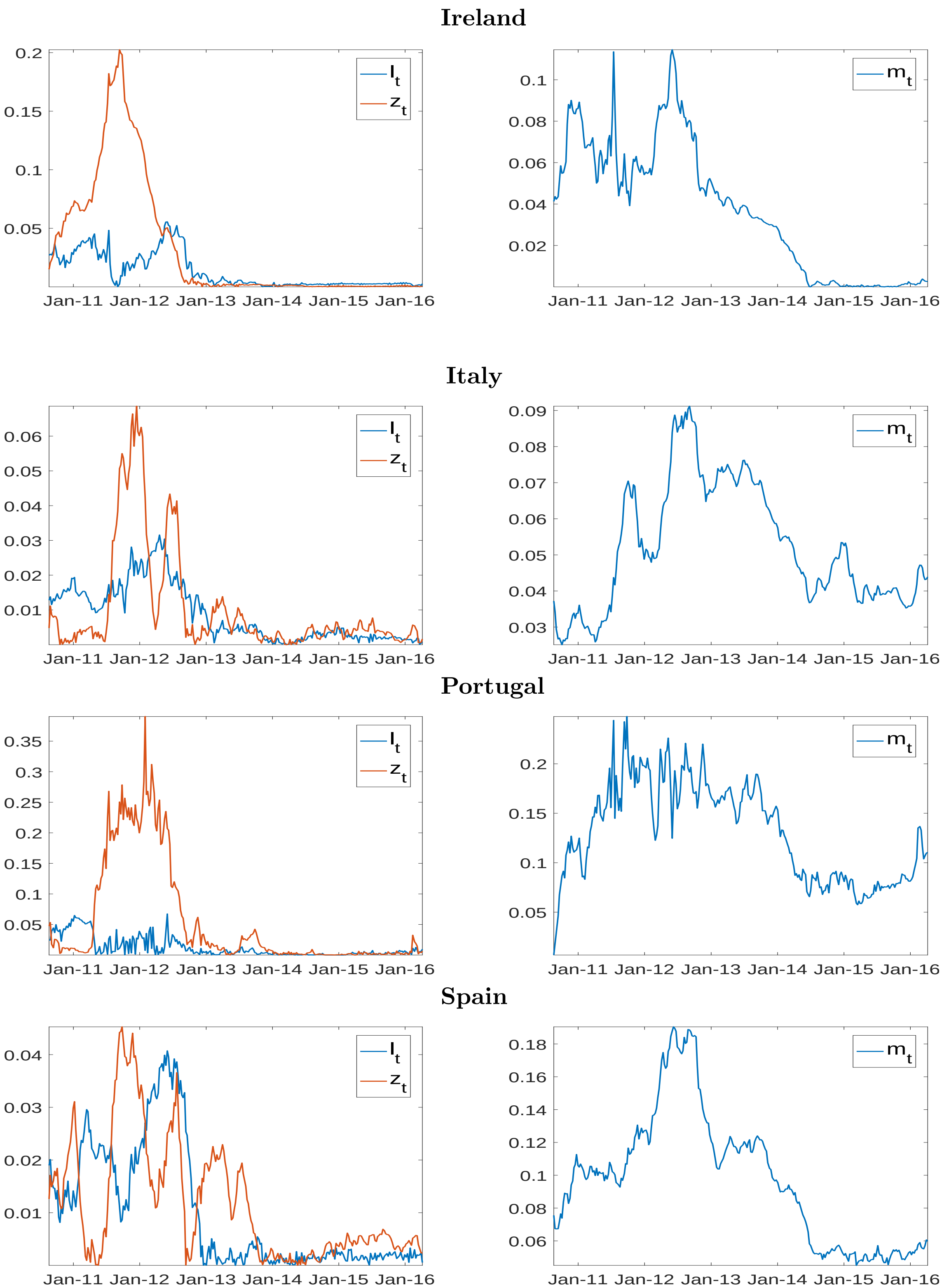


Figure 7: **Estimated time series of state variables.** This figure shows the time series of the estimated state variables. The left panel shows the state variables l_t and z_t and the right panel shows m_t. The model is estimated via maximum likelihood estimation in conjunction with the unscented Kalman filter. The sample period is August 2010 to April 2016 and each time series consists of 281 weekly observations.

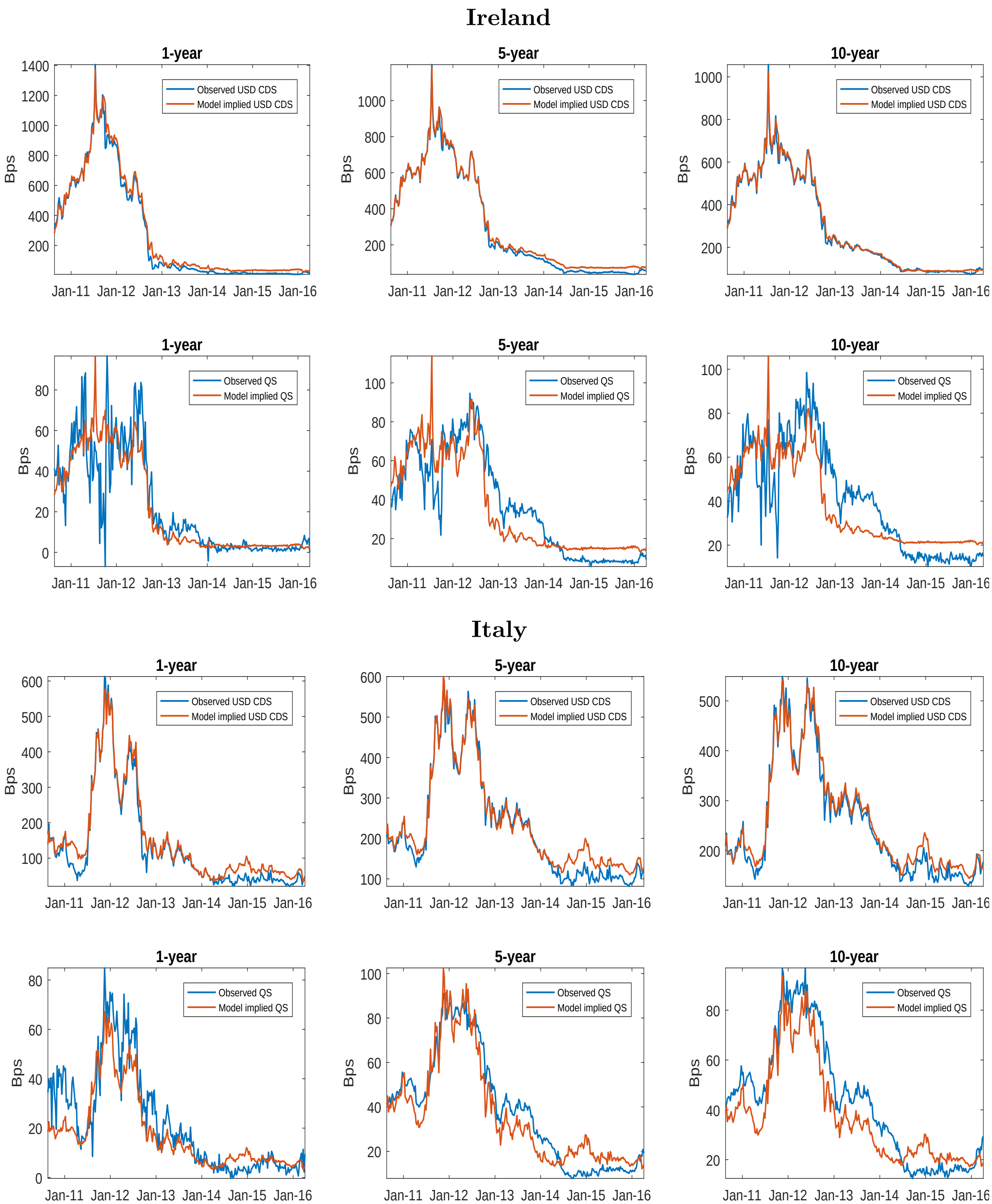
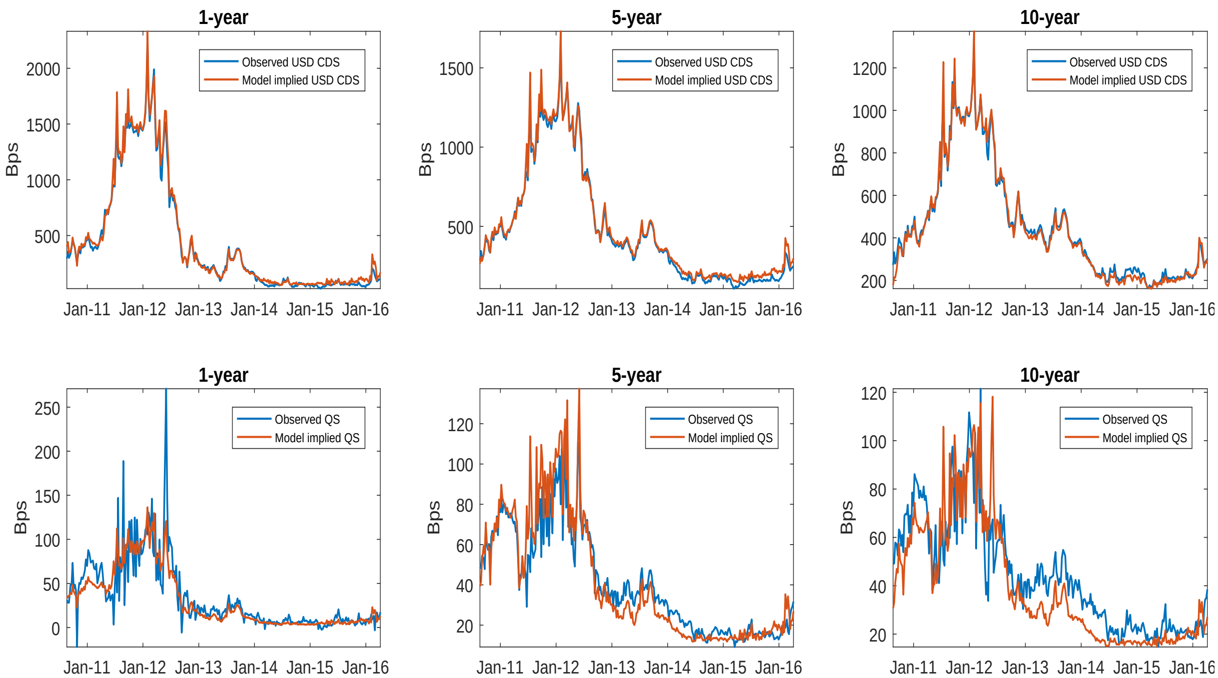


Figure 8: **Model fit for Ireland and Italy.** This figure shows the time series of the model-fitted versus the observed USD CDS premiums and quanto CDS spreads for Ireland and Italy. The illustrated maturities are 1, 5, and 10 years. The model is estimated via maximum likelihood estimation in conjunction with the unscented Kalman filter. The sample period is August 2010 to April 2016 and each time series consists of 281 weekly observations.

Portugal



Spain

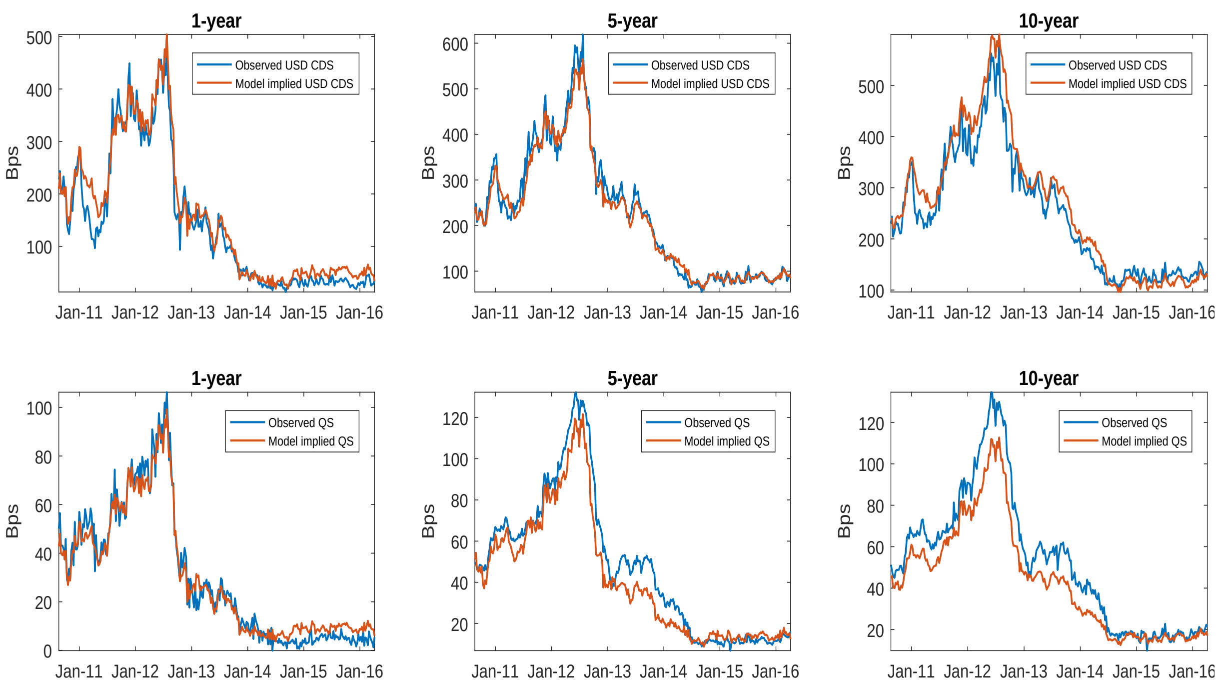


Figure 9: **Model fit for Portugal and Spain.** This figure shows the time series of the model-fitted versus the observed USD CDS premiums and quanto CDS spreads for Portugal and Spain. The illustrated maturities are 1, 5, and 10 years. The model is estimated via maximum likelihood estimation in conjunction with the unscented Kalman filter. The sample period is August 2010 to April 2016 and each time series consists of 281 weekly observations.

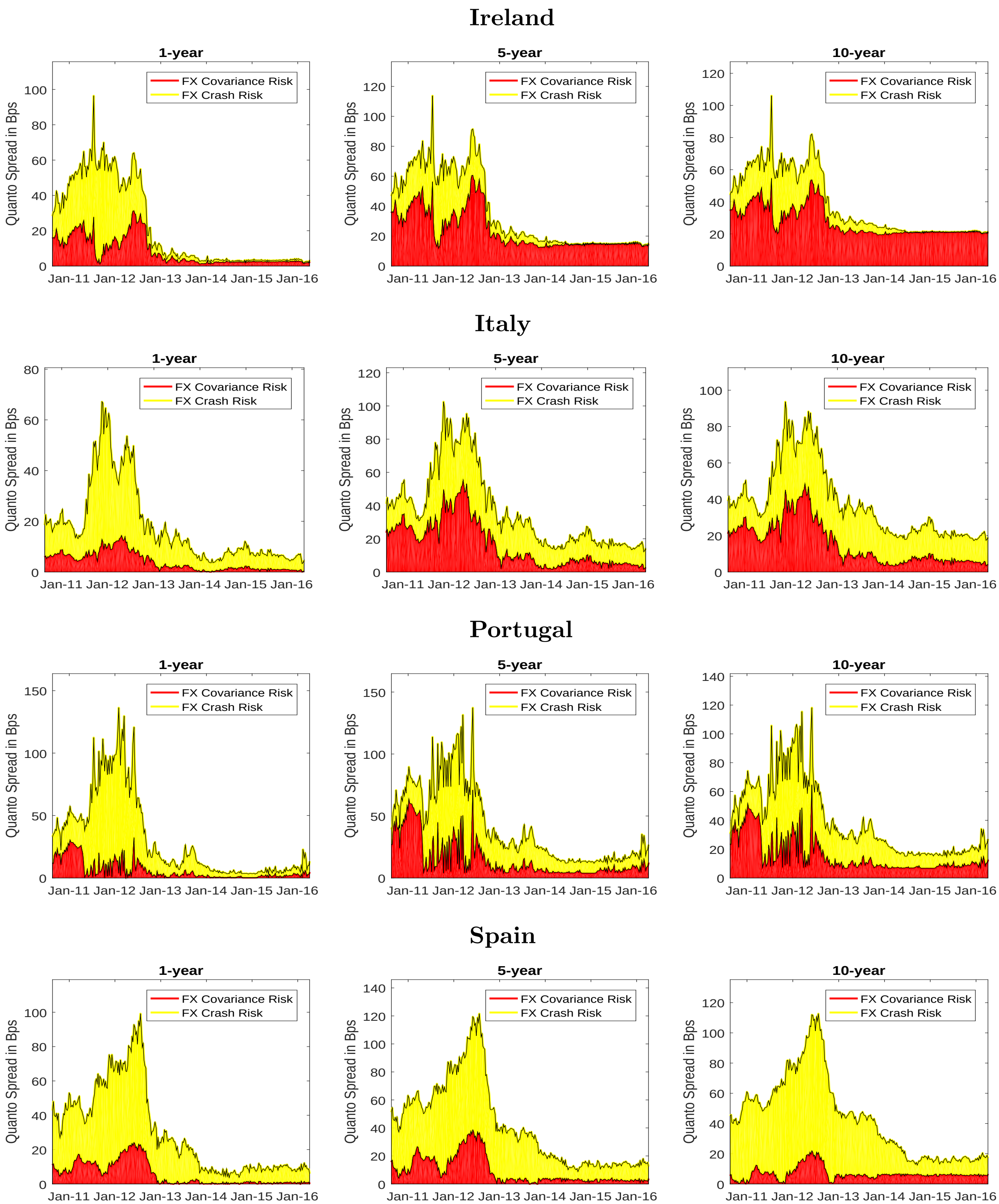


Figure 10: **Quanto spreads decomposed into covariance risk and currency crash risk.** This figure illustrates model decompositions of quanto CDS spreads—defined as the difference between USD and EUR-denominated CDS premiums—into a component driven by covariance between the exchange rate and default risk (orange) and a EURUSD jump risk component triggered by sovereign default (yellow). The illustrated maturities are 1, 5, and 10 years. The model is estimated via maximum likelihood estimation in conjunction with the unscented Kalman filter. The sample period is August 2010 to April 2016 and each time series consists of 281 weekly observations.

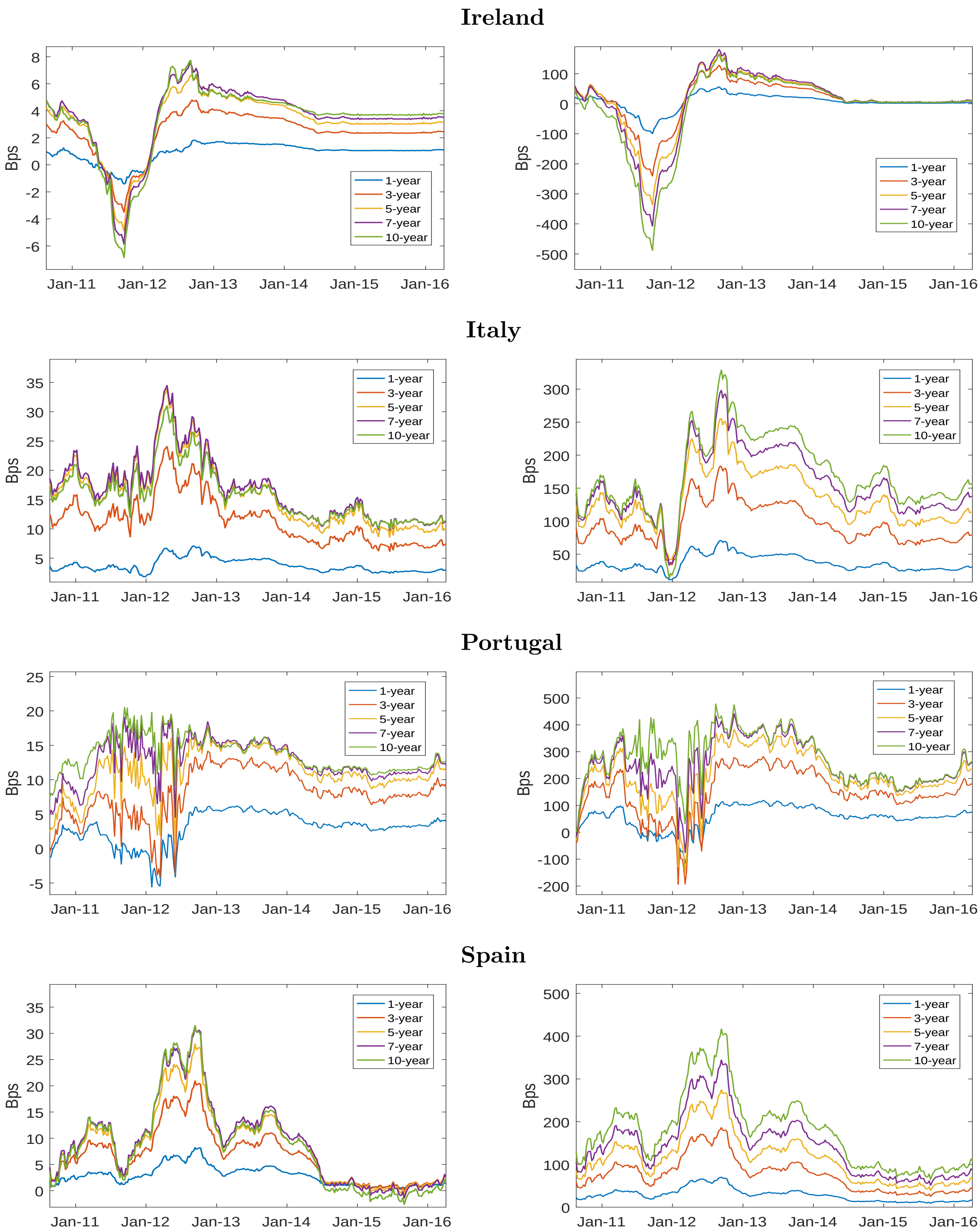


Figure 11: **Risk premiums for USD CDS and quanto CDS.** This figure shows the risk premiums associated with selling USD-denominated CDS (right panel) and the risk premiums associated with selling quanto CDS—defined as the difference between USD and EUR-denominated CDS premiums (left panel). The model is estimated via maximum likelihood estimation in conjunction with the unscented Kalman filter. The sample period is August 2010 to April 2016 and each time series consists of 281 weekly observations.

10 Appendix: Discrete-Time Model

10.1 Crash Risk Consistent with No-Arbitrage

Define the time t risk-neutral expectation of the exchange rate, i.e., the time t forward price:

$$E_t^Q(X_{t+1}) = F \tag{39}$$

Crash risk in the exchange rate upon default is modeled as follows. If default occurs between t and $t+1$, the exchange rate takes a hit of δ compared to the time t forward price

$$E_t^Q(X_{t+1}|\tau = t+1) = \delta E_t^Q(X_{t+1}) = \delta \cdot F \tag{40}$$

We refer to δ as the expected depreciation upon default or the crash risk parameter. Combining equations (39) and (40) gives

$$F = E_t^Q(X_{t+1}|\tau > t)Q(\tau > t+1|\tau > t) + E_t^Q(X_{t+1}|\tau = t)Q(\tau = t+1|\tau > t) \tag{41}$$

Rearranging,

$$E_t^Q(X_{t+1}|\tau > t+1) = \frac{1 - \delta Q(\tau = t+1|\tau > t)}{1 - Q(\tau = t+1|\tau > t)} F = \frac{1 - \delta\lambda_t}{1 - \lambda_t} F \tag{42}$$

Assume the exchange rate appreciates unconditionally with u with probability q and depreciates u^{-1} with probability $1-q$. Then we obtain an arbitrage-free model in each node by scaling the states of the exchange rate conditional on default with δ and the states of the exchange rate conditional on survival with $C(\lambda_t)$:

$$\frac{F}{X_t} = qu + (1-q)u^{-1} = \lambda_t\delta\Big(qu + (1-q)u^{-1}\Big) + (1-\lambda_t)C(\lambda_t)\Big(qu + (1-q)u^{-1}\Big)$$

Since each node is free of arbitrage, the entire model is free of arbitrage. Furthermore, for a given forward price, we see that

$$q = \frac{\frac{F}{X_t} - u}{u - u^{-1}}$$

10.2 Proofs in the Discrete-Time Model

10.2.1 Domestic CDS Premium

Define the unconditional mean default probability that prevails in the next period as $\bar{\lambda} = q^{\lambda}\lambda^{U} + (1 - q^{\lambda})\lambda^{D}$. Then we can express the CDS premium as:

$$S^d(0,T) = (1-R)\frac{P_d^2\bar{\lambda}\left(1-\bar{\lambda}\right)\left(\frac{1-\left(\left(1-\bar{\lambda}\right)P_d\right)^{T-1}}{1-\left(1-\bar{\lambda}\right)P_d}\right) + P_d\lambda_0}{\left(1-\lambda_0\right)P_d\left(\frac{\left(\left(1-\bar{\lambda}\right)P_d\right)^{T}}{1-\left(1-\bar{\lambda}\right)P_d}\right)} \tag{43}$$

Proof. In general, the discrete-time CDS premium in domestic currency with maturity T, $S^d(0,T)$, is given by:

$$S^d(0,T) = (1-R)\frac{\sum_{t=1}^{N} P_d(0,t)E_0^Q\left(1_{\tau=t}\right)}{\sum_{i=1}^{N} P_d(0,t)E_0^Q\left(1_{\tau>t}\right)} = \frac{\sum_{t=1}^{N} P_d^t E_0^Q\left(1_{\tau=t}\right)}{\sum_{i=1}^{N} P_d^t E_0^Q\left(1_{\tau>t}\right)} \tag{44}$$

The last equal sign follows from the assumption of a flat interest rate term structure such that $P_d(0,t) = P_d^t$, where P_d is a one-period domestic discount bond.

The survival probability up and until time t is straightforward to compute in the model, since the one-period survival probabilities are independent across time:

$$
\begin{aligned}
E_0^Q\left(1_{\tau>t}\right) &= Q_0\left(\tau>t\right) = (1-\lambda_0)E_0^Q\left(\prod_{i=1}^{t-1} 1-\lambda_i\right) = (1-\lambda_0)\prod_{i=1}^{t-1} E_0^Q\left(1-\lambda_i\right) \\
&= (1-\lambda_0)\prod_{i=1}^{t-1} 1-\left(q^{\lambda}\lambda^U + (1-q^{\lambda})\lambda^D\right) = (1-\lambda_0)\prod_{i=1}^{t-1}\left(1-\bar{\lambda}\right) \\
&= (1-\lambda_0)\left(1-\bar{\lambda}\right)^{t-1}
\end{aligned}
$$

For two periods or longer, we can express the default probability in terms of the difference between the survival probability up and until time $t-1$ and survival probability up to time t:

$$
\begin{aligned}
E_0^Q\left(1_{\tau=t}\right) &= Q_0\left(\tau>t-1\right) - Q_0\left(\tau>t\right) = (1-\lambda_0)\left(\left(1-\bar{\lambda}\right)^{t-2} - \left(1-\bar{\lambda}\right)^{t-1}\right) \\
&= \bar{\lambda}\cdot(1-\lambda_0)\left(1-\bar{\lambda}\right)^{t-2} \quad \text{for} \quad t\geq 2
\end{aligned}
$$

Plugging the premium and protection leg payments into (44) and by using the expression of a geometric series, we get

$$
\begin{aligned}
S^d(0,T) &= (1-R)\frac{P_d^2\bar{\lambda}\left(1-\lambda_0\right)\sum_{t=0}^{T-2} P_d^t\left(1-\bar{\lambda}\right)^t + P_d\lambda_0}{\left(1-\lambda_0\right)P_d\sum_{t=0}^{T-1} P_d^t\left(1-\bar{\lambda}\right)^t} \\
&= (1-R)\frac{P_d^2\bar{\lambda}\left(1-\bar{\lambda}\right)\left(\frac{1-\left(\left(1-\bar{\lambda}\right)P_d\right)^{T-1}}{1-\left(1-\bar{\lambda}\right)P_d}\right) + P_d\lambda_0}{\left(1-\lambda_0\right)P_d\left(\frac{\left(\left(1-\bar{\lambda}\right)P_d\right)^{T}}{1-\left(1-\bar{\lambda}\right)P_d}\right)}
\end{aligned}
$$

□

In the specific case when $\lambda_0 = \bar{\lambda}$, we have:

$$
\begin{aligned}
S^d(0,T) &= (1-R)\frac{P_d^2 \bar{\lambda}\left(1-\lambda_0\right)\sum_{t=0}^{T-2} P_d^t \left(1-\bar{\lambda}\right)^t + P_d \lambda_0}{\left(1-\bar{\lambda}\right) P_d \sum_{t=0}^{T-1} P_d^t \left(1-\bar{\lambda}\right)^t} \\
&= (1-R)\frac{P_d^2 \bar{\lambda}\left(1-\bar{\lambda}\right)\left(\frac{1-\left(\left(1-\bar{\lambda}\right)P_d\right)^{T-1}}{1-\left(1-\bar{\lambda}\right)P_d}\right) + P_d \lambda_0}{\left(1-\lambda_0\right) P_d \left(\frac{\left(\left(1-\bar{\lambda}\right)P_d\right)^{T}}{1-\left(1-\bar{\lambda}\right)P_d}\right)} \\
&= (1-R)\frac{P_d^2 \bar{\lambda}(1-\bar{\lambda})\left(1-\left(\left(1-\bar{\lambda}\right)P_d\right)^{T-1}\right) + P_d \bar{\lambda}\left(1-(1-\bar{\lambda})P_d\right)}{(1-\bar{\lambda})P_d\left(1-((1-\bar{\lambda})P_d)^T\right)} \\
&= (1-R)\frac{P_d \bar{\lambda}\left(P_d(1-\bar{\lambda}) - \left(\left(1-\bar{\lambda}\right)P_d\right)^T + (1-(1-\bar{\lambda})P_d)\right)}{P_d(1-\bar{\lambda})\left(1-\left(\left(1-\bar{\lambda}\right)P_d\right)^T\right)} \\
&= (1-R)\frac{\bar{\lambda}}{1-\bar{\lambda}}
\end{aligned}
\tag{45}
$$

10.2.2 Derivation of Foreign CDS Premium

In this section, we show that the expression for the discrete-time foreign CDS premium is:

$$
S^f(0,t) = (1-R)\frac{P_d^2\left(F-L\right)L_0\frac{1-(LP_d)^{T-1}}{1-LP_d} + P_d(F-L_0)}{P_d L_0 \frac{1-(LP_d)^T}{1-LP_d}} \tag{46}
$$

where $L_0 = F\left(1-\delta\lambda_0\right)$, $L = F(1-\delta\bar{\lambda}) - K\delta\rho\left(u-u^{-1}\right)\left(\lambda^U - \lambda^D\right)$ and $K = \sqrt{qq^{\lambda}(1-q)(1-q^{\lambda})}$.

Proof. When determining the foreign CDS premium, $S^f(0,T)$, we exchange the payment stream of the premium leg and the protection leg into units of domestic currency using M_t defined in (8):

$$
S^f(0,T) = (1-R)\frac{\sum_{t=1}^N P_f^t E_0^{Q^f}\left(1_{\tau=t}\right)}{\sum_{t=1}^N P_f^t E_0^{Q^f}\left(1_{\tau>t}\right)} = (1-R)\frac{\sum_{t=1}^N P_d^t E_0^{Q}\left(\frac{X_t}{X_0}1_{\tau=i}\right)}{\sum_{t=1}^N P_d^t E_0^{Q}\left(\frac{X_t}{X_0}1_{\tau>t}\right)}
$$

At each point in time, there are 4 possible states for the default probability, λ_t and the one-period relate changes in the exchange rate $\frac{X_{t+1}}{X_t}$: $((u, \tilde{\lambda}^1), (u, \tilde{\lambda}^0), (u^{-1}, \tilde{\lambda}^1), (u^{-1}, \tilde{\lambda}^0))$, which are reached with respective probabilities $(Q_{11}, Q_{10}, Q_{01}, Q_{00})$, where we have used the notation $\tilde{\lambda}^1 = \lambda^U$ and $\tilde{\lambda}^0 = \lambda^D$.

For each survival step, the exchange rate needs to be adjusted for the compensating factor defined as: $C(\lambda) = \frac{1-\delta\lambda}{1-\lambda}$ in order to preclude arbitrage opportunities. Important to mention is that the levels of the one-step survival probabilities are independent of one another, and so are the relative changes in the exchange rate. In summary, only the one-step changes in the exchange rate from t to $t+1$ and the default probability at time t are correlated (this is what gives us the FX/default covariance risk effect). These assumptions give us the following expression for the price of a defaultable foreign bond in terms of domestic currency:

$$
\begin{aligned}
P_f^t E_0^{Q^f}\left(1_{\tau>t}\right) &= P_d^t E_0^Q\left(X_t 1_{\tau>t}\right) \\
&= P_d^t E_0^Q\left(\prod_{k=0}^{t-1}(1-\lambda_k)C(\lambda_k)\frac{X_{k+1}}{X_k}\right) \\
&= P_d^t E_0^Q\left(\prod_{k=0}^{t-1}(1-\delta\lambda_k)\frac{X_{k+1}}{X_k}\right) \\
&= P_d^t\left(1-\delta\lambda_0\right) E_0^Q\left(\frac{X_1}{X_0}\right)\prod_{i=1}^{t-1} E_0^Q\left((1-\delta\lambda_k)\frac{X_{k+1}}{X_k}\right) \\
&= P_d^t\left(1-\delta\lambda_0\right) F\prod_{i=1}^{t-1}\left(\sum_{i,j=0,1} Q_{ij}(1-\delta\tilde{\lambda}^j)u^{2i-1}\right) \\
&= P_d^t\left(1-\delta\lambda_0\right) F\left(\sum_{i,j=0,1} Q_{ij}(1-\delta\tilde{\lambda}^j)u^{2i-1}\right)^{t-1}
\end{aligned}
\tag{47}
$$

Next, we calculate an expression for the last term in equation (47) by plugging in the

Q_{ij}s:

$$\begin{aligned}\left(\sum_{i,j=0,1} Q_{ij}(1-\delta\tilde{\lambda^j})u^{2i-1}\right) &= qu\Big(q^\lambda\left(1-\delta\lambda^U\right)+(1-q^\lambda)\left(1-\delta\lambda^D\right)\Big) \\ &+ (1-q)u^{-1}\Big(q^\lambda\left(1-\delta\lambda^U\right)+(1-q^\lambda)\left(1-\delta\lambda^D\right)\Big) \\ &+ quA_1\Big(\left(1-\delta\lambda^U\right)-\left(1-\delta\lambda^D\right)\Big) \\ &+ (1-q)u^{-1}A_0\Big(\left(1-\delta\lambda^D\right)-\left(1-\delta\lambda^U\right)\Big) \\ &= \Big(qu+(1-q)u^{-1}\Big)\Big(q^\lambda\left(1-\delta\lambda^U\right)+(1-q^\lambda)\left(1-\delta\lambda^D\right)\Big) \\ &+ quA_1\delta(\lambda^D-\lambda^U)-(1-q)u^{-1}A_0\delta(\lambda^D-\lambda^U) \\ &= F(1-\delta\bar{\lambda})-K\delta\rho\left(u-u^{-1}\right)\left(\lambda^U-\lambda^D\right)\equiv L \qquad (48)\end{aligned}$$

where $K=\sqrt{qq^\lambda(1-q)(1-q^\lambda)}$. In the last equal sign, we use the no-arbitrage condition of a one-period forward contract, $F=qu+(1-q)u^{-1}$, and the fact that $qA_1=(1-q)A_0=\rho\sqrt{qq^\lambda(1-q)(1-q^\lambda)}$.

Next step is to express $E_0^{Qf}\left(1_{\tau=t}\right)$ in terms of $E_0^{Qf}\left(1_{\tau>t}\right)$. First, from the derivations above, we can express the premium payments on the compact form:

$$P_f^t E_0^{Qf}\left(1_{\tau>t}\right)=\begin{cases}P_d^t L_0 & \text{if } t=1\\ P_d^t L_0 L^{t-1} & \text{if } t\geq 2\end{cases}$$

where $L_0=F\left(1-\delta\lambda_0\right)$ and L and K are defined above. In order to compute $E_0^{Qf}\left(1_{\tau=t}\right)$ for $t\geq 2$, in terms of domestic currency, we express it in terms of differences between defaultable zero-coupon bonds in foreign currency:

$$
\begin{aligned}
P_f^t E_0^{Q^f}\left(1_{\tau=t}\right) &= P_d^t E_0^Q\left(\frac{X_t}{X_0}1_{\tau=t}\right) \\
&= P_d^t E_0^Q\left(\frac{X_t}{X_0}1_{\tau>t-1}\right) - P_d^t E_0^Q\left(\frac{X_t}{X_0}1_{\tau>t}\right) \\
&= P_d^t L_0\big(F\cdot L^{t-2} - L^{t-1}\big) = P_d^t L_0 L^{t-2}\big(F-L\big)
\end{aligned}
$$

Above, we have used $E_0^Q\left(\frac{X_t}{X_0}1_{\tau>t-1}\right) = FE_0^Q\left(\frac{X_{t-1}}{X_0}1_{\tau>t-1}\right)$. Thus, we can express the protection leg payments on the following compact form:

$$
P_f^t E_0^{Q^f}\left(1_{\tau=t}\right) = \begin{cases} P_d^t \delta\lambda_0 F & \text{if } t=1 \\ P_d^t L_0 L^{t-2}\left(F-L\right) & \text{if } t\geq 2 \end{cases} \tag{49}
$$

We then obtain the expression for the foreign CDS premium in (46) by plugging in the compact form expressions for the premium and protection leg payments, and make use of the expression for a geometric series:

$$
\begin{aligned}
S^f(0,t) &= (1-R)\frac{\sum_{t=2}^T P_d^t L_0 L^{t-2}\left(F-L\right) + P_d\delta\lambda_0 F}{\sum_{t=1}^T P_d^t L_0 L^{t-1}} \\
&= (1-R)\frac{P_d^2\left(F-L\right)L_0\sum_{t=2}^T (P_d L)^{t-2} + P_d\left(F-L_0\right)}{L_0 P_d\sum_{t=1}^T (P_d L)^{t-1}} \\
&= (1-R)\frac{P_d^2\left(F-L\right)L_0\sum_{t=2}^T (P_d L)^{t-2} + P_d\left(F-L_0\right)}{L_0 P_d\sum_{t=1}^T (P_d L)^{t-1}} \\
&= (1-R)\frac{P_d^2\left(F-L\right)L_0\frac{1-(LP_d)^{T-1}}{1-LP_d} + P_d(F-L_0)}{P_d L_0\frac{1-(LP_d)^T}{1-LP_d}}
\end{aligned}
$$

□

10.2.3 Proof of Proposition 1 and 2

The domestic CDS premium is unaffected by changes in the severity in foreign currency at default, δ, hence all we need to show is that the foreign CDS premium in (46) is **increasing** in δ such that the quanto spread, $QS(0,T) = S^d(0,T) - S^f(0,T)$, is **decreasing** in δ.

Evidently both L_0 and L are decreasing functions in δ (holding any other parameters fixed), so if we can show that the CDS premium is decreasing in L_0 and L, we are done. First, we split the CDS premium up in two expressions:

$$\begin{aligned} S^f(0,T) &= (1-R)\frac{P_d^2\,(F-L)\,L_0\frac{1-(LP_d)^{T-1}}{1-LP_d} + P_d(F-L_0)}{L_0 P_d \frac{1-(LP_d)^T}{1-LP_d}} \\ &= \frac{(1-R)}{P_d}\left(\underbrace{(F-L)\,\frac{P_d^2\left(1-(LP_d)^{T-1}\right)}{1-(LP_d)^T}}_{A} + \underbrace{\frac{P_d\left(\frac{F}{L_0}-1\right)}{\frac{1-(LP_d)^T}{1-LP_d}}}_{B}\right) \end{aligned}$$

Next, we show that both A and B are decreasing in δ. Consider the expression A. Since the riskless bond is assumed to be more expensive than a risky bond, we have $F - L > 0$ and $1 - LP_d > 0$. This implies that A is decreasing in L if and only if $\frac{\left(1-(LP_d)^{T-1}\right)}{1-(LP_d)^T}$ is decreasing in L, since $F - L$ obviously is decreasing in L. We show that $\frac{\left(1-(LP_d)^{T-1}\right)}{1-(LP_d)^T}$ is indeed decreasing in L by defining the function:

$$f(m) = \frac{\left(1-(mP_d)^{T-1}\right)}{1-(mP_d)^T}$$

Differentiating f with respect to m yields

$$f'(m) = -\frac{(P_d m)^t\left((P_d m)^t - tP_d m + t - 1\right)}{P_d m^2\left((P_d m)^t - 1\right)^2}$$

From this expression, we see that f' is negative if and only if $(P_d m)^t - tP_d m + t - 1$ is positive, which is indeed the case, since this function is strictly convex with a minimum of 0 at $m = \frac{1}{P_d}$. Hence, we showed that the expression A is decreasing in M and hereby in δ as well.

An analogue argument can be used to show that $\left(\frac{1-(LP_d)^T}{1-LP_d}\right)^{-1} > 0$ is decreasing in L and hence in δ. Likewise is $\frac{F}{L_0} - 1 > 0$ and decreasing in L_0 and therefore in δ. Hence, the expression B is decreasing in δ as a product of two positive monotonically decreasing functions in δ.

The proof for Proposition 2 is conducted in an analogous manner to the proof of Proposition 1. In Proposition 1, we show that the quanto spread is decreasing in L, and since $L = F(1-\delta\lambda) - K\rho\,(u - u^{-1})\,\delta\left(\lambda^U - \lambda^D\right)$ is decreasing in ρ, then the foreign CDS premium increases in ρ. Evidently from the expression of L, L is increasing in $\lambda^U - \lambda^D$ if $\rho < 0$. The foreign CDS premium is therefore decreasing (increasing) in $\lambda^U - \lambda^D$ when $\rho < 0$ ($\rho > 0$).

10.2.4 Derivation of the Expressions (9)-(10)

First, the expression (9) follows immediately from (45) with $\lambda^U = \lambda^D = \lambda$ and $\bar{\lambda} = \lambda$, where λ is the fixed probability of default. Hence it follows that for any maturity T:

$$S^d(0,T) = (1-R)\frac{\lambda}{1-\lambda}$$

In order to derive the foreign-denominated CDS premium in the presence of crash risk and fixed default risk, we first notice that $L_0 = L = (1-\delta\lambda)\,F$. Inserting this into (46) gives

$$S^f(0,T) = (1-R)\frac{P_d^2 F\Big(1-\big(1-\delta\lambda\big)\Big)F\Big(1-\delta\lambda\Big)\frac{1-\Big(F(1-\delta\lambda)P_d\Big)^{T-1}}{1-\Big(F(1-\delta\lambda)P_d\Big)} + P_d\Big(F - F(1-\delta\lambda)\Big)}{F\Big(1-\delta\lambda\Big)\frac{1-\Big(F(1-\delta\lambda)P_d\Big)^{T-1}}{1-\Big(F(1-\delta\lambda)P_d\Big)}}$$

$$= (1-R)\frac{P_f^2\delta\lambda\,(1-\delta\lambda)\left(\frac{1-\big((1-\delta\lambda)P_f\big)^{T-1}}{1-(1-\delta\lambda)P_f}\right) + P_f\delta\lambda}{(1-\delta\lambda)\,P_f\left(\frac{\big((1-\delta\lambda)P_f\big)^{T}}{1-(1-\delta\lambda)P_f}\right)} = (1-R)\frac{\delta\lambda}{1-\delta\lambda}$$

The last equal sign follows from repeating the exact same calculations that led us to equation (45), with P_d replaced with P_f and $\bar{\lambda}$ replaced with $\delta\lambda$.

11 Appendix: Affine Model

11.1 Market Price of Risk

In this section we provide a proposition which help us specify the pricing kernel between the data-generating measure, the domestic measure, and the foreign measure. Cheridito et al. (2007) show that if an affine diffusion exists under a measure M^0, and it does not hit the boundary of the state space, then there also exists an affine diffusion under a measure M^1 which does not hit the boundary of the state space. More formally, they show (in the case of affine models without jumps) that if the drift and diffusion functions under M^0 and M^1 both satisfy the boundary non-attainment condition and the existence condition[9], then a true martingale exists defining the measure change from M^0 to M^1.

Lemma 1. *Assume that $\left(\mu^{M^0}, \sigma\right)$ and $\left(\mu^{M^1}, \sigma\right)$ satisfy the boundary non-attainment condition and the existence condition. Define the Radon-Nikodym derivative from M^0 to M^1:*

$$L_t = -L_{t-}\gamma_t dW_t^{M^0} + L_{t-}\sum_{i=1}^{K}(dZ_{i,t}^{M^0} + \lambda_{i,t}^{M^0}\zeta_i dt)$$

Where $dZ_{i,t}^{M^0}$ is a pure jump process with intensity λ_i and the jump size distribution with mean jump size ζ_i. The jump times for Z_i are serially and cross-sectionally independent. Define for $j = 0, 1$:

$$\mu^{M^j} : D \rightarrow \mathbb{R}^n,\ \mu^{M^j}(y) = a^{M^j} + b^{M^j}y,\ \sigma : D \rightarrow \mathbb{R}^{n\times n},\ \sigma(y)\sigma^T(y) = a_{ij} + b_{ij}y$$
$$L^{M^j} : D \rightarrow \mathbb{R}^n,\ L^{M^j}(y) = l_0^{M^j} + l_1^{M^j}y$$

[9] The existence criterium is a necessary restriction on μ, σ, λ and D in order for an SDE to have a solution. Essentially the matrix $\sigma(Y_t)\sigma^T(Y_t)$ has to be positive definite on the interior of the state space and positive semi-definite on the closure of state space. In order for the latter to be fulfilled the drift term has to be positive on the closure of D and $\sigma(Y_t)\sigma^T(Y_t)$ has to approach the 0-matrix. These two requirements make sure that $\sigma(Y_t)\sigma^T(Y_t)$ is positive definite on D and does not fail to be positive semi-definite on the closure of D. The boundary non attainment condition makes sure that the volatility for each coordinate in Y_t remains strictly positive. For a detailed discussion of the existence of a solution to SDEs, see Duffie and Kan (1996) and Cheridito et al. (2007).

Then the following three statements hold:

1. *There exists a stochastic process Y_t that solves the SDE:*

$$Y_t = Y_0 + \mu^{M^0}(Y_t)dt + \sigma(Y_t)dW_t^{M^0}$$

2. *There exists a measure M^1 equivalent to M^0 such that:*

$$Y_t = Y_0 + \mu^{M^1}(Y_t)dt + \sigma(Y_t)dW_t^{M^1}$$

3. *The jump intensities and drifts under M^0 and M^1 are related as:*

$$\lambda_i^{M^1}(Y_t) = (1+\zeta_i)\lambda_i^{M^0}(Y_t), \quad \mu^{M^1}(Y_t) = \mu^{M^0}(Y_t) - \sigma(Y_t)\gamma_t$$

Proof. Cheridito et al. (2007) show that continuous process: $dL_t^C = -\gamma_t dW_t^{M^0}$ is indeed a true martingale with $E_t^{M^0}(L_T^C) = 1$, provided that the existence and boundary non-attainment condition holds under both M^0 and M^1. The compensated jump process $Z_{i,t} + \lambda_{i,t}\zeta_{i,t}$ is also a true $\mathbb{M}^0$- martingale, since the mean jump size for each $Z_{i,t}$ is bounded and only exhibits a finite number of jumps. Hence the process $\sum_{i=1}^{K}(dZ_{t,i}^{M^0} + \lambda_{i,t}^{M^0}\zeta_i dt)$ is a true M^0-martingale since it is a finite sum of martingales. The process L_t is therefore a true martingale and hence 1.-3. follows from Girsanov's theorem for jump processes. □

11.2 Pricing of CDS in Affine Framework

11.2.1 Pricing of Domestic CDS

All the state-variables that are used to price the domestic CDS premium are independent. This makes the expressions for the ordinary differential much more simple, since the variance-covariance structure of the state-variables is a diagonal matrix. Therefore, we can represent the system of ordinary differential equations used for computing (17)-(18) for the domestic denominated CDS as

$$\frac{\partial\beta(t,T)}{\partial t} = \omega - K_1^T\beta(t,T) - \frac{1}{2}H\beta(t,T)\circ\beta(t,T), \quad \frac{\alpha(t,T)}{\partial t} = -K_0^T\beta(t,T) \tag{50}$$

$$\frac{\partial B(t,T)}{\partial t} = -K_1^T B(t,T) - \frac{1}{2}H\beta(t,T)\circ B(t,T), \quad \frac{A(t,T)}{\partial t} = -K_0^T B(t,T) \tag{51}$$

Where $\circ$ is the Hadamard product, and

$$\omega = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix},\ K_0 = \begin{bmatrix} \kappa_l\theta_l \\ 0 \\ \kappa_m\theta_m \end{bmatrix},\ K_1 = \begin{bmatrix} -\kappa_l & 0 & 0 \\ 0 & -\kappa_z & \kappa_z \\ 0 & 0 & -\kappa_m \end{bmatrix},\ H = \begin{bmatrix} \sigma_l^2 & 0 & 0 \\ 0 & \sigma_z^2 & 0 \\ 0 & 0 & \sigma_m^2 \end{bmatrix}$$

The boundary conditions are $\alpha(T,T) = 0$, $\beta(T,T) = [0,0,0]$, $A(T,T) = 0$ and $B(T,T) = [1,1,0]$

11.2.2 Pricing of Foreign CDS

The foreign CDS premium is a bit more involved than the domestic CDS premiums but also fits into the affine framework. Define the vector $\beta_j(t,T) = [\beta_v(t,T), \beta_l(t,T), \beta_z(t,T), \beta_m(t,T)]$, where $\beta_j(t,T)$ corresponds to the beta for state variable j, then the ordinary differential equation for state variable j is given by:

$$\frac{\partial\beta_j(t,T)}{\partial t} = \omega - K_1^T\beta_j(t,T) - \frac{1}{2}\beta_j(t,T)H_j\beta_j(t,T), \quad \frac{\alpha_j(t,T)}{\partial t} = -K_0^T\beta_j(t,T) \tag{52}$$

$$\frac{\partial B_j(t,T)}{\partial t} = -K_1^T B_j(t,T) - \frac{1}{2}\beta_j(t,T)H_j B_j(t,T), \quad \frac{A_j(t,T)}{\partial t} = -K_0^T B_j(t,T) \tag{53}$$

where:

$$\omega = \begin{bmatrix} 0 \\ (1+\zeta) \\ (1+\zeta) \\ 0 \end{bmatrix}, \; K_0 = \begin{bmatrix} \kappa_v^f \theta_v^f \\ \kappa_l \theta_l \\ 0 \\ \kappa_m \theta_m \end{bmatrix}, \; K_1 = \begin{bmatrix} -\kappa_v^f & 0 & 0 & 0 \\ \frac{1}{2}\sigma_l \rho \left(\frac{\theta_l}{\theta_v}\right)^{\frac{1}{2}} & \frac{1}{2}\sigma_l \rho \left(\frac{\theta_v}{\theta_l}\right)^{\frac{1}{2}} - \kappa_l & 0 & 0 \\ 0 & 0 & -\kappa_z & \kappa_z \\ 0 & 0 & 0 & -\kappa_m \end{bmatrix}$$

$$H_v = \begin{bmatrix} \sigma_v^2 & \frac{1}{2}\sigma_l \sigma_v \left(\frac{\theta_l}{\theta_v}\right)^{\frac{1}{2}} & 0 & 0 \\ \frac{1}{2}\sigma_l \sigma_v \left(\frac{\theta_l}{\theta_v}\right)^{\frac{1}{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \; H_l = \begin{bmatrix} 0 & \frac{1}{2}\sigma_l \sigma_v \left(\frac{\theta_v}{\theta_l}\right)^{\frac{1}{2}} & 0 & 0 \\ \frac{1}{2}\sigma_l \sigma_v \left(\frac{\theta_v}{\theta_l}\right)^{\frac{1}{2}} & \sigma_l^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$H_z = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_z^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \; H_m = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_m^2 \end{bmatrix}$$

The boundary conditions are $\alpha(T,T) = 0$, $\beta(T,T) = [0,0,0,0]$, $A(T,T) = 0$, and $B(T,T) = [0,(1+\zeta),(1+\zeta),0,0]$

12 Appendix: Estimation Approach

We estimate the model in two steps. In the first step, we apply maximum likelihood estimation (MLE) in conjunction with the unscented Kalman filter to infer a time-series of the instantaneous currency volatility process v_t and estimates of its risk-neutral and objective parameters ($[\kappa_v, \theta_v, \sigma_v, \kappa_v^P, \theta_v^P]$). We refer to section 12.1 for details on the Unscented Kalman filter and why we use this estimation approach. In this step, we only have one state variable, and the measurements consist of currency implied volatilities. We use a stochastic volatility model a la Heston (1993) as the currency options model, i.e, we assume that instantaneous currency volatility dynamics are unaffected by the jump components in the exchange rate arising from sovereign defaults specified in (15). Importantly, this does not mean that we ignore the correlation between sovereign credit and currency risk or the jump risk when pricing the sovereign CDS contracts, which is the focus of the analysis.

For pricing the currency options and CDS premiums, the discount factors in Euro and U.S. dollar are needed, which we bootstrap from their respective overnight index swap rates. The model-implied option prices are derived using the Fast Fourier Transform of Carr and Madan (1999) which we then transform into implied volatilities using the Garman and Kohlhagen (1983) formula such that they are comparable to the observables. We use implied volatilities rather than option prices since these are more stable than option prices along the moneyness and maturity dimension (see e.g., Schwartz and Trolle (2009)). Denoting x_t the time t state variable vector, then the measurement equation in the Kalman filter is given by

$$y_t = h(x_t) + e_t \tag{54}$$

where y_t is the vector of observables, $h(x_t)$ is the pricing function at state x_t, and e_t is the vector of measurement errors. In this particular case: $x_t = v_t$, y_t is the vector of observed implied volatilities, $h(x_t)$ is the vector of corresponding Heston (1993) implied volatilities, and e_t is a vector of IID Gaussian measurement errors with covariance matrix R. To reduce the number of parameters, we make the common assumption that the measurement errors are cross-sectionally uncorrelated (i.e., R is a diagonal matrix), and furthermore, we assume that the standard deviations of the measurement errors are identical for all options, σ_O.

We approximate the distribution of v_t with a Gaussian distribution such that the moments of the Gaussian distribution match the first two moments of v_t. All moments are computed by means of an Euler discretization, and we then cast the model into state space form

$$x_t = A + \phi x_{t-1} + \sqrt{Q_{t-1}}\varepsilon_t, \qquad \varepsilon_t \sim \mathrm{N(0,I)} \tag{55}$$

where in this particular case

$$A = \kappa_v^P \theta_v^P \cdot dt, \qquad \phi = e^{-\kappa^P dt}, \qquad Q_t = \sigma_v^2 v_t \cdot dt \tag{56}$$

Through the UKF iterations, we obtain $t-1$ predictions of the observables at time t, $\bar{y}_t$, and the corresponding prediction error covariance matrix $\bar{\Sigma}_{yy,t}$. With those at hand, we can then express the log-likelihood function using the prediction error decomposition

$$l(\Theta) = \sum_{t=1}^{N} -\frac{1}{2}\log|\bar{\Sigma}_{yy,t}| - \frac{1}{2}(y_t - \bar{y}_t)^T \bar{\Sigma}_{yy,t}^{-1}(y_t - \bar{y}_t) \tag{57}$$

where N is the number of observations, using weekly sampling we have $N = 281$ observations. We then find the maximum likelihood estimate of the parameters by maximizing (57).

In the second step, we estimate the parameters of the default intensities for one sovereign at the time using CDS premiums denominated in EUR and USD, now treating v_t as observable and its parameters as given. In this step, we use MLE in conjunction with the UKF to filter out the default intensity state variables, $[l_t, z_t, m_t]$, and to estimate their objective and risk-neutral parameters.

The measurements are the CDS premiums denominated in USD and the quanto CDS spread. In the pricing model, the USD contract is taken to be the domestic CDS contract, and the EUR contract is considered to be the foreign-denominated CDS contract. Their respective model-implied CDS premiums are henceforth derived according to (20), with the relevant transforms reported in Appendix, equations (51) and (53), respectively.

We assume that the measurement errors are the same for all maturities and for each type of contract, and we denote them σ_U and σ_{UE} for the USD-denominated CDS and the quanto CDS spread, respectively. The state space form of the discretized state variable dynamics, i.e., equation (55), is represented by the transition matrices

$$A = \begin{bmatrix} \kappa_l^P \theta_l^P \\ 0 \\ \kappa_m^P \theta_m^P \end{bmatrix} dt, \quad \phi = \begin{bmatrix} e^{-\kappa_l^P dt} & 0 & 0 \\ 0 & e^{-\kappa_z^P dt} & -\kappa_z^P dt \\ 0 & 0 & e^{-\kappa_m^P dt} \end{bmatrix}, \quad Q_t = \begin{bmatrix} \sigma_l^2 l_t & \sigma_v \sigma_l \sqrt{l_t v_t} & 0 \\ \sigma_v \sigma_l \sqrt{l_t v_t} & \sigma_z^2 z_t & 0 \\ 0 & 0 & \sigma_m^2 m_t \end{bmatrix} dt \tag{58}$$

With the model represented on state space form, we can then compute the maximum likelihood estimates by maximizing the log-likelihood function in (57).

Various specifications of the model above have been implemented, and our estimations reveal that it is important that the model allows for a drift adjustment for currency/default covariance risk which depends on the level of the default intensity. For instance, we implemented a simple affine model capturing default/currency covariance risk in which the systematic default intensity is a fixed fraction of the currency volatility: $\beta_i v_t$. This model has a closed form solution for the foreign CDS premium, without using any approximations. The problem with this specification, however, is that is not well-suited for handling differences in time trends in the credit spreads and the currency volatility. In the sample, the EURUSD currency volatility is persistent and exhibits strong mean-reversion, while sovereign eurozone default risk unambiguously trends downward during the latter period of the sample period.

12.1 The Unscented Kalman Filter

In the standard Kalman filter both the state vector equation and the measurement equation are linear in the state variables and both have Gaussian noise. To be specific, the (Gaussian) state space representation of such a system is:

$$x_t = A + \phi x_{t-1} + \sqrt{Q_{t-1}}\varepsilon_t, \qquad \varepsilon_t \sim \mathrm{N}(0,\mathrm{I}) \tag{59}$$

$$y_t = Hx_t + e_t, \qquad e_t \sim \mathrm{N}(0,\mathrm{R}) \tag{60}$$

x_t is the state vector and y_t are the measurements (in our case CDS premiums and option implied volatilities). We denote the forecasts at time $t-1$ of the state variables at time t and their covariance matrix as $\bar{x}_t$ and $\bar{\Sigma}_{xx,t}$, and $\hat{x}_t$ and $\hat{\Sigma}_{xx,t}$ are their updates at time t (updated based on new information inherit in y_t). $\bar{y}_t$ and $\bar{\Sigma}_{yy,t}$ represent the $t-1$ model forecast errors of the measurements at time t and their covariance matrix. The forecasts of the state variables and their covariance matrix are given by

$$\bar{x}_t = A + \phi \hat{x}_{t-1},\ \bar{\Sigma}_{xx,t} = \phi \hat{\Sigma}_{xx,t-1} \phi^T + Q_{t-1} \tag{61}$$

and the forecasts of the measurements and their covariance matrix, and their covariance with the state variables are given by:

$$\bar{y}_t = H \bar{x}_t,\ \bar{\Sigma}_{yy,t} = H \bar{\Sigma}_{xx,t} H^T + R,\ \bar{\Sigma}_{xy,t} = \bar{\Sigma}_{xx,t} H^T \tag{62}$$

The updated state variables and their covariance are calculated as

$$\hat{x}_t = \bar{x}_t + K_t(y_t - \bar{y}_t),\ \hat{\Sigma}_{xx,t} = \bar{\Sigma}_{xx,t} - K_t \bar{\Sigma}_{yy,t} K_t^T \tag{63}$$

where $K_t = \bar{\Sigma}_{xy,t} \bar{\Sigma}_{yy,t}^{-1}$. Given the (exponential) affine structure of the dynamics of the state vector, we can represent the discretized dynamics of the state variables as in (64) below with system matrices as specified in (58). Since neither the CDS premiums or options are linear in the state variables, the measurement equation (65) is governed by a non-linear function h:

$$x_t = A + \phi x_{t-1} + \sqrt{Q_{t-1}} \varepsilon_t, \qquad \varepsilon_t \sim \text{N(0,I)} \tag{64}$$

$$y_t = h(x_t) + e_t, \qquad e_t \sim \text{N(0,R)} \tag{65}$$

The UKF is one method for handling this non-linearity. In the UKF, the mean and covariance matrix of the forecasts of the measurement series and its covariance with the state variables are derived using a set of deterministic sampling points denoted sigma points, $\mathcal{X}_{t,i}$. The sigma points are chosen such that their mean and covariance match $\bar{x}_t$ and $\bar{\Sigma}_{xx,t}$, respectively. Based on the sigma points, new measurements, $\mathcal{Y}_{t,i}$, are generated $h(\mathcal{X}_{t,i}) = \mathcal{Y}_{t,i}$. From $\mathcal{Y}_{t,i}$, we then estimate the moments of the forecasts of the measurements as:

$$\bar{y}_t = \sum_{i=0}^{2p} w_i \mathcal{Y}_{t,i},\ \bar{\Sigma}_{yy,t} = \sum_{i=0}^{2p} w_i \left[\mathcal{Y}_{t,i} - \bar{y}_t\right] \left[\mathcal{Y}_{t,i} - \bar{y}_t\right]^T + R,\ \bar{\Sigma}_{xy,t} = \sum_{i=0}^{2p} w_i [\mathcal{X}_{t,i} - \bar{x}_t] \left[\mathcal{Y}_{t,i} - \bar{y}_t\right]^T$$

where the sigma points and the weights are defined as

$$\mathcal{X}_{t,0} = \bar{x}_t, \quad \mathcal{X}_{t,i} = \bar{x}_t \pm \sqrt{(p+\delta)\left(\Sigma_{xx,t}\right)_j} \ \ j = 1, \cdots, p,\ i = 1, \cdots, 2p$$
$$w_0 = \frac{\delta}{p+\delta}, \quad w_i = \frac{1}{2(p+\delta)}, \ \ j = 1, \cdots, 2p$$

where p is the dimension of the state vector and $\delta > 0$. We then use the Kalman filter as described above to obtain forecasts and updates of the state variables. Assuming normality of the forecast errors, we can use the forecast error decomposition of the log-likelihood function for the sample:

$$l(\Theta) = \sum_{t=1}^{N} -\frac{1}{2}\log|\bar{\Sigma}_{yy,t}| - \frac{1}{2}(y_t - \bar{y}_t)^T \bar{\Sigma}_{yy,t}^{-1}(y_t - \bar{y}_t)$$