

Quantessence

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Higher-Moment Models

One approach to the presence of skew in equity markets is to state, empirically, that the asset distribution at some maturity is not lognormal, and so characterised by just two parameters, but must as a minimum be described by a third moment ('skew'), a fourth ('kurtosis') and optionally a fifth. This is not a *model* in the sense that the shape of the implied volatility surface is explained by underlying processes (such as stochastic volatility and/or jump diffusions) but is just an empirical description of the asset distribution.

The usefulness of such a description lies in listed option markets, where a trader can know the typical range of parameters in which the market trades and use this to monitor changes. For example it might be observed that the kurtosis is relatively stable and typical market moves correspond largely to changes in volatility and skew.

For this reason, the [Five Moment](#) model is being incorporated into Option Viewer, a system developed in Sydney for market-making in electronic option markets. Already live in Sydney, shortly in Tokyo and later this year in New York, Option Viewer delivers real-time comparison of theoretical vs. market prices and automatic generation of orders based on user-defined spread/vol parameters. Global Quantitative Research has collaborated actively with the Sydney development team in model integration and other functional areas.

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Publications and Presentations

BOOKS

[Equity Derivatives and Market Risk Models](#)

Oliver Brockhaus, Michael Farkas, Andrew Ferraris, Douglas Long, and Marcus Overhaus
Risk Books, 2000.

[Modelling and Hedging Equity Derivatives](#)

Oliver Brockhaus, Andrew Ferraris, Christoph Gallus, Douglas Long, Reiner Martin, and Marcus Overhaus
Risk Books, 1999.

ARTICLES

[Pricing Quanto and Composite Equity Derivatives in Skewed Markets with Discrete Dividends](#)

Gero Schindlmayr
In preparation, 2000.

[Volatility Swaps Made Simple](#)

Oliver Brockhaus and Douglas Long
Risk, January 2000.

[Modelling, Hedging and Pricing Volatility](#)

Oliver Brockhaus and Douglas Long
J. Derivatives, submitted.

[Stochastic Volatility and Jump Processes](#)

Marcus Overhaus
In preparation, 2000.

[Generic Pricing Framework for Derivatives](#)

Michael Farkas, Andrew Ferraris, and Reiner Martin
J. Computational Finance, submitted.

CONFERENCES

[Latest Advances for Modelling and Hedging Volatility](#)

Marcus Overhaus
Risk 2000 Conference, April.

SEMINARS

Bath, Jun 99 & Dec 99 (workshop in London); C. Rogers.

Oxford, July 00 (workshop in London); N. Johnson.

Berlin Humboldt, April 00; H. Foellmer, M. Schweizer.

Bonn, Jul 99; S. Albeverio.

D'Evry, Feb 00; M. Jeanblanc

London Birkbeck, Feb 00; R. Kiesel.

Paris P&M Curie, Nov 99 & Jan 00; M. Yor, N. El-Karoui.



Quanto and Composite Options

Equity option contracts which make a payment in a currency different from the currency of the underlying are not uncommon in the OTC market.

The two principal types are the *quanto* and *composite* options. The quanto option pays out the payoff of the corresponding non-quanto option, converted into another currency at a pre-specified exchange rate. The composite option, by contrast, pays off an amount which depends on the value of the underlying converted into a second currency at the prevailing market rate.

More precisely, defining a functional $f(S)$ of the *path* of the asset which yields the payout of a single-currency (neither quanto nor composite) option of some type: this may be American, Asian or anything else. Since f is defined on the path of the asset, the approach applies fully to path-dependent options. We then assume that, for this single-currency case, we have an algorithm for pricing the option in the form of some function $\pi_f(S_0, r^d, d, \sigma^s)$ in terms of the domestic interest rate, dividend yield and volatility.

Given this, we define the composite option as paying $f(S/X)$ in which X is the exchange rate converting the domestic currency into foreign currency, and the quanto variant as paying $f(S)/x$ in which x is some fixed exchange rate (often the spot exchange rate at the trade date).

With these definitions, we may proceed to derive algorithms for the prices of the quanto and composite options. It transpires that these can be achieved by applying modified arguments to the π_f . This conclusion will be valid when volatility and interest rates are modelled as deterministic. See, for example, [Modelling and Hedging Equity Derivatives](#). A very similar approach can be taken when *local volatility* is deterministic (as in the Finite Difference framework). It requires extension for hybrid models (eg. Vasicek model for interest rates), and this has been carried out and is available

In the Black-Scholes framework, we have two processes to model – the equity, denoted by S , and the exchange rate, denoted by X . We model each process as a geometric Brownian motion:

$$\begin{aligned}\frac{dS}{S} &= \mu_t^S dt + \sigma_t^S dW^S \\ \frac{dX}{X} &= \mu_t^X dt + \sigma_t^X dW^X\end{aligned}$$

in which the drifts μ and volatilities σ are time-dependent and deterministic and W are standard Brownian motions, correlated with coefficient ρ .

From this it can be shown that the quanto and composite pricing formulae which result are given by the following well-known results:

$$\text{Composite: } \pi_f\left(\frac{S_0}{X_0}, r^f, d, \sigma^f\right)$$

Quanto:

$$\frac{1}{x} \pi_f(S_0, r^f, r^f - r^d + d - \rho \sigma^s \sigma^x, \sigma^s)$$

The fact that the algorithm π_f can, in many cases, be re-used, has a great impact on the tractability of implementing quanto and composite as features which can be applied to a wide range of models, as opposed to being developed separately from the corresponding single-currency model (compare the Imagine system). It transpires that, for the class of models to which this approach applies, a framework can be implemented which pre-processes the market information in such a way that the model implementations are blind as to whether the option priced is quanto, composite or vanilla. In fact, it is precisely the corrections to the drift

$$-\rho \sigma^s \sigma^x$$

for the quanto case, and to the volatility

$$(\sigma^f)^2 = (\sigma^x)^2 + (\sigma^s)^2 - 2\rho \sigma^x \sigma^s$$

which the pre-processing performs. There is no special logic for changing from domestic to foreign discounting, as we always discount in the currency of the option, we regard quanto and composite options as being denominated in the foreign currency, and it is in this currency that the result is quoted.

Library Support

Virtually all structures incorporated by the library can be priced in quanto and composite variants. The appropriate optional arguments are [Quanto=>](#) and [Composite=>](#), which are used in conjunction with named correlation, FX rate and FX Vol matrices (see [GED Set Matrix](#) and [CorrelationMatrix=](#) etc). This interface is very general and suited to books with a large number of positions: for quick-and-easy quanto and composite pricing, use [Quanto_](#) and [Composite_](#). Note particularly that, since the composite option sees the ADR process SX , its strike is entered in composite currency so that the payoff $\max(SX - K, 0)$ makes sense.

These can be applied to Black-Scholes and Finite Difference models. For quanto and composite pricing under Heston or hybrid models, contact Global Quantitative Research.



Hedging Composite Options

We can regard a composite option as a vanilla option on an ADR. Thus a US investor buying Deutsche Bank composite USD buys a true composite option, whereas for Nokia an ADR is listed in the US, so that an option trader on that stock in fact trades vanilla options, and has no awareness of EUR at all.

If no listed ADR exists, but a desk exists which provides the ADR to the trader, then this has the same effect. Consider the case, for illustration, of S&P composite in GBP: the option trader pays GBP for the S&P ADR; that amount (GBP) is immediately used to buy equivalent USD, which is used to buy S&P futures.

If we sell a call on S&P composite GBP, then regarding it as a vanilla option on an ADR, our delta-cash is (say) -50M GBP. Thus:

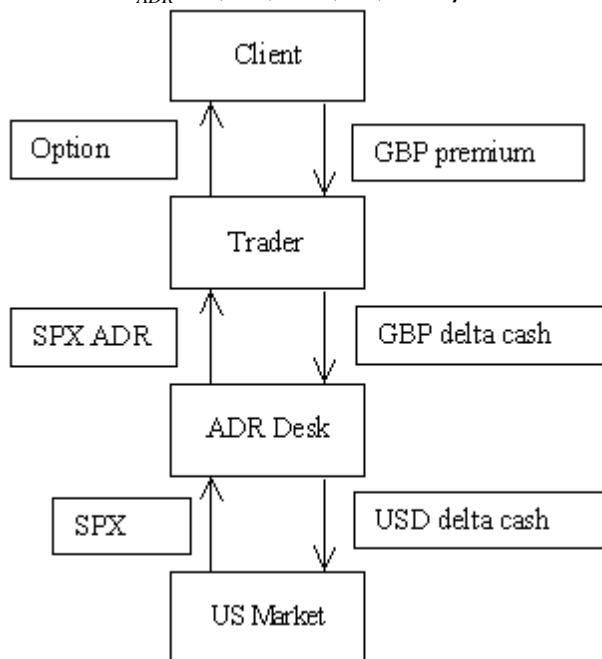
Option: Delta Cash = -50 M GBP

Hedge: + 50M GBP of S&P ADR
 = + 50M GBP equivalent of S&P
 and -50M GBP + USD

and it follows that we have an FX exposure on the option of + 50M GBP – USD.

Since the ADR volatility is different from the plain underlying volatility, the delta calculation must be carried out with the ADR volatility given, as above, by:

$$\sigma_{ADR}^2 = (\sigma^X)^2 + (\sigma^S)^2 - 2\rho\sigma^X\sigma^S$$



Hedging Quanto Options

Compared to the hedging of composites, the hedging of quantos is in some ways more complex. Consider the case of a trader who has sold a FTSE quanto USD call. He sells at fair price, and his P/L is zero USD. His delta hedge is bought in GBP, again at fair price, and again his P/L is zero.

Now consider a strengthening of the dollar, with FTSE unchanged: on the quanto USD option, the P/L is zero, as it is on the hedge. Surprisingly, therefore, there is no FX risk in this position. Of course, the hedge is now too small and must be increased.

Secondly, consider a 1% upward move in the FTSE, supposing the delta on the quanto option to be -100M USD. We lose 1M USD on the option, but since the hedge is +100M USD equivalent of FTSE in GBP, we make +1M USD equivalent of GBP. Our hedge is such that there is no overall P/L. *However*, there is now an FX exposure from the marked-to-market positions in dollars and sterling.

Thus:

- as the market moves up, the hedger sells local currency and buys quanto currency,
- as FX changes, he re-adjusts his delta hedge to have equivalent delta exposure.

P/L only arises from cross moves of the FX and equity, hence the importance of the equity/FX correlation in quanto option pricing.

To consider interest rate exposure, we consider our dollar liability (again in our example of FTSE quanto USD). This is given by the expected value of the option payout, and occurs at maturity. However the trader has been paid the discounted value of this expectation at trade date. The natural hedge is a zero coupon bond to maturity.

When the market moves, say by 1% up, the USD liability at maturity increases by $\Delta^*1\%$, compounded at US rates. A spot FX hedge, however, realises $\Delta^*1\%$ today, which again could be invested in a zero coupon bond to match the change in liability. The alternative is a forward FX trade on the increase in liability, which works well if the hedge maturity equals the option maturity. The problem with the forward FX hedge is that the spreads on FX forwards tend to be too large to make this competitive with the spot & “bond” hedge.

In a large book with many maturities, in practice it is most effective to hedge with swaps and spot FX trades, so the individual maturities are not matched but a macro-hedge done instead.

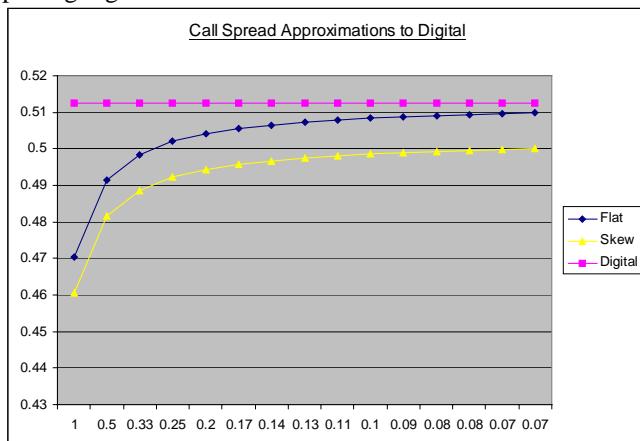


Option

Digital: Superficially one of the simplest options, the European Digital call (put) pays \$1 at maturity if the asset closes above (below) the barrier on that date. The Black-Scholes solution for the digital is well-known and implemented as [GED Digital](#). The most notable feature of the structure is, of course, the steep gradients which arise if the asset is in the neighbourhood of the barrier near the maturity date.

However, there are two reasons why a digital is never priced with the plain Black-Scholes solution, both related to the fact that it can be regarded as the limit of a narrow call spread.

Firstly, a desk wishing to avoid the extreme deltas generated by an actual digital, will often actually book and hedge a call spread. The second reason is that, in a skewed market, the limit of a narrow call spread gives a price which is *not* the same as in a non-skewed market (see graph below). Since this price is model-independent, it is an arbitrage price and for this reason the skew is always taken into account when pricing digitals.



The other type of digital, the American Digital, pays out if the barrier is hit at any time and is also skew-sensitive. The [Finite Difference](#) functions can be used to price this.

Utility "Store And Name"

The library incorporates a little-known but powerful utility which helps with handling large numbers of positions on spreadsheets, which traders and controllers routinely need to do. It allows users to decouple the process of specifying the static data needed to describe an option from the processes of using it for risk management, P&L reporting etc. In particular, it allows us to establish, in effect, a database of option definitions which may be shared across a desk (or more widely), while allowing spreadsheet users to build sheets which use that information in different ways. Without this, it would be necessary to have some mechanism for synchronising static data between many application spreadsheets. The intent is to allow spreadsheet + library to function much more like a complete application than a simple option calculator.

This is achieved through the optional argument [StoreAndName](#), which may be included in any GED pricing function (European, American, Bond, Convertible Bond, whatever). Its effect is to cause the library not to price the structure, but instead to remember its definition and associate it with a name. It can then be recalled and calculations performed using the generic pricing function [GED Price Product](#), which accepts all the usual optional arguments [Delta](#), [Gamma](#), etc.

This process of establishing a database of option definitions can be compared to the familiar Yield Curve and Underlyings spreadsheets, which establish definitions of market data objects in the memory of the library, and these definitions persist unchanged until Excel is closed or the definition is modified and the object re-created. (Note that the Yield Curve or Underlyings sheets can themselves be closed without affecting the objects, although real-time updates for interest rates would no longer be received). There is no real difference between this and storing static data defining options.

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Futures

Transactions in futures involve the obligation to make, or to take, delivery of the underlying asset of the contract at a future date, or in some cases to settle your position with cash. **They carry a high degree of risk.** The "gearing" or "leverage" often obtainable in futures trading means that a small deposit or down payment can lead to large losses as well as gains. It also means that a relatively small market movement can lead to a proportionately much larger movement in the value of your investment, and this can work against you as well as for you. **Futures transactions have a contingent liability, and you should be aware of the implications of this, in particular the margining requirements, which are set out in the paragraph below.**

Contingent liability transactions

Contingent liability transactions which are margined require you to make a series of payments against the purchase price, instead of paying the whole purchase price immediately.

If you trade in futures, contracts for differences or sell options you may sustain a total loss of the margin you deposit with your broker to establish or maintain a position. If the market moves against you, you may be called upon to pay substantial additional margin at short notice to maintain the position. If you fail to do so within the time required, your position may be liquidated at a loss and you will be liable for any resulting deficit.

Even if a transaction is not margined, it may still carry an obligation to make further payments in certain circumstances over and above any amount paid when you entered the contract.

Except in specific circumstances under SFA rules, your broker may only carry out margined or other contingent liability transactions with or for you if they are traded on or under the rules of a recognised or designated investment exchange. Contingent liability transactions which are not traded on or under the rules of a recognised or designated investment exchange may expose you to substantially greater risks.

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