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Pricing Default-Contingent Contracts with Deterministic Credit

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Abstract

We describe a simple framework to incorporate credit risk from a specific reference entity into the valuation of exotic or hybrid structured products. We also show how quanto credit-linked notes can be priced in this framework.

Keywords. Deterministic credit, credit-linked note, quanto, jump diffusion.

Contents

1 The Deterministic Credit Framework

A simple instrument such as a callable corporate bond is naturally susceptible to credit risk, and structured products like credit-linked notes explicitly embed default-contingent cashflows. A simple and generic approach to incorporating credit risk in the valuation of these instruments is to pick a suitable model for a hypothetically default-free version of the instrument, and to augment its state space by introducing an absorbing default state, whose probability of entering can be implied from a deterministic term-structure of CDS spreads. This is easily understood with a simple example. Suppose we have valued a callable bond using a Hull-White model for the interest rate risk. Let us denote this (default-free) value by $V(t)$. Now to incorporate credit risk into the valuation, we can strip out the cumulative risk-neutral probability of default $Q(\tau \leq t)$ from the issuer CDS curve, and assume a fixed recovery value V_R upon default as a fraction of par. In the deterministic credit framework, default is independent of the other risk factors, so we may write the default-able value $V_D(t)$ as the sum of the survival and recovery prices:

$$V_D(t) = Q(\tau > t)\, V(t) + Q(\tau \leq t)\, V_R\ . \tag{1.1}$$

In a more complex structured product, the V_R in the above equation could be replaced by a series of credit-riskless cashflows.

This simple, yet powerful, recipe is superior to credit risk-adjusted discounting with a funding spread (in which, for example, the credit risk does not impact exercise decisions of embedded call options), yet leverages existing model calibration procedures, and also circumvents the estimation of model parameters that are not readily priced in the market, *e.g.* credit volatility or credit correlation to other risk factors.

One can also price quanto credit-linked notes within the pricing framework (1.1). In this case, the default probability $Q(\tau \leq t)$ corresponds to the quanto-adjusted default probability. The details on how one implies this quanto-adjusted default probability from the standard CDS curve are given in the following section.

2 Quanto Adjustment

The vast majority of CDS trading is in USD, EUR and JPY, although a credit-linked structured product may be issued in different, local currency. For example, consider a credit-linked note denominated in IDR (Indonesian Rupiah) and credit-linked to the Republic of Indonesia. The note pays a potentially complex IDR coupon stream, which would be terminated prematurely if Indonesia defaults prior to maturity of the note. In the event of default, the note is redeemed at some assumed recovery rate.

Since CDS on Indonesia is natively quoted and traded in USD, one cannot use the default distribution in the USD numeraire (stripped out of USD CDS quotes) to compute a risk-neutral expectation of IDR cash-flows. This would be tantamount to assuming that the default of Indonesia, the USDIDR exchange rate, and the interest rates are all independent. However, we should expect an Indonesian default to cause the USDIDR exchange rate and the IDR interest rates to spike. This *quanto adjustment* requires a model in which the default event is dependent on the other market factors. In practice, the difference between the native CDS spreads and their quanto-adjusted versions is too large to be captured by purely diffusive processes.

Next, we describe a model in which default causes the exchange rate to jump. Then, we relate the quanto-adjusted default probability curve with the quanto CDS spreads. Thus, we use the model to fabricate quanto CDS spreads in which the corresponding quanto-adjusted default probability curve is input into the deterministic credit valuation framework for pricing quanto credit-linked notes.

2.1 Quanto Model

We consider a lognormal diffusion model for the exchange rate with a default-induced jump. This model was first studied in [KHFB], and more recently in [WC, BPP].

Let τ be the random default time, and $\mathcal{F}_t$ be the filtration generated by the standard Brownian motion $W_X(t)$. Let $\mathcal{G}_t = \sigma(\tau \le t) \vee \mathcal{F}_t$ denote the enlarged filtration that includes information about the default event. In the $\mathcal{P}$–measure denominated by the native currency of the standard CDS, if we define the *cumulative hazard rate* $\Lambda(t)$ as

$$\exp(-\Lambda(t)) = \mathrm{E}\left\{ \mathbf{1}_{\{\tau \le t\}} \,|\, \mathcal{F}_t \right\},$$

then the compensated default process given by

$$M(t) \;=\; \mathbf{1}_{\{\tau \le t\}} - \Lambda(t \wedge \tau), \tag{2.1}$$

is a $\mathcal{G}$-martingale under $\mathcal{P}$. We posit the following dynamics for the exchange rate process $X(t)$:

$$\frac{dX(t)}{X(t)} = (\, r(t) - r_q(t)\,)\; dt + \sigma_X(t)\, dW_X(t) + \alpha(t)\, dM(t), \tag{2.2}$$

where r and r_q are respectively the interest rates for the native currency and the quanto currency comprising the foreign exchange rate $X(t)$, $\sigma_X(t)$ is the time-dependent volatility of $X(t)$, $W_X(t)$ is the standard Brownian motion driving $X(t)$, and $\alpha(t) > -1$ is the deterministic fractional jump in $X(t)$ triggered by a default event. The solution to (2.2) is given by

$$X(t) = X(0)\, \exp\left(\int_0^t (\, r(u) - r_q(u)\,)\, du \right)\, M_X(t)\, M_\alpha(t),$$

where the continuous and jump components are given by

$$\begin{aligned} M_X(t) \;&=\; \exp\left(\int_0^t \sigma_X(u)\, dW_X(u) - \frac{1}{2} \int_0^t \sigma_X^2(u)\, du \right), \\ M_\alpha(t) \;&=\; \left(1 + \alpha_\tau \mathbf{1}_{\{\tau \le t\}} \right)\, \exp\left(- \int_0^{t \wedge \tau} \alpha_u\, d\Lambda(u) \right). \end{aligned}$$

In our setup, the native CDS currency is the quote currency and the quanto currency q is the base currency of the exchange rate $X(t)$. Denote by $\mathcal{P}^q$ the risk-neutral probability measure under the quanto currency. The change of measure from $\mathcal{P}$ to $\mathcal{P}^q$ is given by

$$\eta^q(t) \;=\; \frac{d\mathcal{P}^q}{d\mathcal{P}}(t) \;=\; \frac{X(t)\, \exp\left(\int_0^t r_q(u)\, du \right)}{X(0)\, \exp\left(\int_0^t r(u)\, du \right)} \;=\; M_X(t)\, M_\alpha(t).$$

Then η^q is a $\mathcal{G}$-martingale under the $\mathcal{P}$ measure as its differential form satisfies

$$\frac{d\eta^q(t)}{\eta^q(t)} \;=\; \sigma_X(t)\,dW_X(t) \;+\; \alpha(t)\,dM(t).$$

2.2 Quanto CDS Spreads

To infer the CDS spreads in the quanto currency q, we need to know the compensated default process M^q in the quanto currency. By change of measure, we have

$$\begin{aligned} dM^q(t) &= dM(t) \,-\, \eta^q(t)^{-1} d\big\langle M, \eta^q \big\rangle(t) \\ &= dM(t) \,-\, \alpha(t)\, d\big\langle M \big\rangle(t) \\ &= dM(t) \,-\, \alpha(t)\, d\Lambda(t \wedge \tau) \\ &= d\mathbf{1}_{\{\tau \le t\}} \,-\, (1+\alpha(t))\, d\Lambda(t \wedge \tau). \end{aligned}$$

Note that we have taken into account the fact that W_X is a continuous Brownian motion such that $d\big\langle M, W_X \big\rangle(t) = 0$. Assuming the cumulative hazard rate $\Lambda(t)$ is continuous, we may define $\lambda(t)$ such that $\Lambda(t) = \int_0^t \lambda(u)du$. Therefore, the hazard rate under the q numeraire is given by

$$\lambda^q(t) \;=\; (\, 1+\alpha(t)\,)\; \lambda(t).$$

By stripping CDS quotes in a bootstrap manner, a term structure of forward hazard rates $\lambda(0, T_i)$ can be obtained for progressively increasing maturities along the credit curve. The corresponding term structure of the quanto-adjusted forward hazard rates is simply

$$\lambda^q(0, T_i) \;=\; (\, 1+\alpha(0, T_i)\,)\; \lambda(0, T_i)\;, \tag{2.3}$$

where $\alpha(0, T)$ is the term structure of jump.

For quanto CDS valuation, let us consider the survival, default and accrual-to-default building block valuation formulas under the quanto currency:

$$\begin{aligned} s(T) &= \mathrm{E}^q \left\{ \exp\left(-\int_0^T r_q(u)\,du \right) \mathbf{1}_{\{\tau > T\}} \right\} \\ &= z^q(0,T)\, \exp\left(-\int_0^T \lambda^q(0,u)\,du \right)\;, && \text{(2.4a)} \\ d(T) &= \mathrm{E}^q \left\{ \exp\left(-\int_0^\tau r_q(u)\,du \right) \mathbf{1}_{\{\tau \le T\}} \right\} \\ &= \int_0^T z^q(0,s)\,\lambda^q(0,s)\, \exp\left(-\int_0^s \lambda^q(0,u)\,du \right) ds\;, && \text{(2.4b)} \\ a(t,T) &= \mathrm{E}^q \left\{ \exp\left(-\int_0^\tau r_q(u)\,du \right) (\tau - t)\, \mathbf{1}_{\{t < \tau \le T\}} \right\} \\ &= \int_t^T z^q(0,s)\,(s-t)\,\lambda^q(0,s)\, \exp\left(-\int_0^s \lambda^q(t,u)\,du \right) ds\;, && \text{(2.4c)} \end{aligned}$$

respectively. Note that $z^q(0,t)$ is the discount factor of time t for the quanto currency.

Thus, the dirty risky annuity $A(T)$ (which includes the valuation of accrual-to-default), and the default-leg $D(T)$, are given by

$$A(T) = \sum_{i=1}^{n} [\, s(T_i) + a(T_{i-1}, T_i) \,] \ , \tag{2.5a}$$

$$D(T) = (\, 1 - R \,)\, d(T) \ , \tag{2.5b}$$

where $T_1, \ldots, T_n$ are the future coupon payment dates, T_0 is the previous coupon payment date, and R is the fixed recovery rate due to default. Denoting by t_v the valuation date, the clean quanto CDS spread can be expressed as the ratio

$$\frac{D(T)}{A(T) - [t_v - T_0]} \, .$$

2.3 Numerical Example

We illustrate the impact on the quanto adjustment of CDS spreads in this model by means of a concrete example. Table 2.1 shows the CDS curve for the Republic of Indonesia in USD.

Maturity	Spread (bp)
06/20/2017	34.475
12/20/2017	39.965
12/20/2018	65.45
12/20/2019	92.575
12/20/2020	124.035
12/20/2021	154.41
12/20/2023	198.28
12/20/2026	221.855

Table 2.1: *USD CDS curve for Republic of Indonesia as of 2016/10/13.*

We want to convert the spreads from USD denomination to IDR denomination. In this case, the quote and base currencies are USD and IDR, respectively, and so the exchange rate is given by X=IDRUSD. We assume a time-independent jump size $\alpha(t) \equiv \alpha$, and perform this conversion for a range of jump sizes using $\alpha = -0.1, -0.2, -0.5$. The negative values for α reflect a depreciation of IDR in the event of an Indonesian default. For a flat credit spread, flat interest rate and continuous coupon, the CDS spread is proportional to the hazard rate. Hence, from (2.3), one expects the relative deviation between the quanto CDS spread and the standard CDS spread to be roughly equal to the constant jump size α. As illustrated below, Figure 2.1 corroborates this expectation.

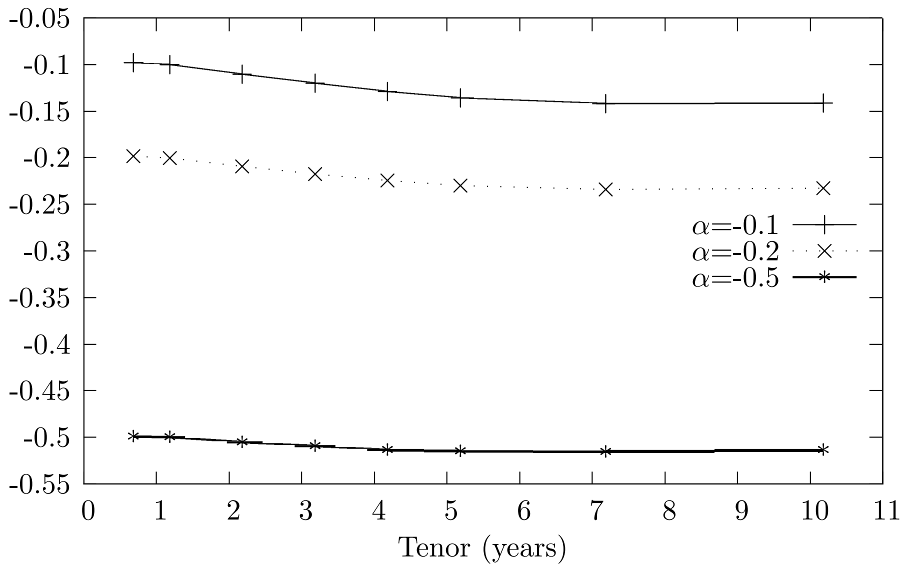

Figure 2.1: *Deviation of quanto-adjusted CDS spread relative to the USD CDS spread to different tenors for different values of α, the default-induced jump size on the IDRUSD exchange rate.*

3 Conclusion

The deterministic credit framework in DLIB currently supports multiple credit events. However, the credit events of different reference entities are assumed to be independent. In future work, we expect to extend the framework to support multi-name credit products with default correlations.

At present, we don't consider multi-currency or multi-entity deals, which is to say, only those quanto-credit deals depending on a single payment currency and a single credit entity are supported. In this pricing environment, we do not need to simulate FX rates. Therefore, the FX default-contingent jump model is currently used only to fabricate the quanto CDS spread data in which the quanto-adjusted default probability curve is passed to the deterministic credit valuation framework. In future releases, we may support general credit-linked products which involve multiple quanto credit entities in a multi-currency pricing environment.

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