

# Low Latency Interest Rate Markets

## Theory, Pricing & Practice



Nicholas Burgess

## PART ONE: Theory

### IR Markets, Products & Models

- Introduction to IR Markets
- Interest Rate Swaps
- IR Products & CDS
- Yield Curves
- IR Risk
- Credit Models

## PART TWO: Pricing & Practice

### Case Studies

- IRS Pricing Formulae
- IRS Pricing Case Study
- Asset Swap Structuring
- Asset Swap Pricing Case Study
- Pricing Tricks & Rules of Thumb

Quant Research Papers

<https://ssrn.com/author=1728976>

Support Materials: Quant Research, C++ and Excel Examples

<https://github.com/nburgessx/SwapsBook>

## PART ONE - THEORY

IR Markets, Products & Models

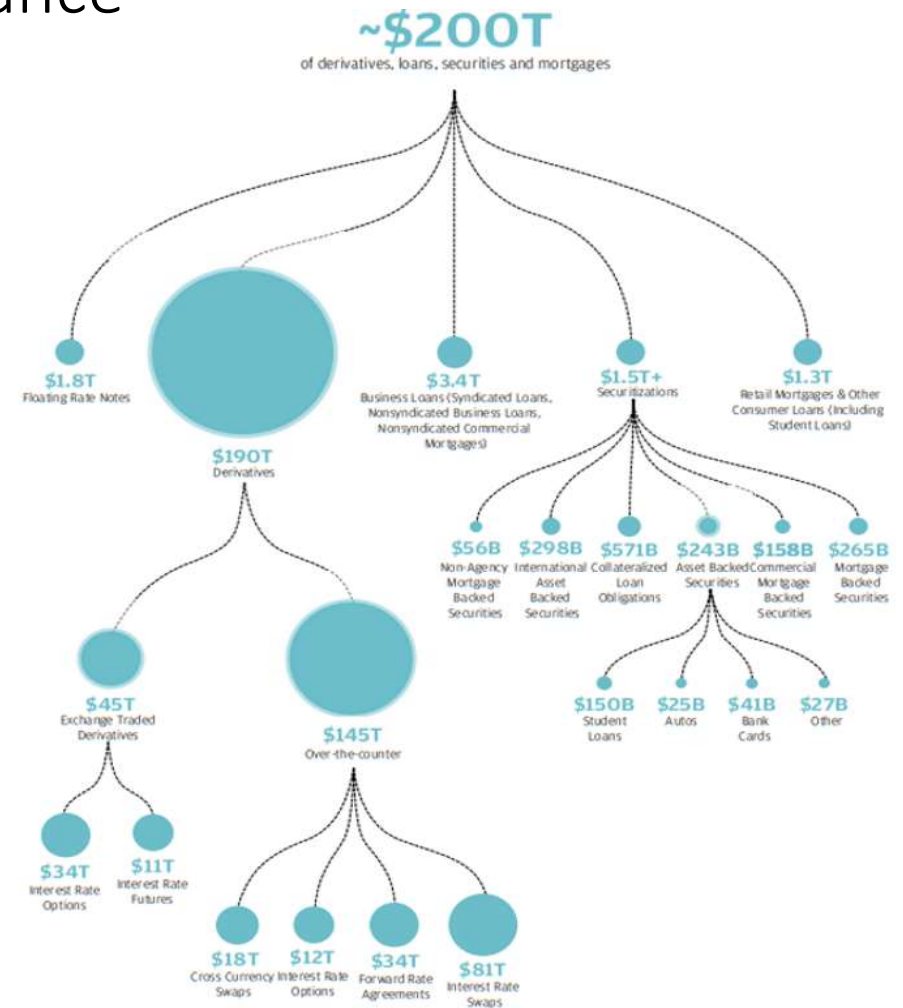
# Interest Rate Markets - Project Finance

## Purpose

- To Facilitate Government, Corporate & Project Finance
- Mortgages, Corporate Loans, Gov Projects & Infrastructure
- e.g. Hospitals, Transport (HS2), Energy & Defence Projects

## Market Size

- Market Size by Notional: \$200T (US) + \$150T (EU)
- Derivatives, Loans & Securities
- All Referencing LIBOR, until Recently



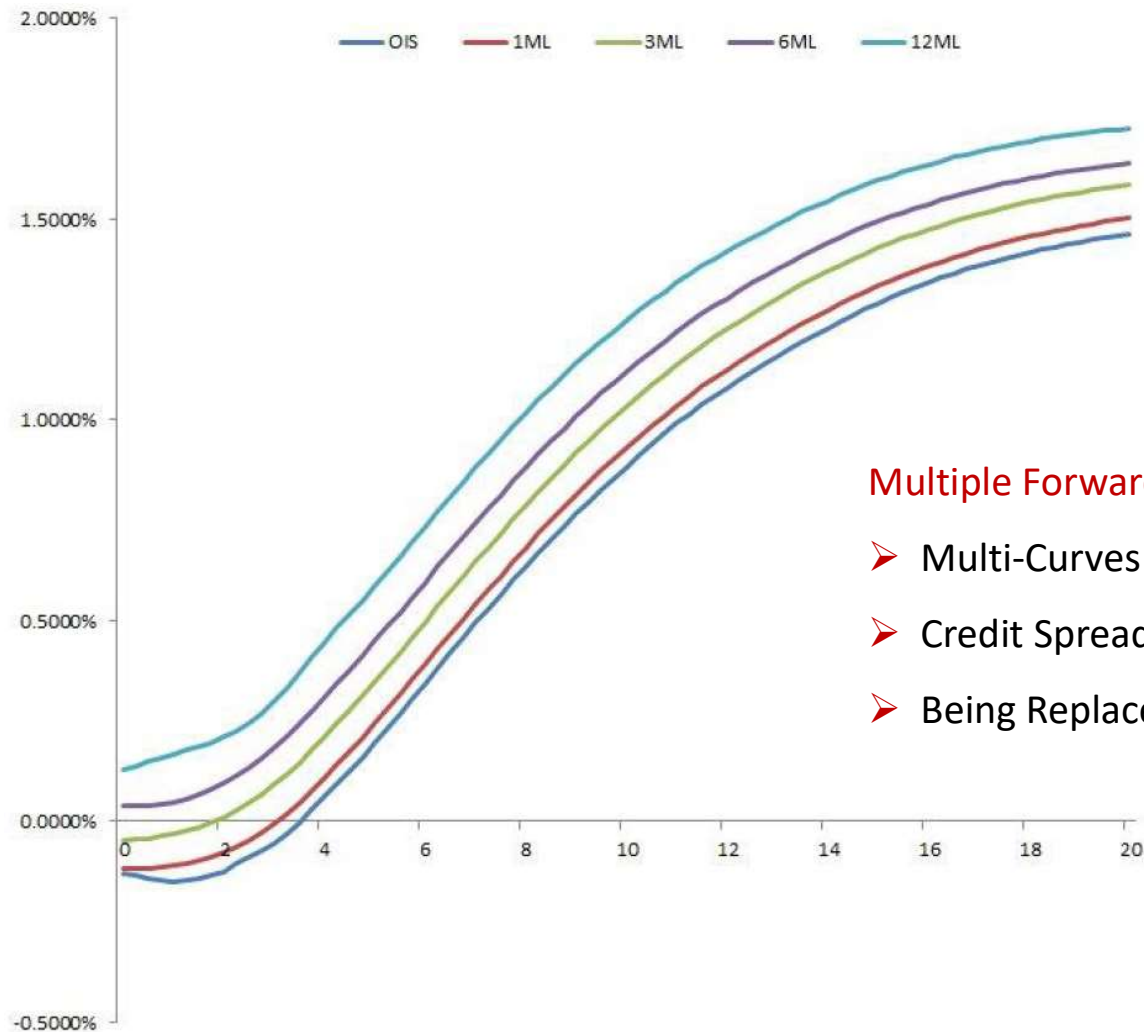


# Interest Rate Markets – Why the need for Speed?

- Cleared **Electronic Trading** & Auto-Hedging
- Real-Time, Highly Liquid & High Precision (Bid-Offer 1/10<sup>th</sup> bps i.e. USD 10 per MM)
- Trading Horizon: **High Frequency Trading** (HFT) vs Long-Term Fund Performance

USD Semi vs 3M Libor					USD Spreads vs Treasuries				
31) 1 Year	0.750	/	0.754	+0.014	71) 1 Year	4.282	/	5.295	+0.687
32) 2 Year	1.045	/	1.049	+0.017	72) 2 Year	10.248	/	10.806	-0.073
33) 3 Year	1.284	/	1.287	+0.018	73) 3 Year	3.337	/	3.895	-0.029
34) 4 Year	1.467	/	1.471	+0.015	74) 4 Year	1.350	/	1.900	+0.161
35) 5 Year	1.617	/	1.621	+0.014	75) 5 Year	-4.020	/	-3.454	+0.138
36) 6 Year	1.750	/	1.754	+0.012	76) 6 Year	-8.100	/	-7.550	+0.157
37) 7 Year	1.866	/	1.870	+0.011	77) 7 Year	-13.577	/	-13.036	+0.382
38) 8 Year	1.966	/	1.970	+0.011	78) 8 Year	-11.100	/	-10.550	+0.335
39) 9 Year	2.052	/	2.056	+0.011	79) 9 Year	-9.888	/	-9.088	+0.492
40) 10 Year	2.126	/	2.129	+0.011	80) 10 Year	-9.775	/	-9.275	+0.537
41) 12 Year	2.250	/	2.254	+0.007	81) 12 Year	2.520	/	3.320	+0.204
42) 15 Year	2.376	/	2.380	+0.006	82) 15 Year	-3.599	/	-2.799	+0.110
43) 20 Year	2.497	/	2.501	+0.002	83) 20 Year	-10.100	/	-9.600	+0.150
44) 25 Year	2.558	/	2.563	+0.003	84) 25 Year	-22.800	/	-22.250	+0.150
45) 30 Year	2.592	/	2.597	+0.000	85) 30 Year	-38.058	/	-37.491	+0.351
46) 40 Year	2.612	/	2.621	+0.003					
47) 50 Year	2.598	/	2.604	+0.004					

# Interest Rate Markets – Yield Curve Models



Required to Forecast Future Interest Rates

- Use **Liquid** Market Instruments
- To Imply Forward Rates & Disc. Factors

Multiple Forward Curves

- Multi-Curves Have In-Built **Credit Spread** (Tenor Homogenous)
- Credit Spread Determined by Loan Repayment Frequency
- Being Replaced by Single RFR Curves (Similar to OIS Curve)

# Interest Rate Markets – The LIBOR Problem

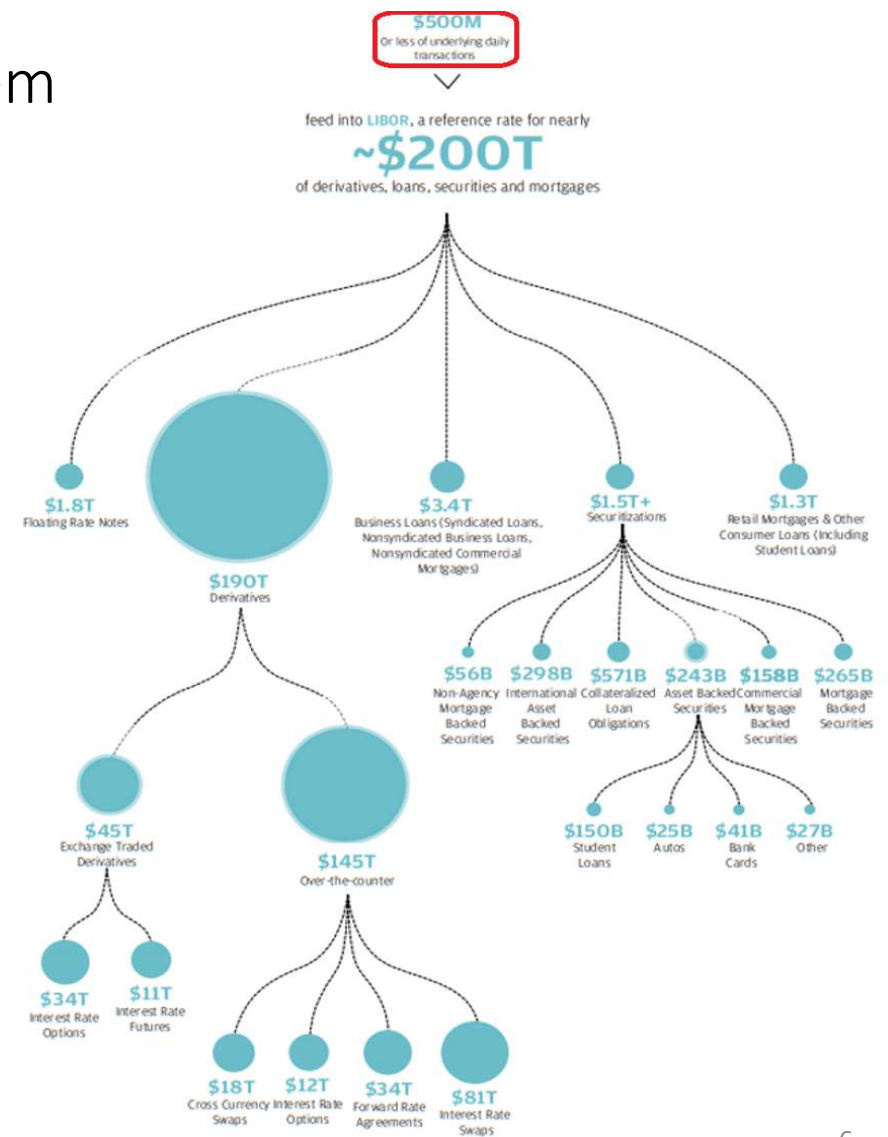
## The Problem with LIBOR

- LIBOR Market Transactions < \$500M
- Rates Do Not Reflect Actual Borrowing Levels
- LIBOR Levels Increasingly Set by Panel/Expert Judgement

## Market Size

- Market Size by Notional: \$200T (US) + \$150T (EU)

Large Market Driven by Small Number of LIBOR Transactions!!!



# Interest Rate Markets – LIBOR Benchmark Replacement

## LIBOR Rates

- Low Transaction Volume / Panel Based
- Forward Looking **Term Rate**, known **In-Advance**
- In Built Credit Risk Component

## Risk-Free Rates (RFRs)

- Transaction Based
- Backward Looking Rate, Known **In-Arrears**
- No Credit Component i.e. Risk-Free

## Market Changes

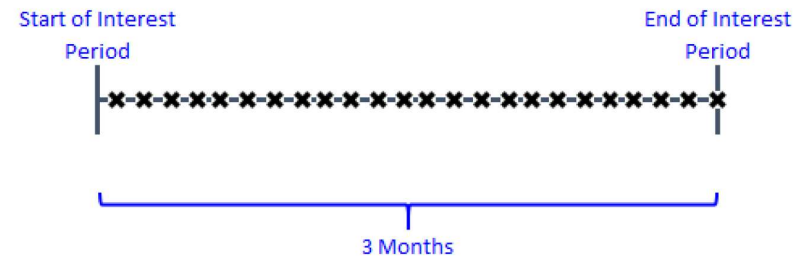
- Legacy LIBOR Contracts, Fall-Back Rates
- New RFR Products & Yield Curve Model Changes



**Rate:** Term Rate Fixed In-Advance

**Coupon:** Determined in Advance

### 3 Month Risk-Free Rate



**Rate:** Daily O/N Fixings leading to an Averaged Effective Rate

**Coupon:** Determined in Arrears



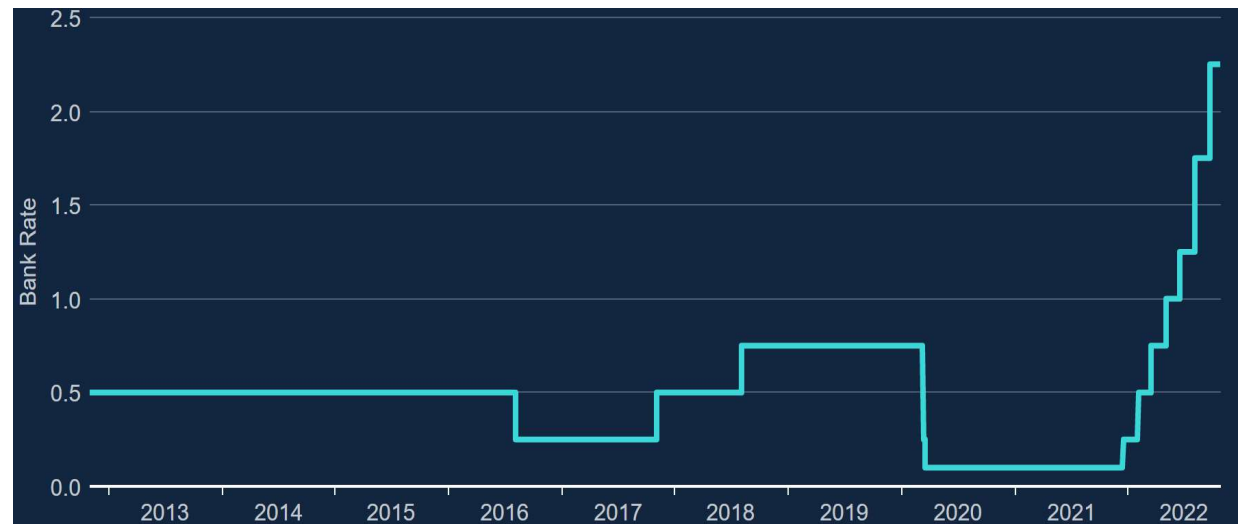
# Interest Rate Markets – Project Finance Risks & Solutions

## 1. Interest Rate Risk

- Finance linked to variable interest rates
- Use IRS to Fix Borrowing Costs

## 2. Foreign Exchange / Currency Risk

- International Finance
- Use Cross Currency Swaps to Fix FX Rates



## 3. Credit Default Risk

- Bonds, Bi-Lateral and Non-Cleared Transactions
- Risk of Counterpart Default
- Credit Default Swaps, Collateral & CSA Agreements

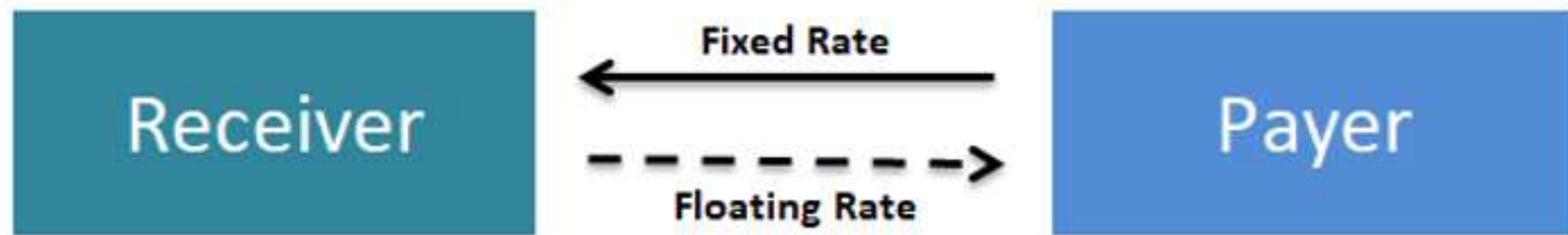
## 4. No money to invest?

- Use Asset Swaps to Borrow Funds to Invest in Bonds
- Pay LIBOR + Spread (Finance) to Receive Bond Coupons
- Floating Spread includes Funding + Credit Costs

# Interest Rate Swaps – Fixed or Variable Borrowing Costs?

## Project Finance

- Project Finance Naturally Incurs Variable Interest Costs (LIBOR + Spread)
- Exposed to Interest Rate Risk (Market may Move Against Us)



## Hedging Interest Rate Risk

- Use IRS to Exchange Floating for Fixed Interest (or Vice Versa)
- We Can Choose to Fix Borrowing Costs
- We Also Trade IRS for Speculative Purposes

## Interest Rate Swaps –Market Quotes & Pricing

USD Semi vs 3M Libor				USD Spreads vs Treasuries			
31) 1 Year	0.750 / 0.754	+0.014	≡	71) 1 Year	4.282 / 5.295	+0.687	
32) 2 Year	1.045 / 1.049	+0.017	≡	72) 2 Year	10.248 / 10.806	-0.073	≡
33) 3 Year	1.284 / 1.287	+0.018	≡	73) 3 Year	3.337 / 3.895	-0.029	≡
34) 4 Year	1.467 / 1.471	+0.015	≡	74) 4 Year	1.350 / 1.900	+0.161	
35) 5 Year	1.617 / 1.621	+0.014	≡	75) 5 Year	-4.020 / -3.454	+0.138	≡
36) 6 Year	1.750 / 1.754	+0.012	≡	76) 6 Year	-8.100 / -7.550	+0.157	
37) 7 Year	1.866 / 1.870	+0.011	≡	77) 7 Year	-13.577 / -13.036	+0.382	≡
38) 8 Year	1.966 / 1.970	+0.011	≡	78) 8 Year	-11.100 / -10.550	+0.335	
39) 9 Year	2.052 / 2.056	+0.011	≡	79) 9 Year	-9.888 / -9.088	+0.492	
40) 10 Year	2.126 / 2.129	+0.011	≡	80) 10 Year	-9.775 / -9.275	+0.537	≡
41) 12 Year	2.250 / 2.254	+0.007	≡	81) 12 Year	2.520 / 3.320	+0.204	
42) 15 Year	2.376 / 2.380	+0.006	≡	82) 15 Year	-3.599 / -2.799	+0.110	
43) 20 Year	2.497 / 2.501	+0.002	≡	83) 20 Year	-10.100 / -9.600	+0.150	
44) 25 Year	2.558 / 2.563	+0.003	≡	84) 25 Year	-22.800 / -22.250	+0.150	
45) 30 Year	2.592 / 2.597	+0.000	≡	85) 30 Year	-38.058 / -37.491	+0.351	≡
46) 40 Year	2.612 / 2.621	+0.003	≡				
47) 50 Year	2.598 / 2.604	+0.004	≡				

- Standard Tenors: Spread Over US Treasury Yields
- New Swaps: Par Rate (%), since PV=0
- Existing Swaps: Present Value (USD)

# Interest Rate Swaps – Present Value

The screenshot displays a financial software interface for configuring and valuing an Interest Rate Swap. The main configuration area is titled "Fixed Float Swap" and includes the following details:

- Deal:** Fixed Float Swap, Counterparty: SWAP CNTRPARTY, Ticker: / SWAP, Properties: 20) Properties
- Swap:**
  - Leg 1: Fixed:** Receive, Notional: 1MM, Currency: USD, Effective: 0D, Maturity: 08/25/2015, Coupon: 5.000000%, Pay Freq: SemiAnnual, Day Count: 30I/360, Calc Basis: Money Mkt.
  - Leg 2: Float:** Pay, Notional: 1MM, Currency: USD, Effective: 0D, Maturity: 08/25/2015, Index: 3M, Spread: 0.000 bp, Latest Index: 0.32910, Day Count: ACT/360, Reset Freq: Quarterly, Pay Freq: Quarterly.
- Market:** Dscnt: 42, M: USD Bloomberg Curv, Fwd: 23, M: USD Bloomberg Curv.
- Valuation Settings:** Curve Date: 08/21/2015, Valuation: 08/25/2015, OIS DC Strip: ON, CSA Coll Ccy: USD.

The bottom section, "Valuation Results", shows the following values:

Item	Value	Item	Value
Par Cpn	1.548250	Premium	16.78921
Principal	167,892.11	BP Value	1678.92112
Accrued	0.00	PV01	486.40
NPV	167,892.11	DV01	532.42
		Gamma (1bp)	0.29

**Present Value** is the Sum of **Discounted** Cash Flows

$$Swap\ PV = \underbrace{\sum_{i=1}^n N r \tau_i P(t_0, t_i)}_{\text{Fixed Cash Flows}} - \underbrace{\sum_{j=1}^m N (l_{j-1} + s) \tau_j P(t_0, t_j)}_{\text{Floating Cash Flows}}$$

## Interest Rate Swaps – Par Rate

- New Swaps Trade at Par i.e.  $PV = 0$
- Consequently such Swaps Quote as a Par Rate
- This is the fixed rate that makes both trade legs equal

$$Swap\ PV = \underbrace{r \sum_{i=1}^n N \tau_i P(t_0, t_i)}_{Fixed\ Cash\ Flows} - \underbrace{\sum_{j=1}^m N (l_{j-1} + s) \tau_j P(t_0, t_j)}_{Floating\ Cash\ Flows} = 0$$

Rearrange for the Fixed Rate  $r$  and call this the Par Rate,  $p$

$$Par\ Rate, p = \frac{PV(Float\ Leg)}{\sum_{i=1}^n N \tau_i P(t_0, t_i)} = \frac{PV(Float\ Leg)}{Annuity(Fixed\ Leg)}^1$$

<sup>1</sup> Par Rates calculated in terms of Annuity or PV01



# Interest Rate Swaps - Specification

- Majority of Swap Booking Schedule Related
- Trading **Templates**, Generators & Static Data

Swap Generator Template			
USD_SWAP_3M			
Dynamic Trade Info	LEG TYPE	LEG1:FIXED	LEG2:FLOAT
	PAY / RECEIVE	PAY	RECEIVE
Static Data + Schedule Info	NOTIONAL	1,000,000	1,000,000
	FIXED RATE (%)	1.00%	-
	FLOAT SPREAD (BPS)	-	0.00
	EFFECTIVE DATE / LAG	2D	2D
	MATURITY DATE / TENOR	2Y	2Y
	LEG CURRENCY	USD	USD
	NOTIONAL EXCHANGE	NONE	NONE
	LEVERAGE	1.00	1.00
	FRONT STUB INDEX	-	NATURAL
	BACK STUB INDEX	-	NATURAL
	VALUATION CURRENCY	USD	USD
	FORECAST INDEX	-	USD3M
	DISCOUNT INDEX	USDOIS	USDOIS
	INDEX COMPOUND METHOD	-	NONE
	SPREAD COMPOUND METHOD	-	NONE
	ROLL DAY	END	END
	STUB TYPE	SHORT START	SHORT START
	FIXING BUS DAY ADJUSTMENT	-	MODIFIED_FOLLOWING
	FIXING CALENDAR	-	NY+LDN
	FIXING LAG	-	2D
	FIXING IN-ADVANCE / IN-ARREARS	-	IN-ADVANCE
	ACCRUAL FREQUENCY	SEMI-ANNUAL	QUARTERLY
	ACCRUAL BUS DAY ADJUSTMENT	MODIFIED_FOLLOWING	MODIFIED_FOLLOWING
	ACCRUAL CALENDAR	NY	NY
	ACCRUAL DAYCOUNT	30/360	ACT/360
	PAYMENT FREQUENCY	SEMI-ANNUAL	QUARTERLY
	PAYMENT BUS DAY ADJUSTMENT	MODIFIED_FOLLOWING	MODIFIED_FOLLOWING
	PAYMENT CALENDAR	NY	NY
	PAYMENT LAG	2D	2D

	TRADE PARAMETERS		LEG1	LEG2
TRADE ECONOMICS	LegType		FLOAT	FLOAT
	Currency		EUR	USD
	Notional		8,769,622	10,000,000
	NotionalExchange		ALL	ALL
	PayReceive		PAY	RECEIVE
	EffectiveDate		Fri, 26-Oct-18	Fri, 26-Oct-18
	MaturityDateOrTenor		1Y	1Y
	FixedRate (%)		-	-
	FloatSpread (Bps)		0.00	0.00
	IndexCompoundMethod		-	NONE
	SpreadCompoundMethod		-	NONE
	Leverage		1.00	1.00
MTM SWAPS	ForecastCurve		EUR3M	USD3M
	DiscountCurve		EURDF_USDCSA	USDDF
	isMTMResetLeg		FALSE	TRUE
	ResetBaseFX		1.00000	1.14030
	ValuationCurrency		USD	USD
COUPON & STUB CONVENTIONS	CouponRollDay		NATURAL	NATURAL
	isEndOfMonth		TRUE	TRUE
	StubType		SHORT_START	SHORT_START
	FrontStubCurveIndex		NATURAL	NATURAL
	BackStubCurveIndex		NATURAL	NATURAL
	FrontStubDate		-	-
	BackStubDate		-	-
SCHEDULE INFORMATION	AccrualFrequency		QUARTERLY	QUARTERLY
	AccrualCalendar		TGT+NY+LON	TGT+NY+LON
	AccrualBusDayConv		MOD_FOLLOWING	MOD_FOLLOWING
	AccrualDaycount		ACT/360	ACT/360
	IRFixingBusDayConv		MOD_FOLLOWING	MOD_FOLLOWING
	IRFixingCalendar		TGT+NY+LON	TGT+NY+LON
	IRFixingLag		2D	2D
	IRFirstFixingLag		-	-
NON-DELIVERABLES	PaymentFrequency		QUARTERLY	QUARTERLY
	PaymentBusDayConv		MOD_FOLLOWING	MOD_FOLLOWING
	PaymentCalendar		TGT+NY+LON	TGT+NY+LON
	PaymentLag		2D	2D
	IsNonDeliverable		FALSE	FALSE
	SettlementCurrency		-	-
	FXFixingLag		-	-
	FXFixingBusDayConv		-	-
	FXFixingCalendar		-	-

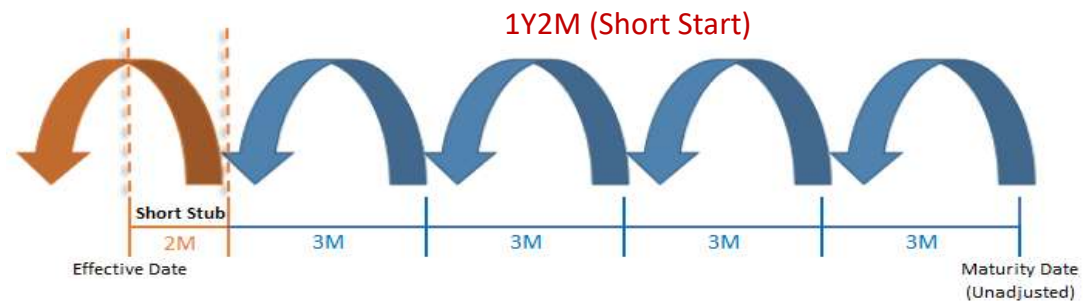
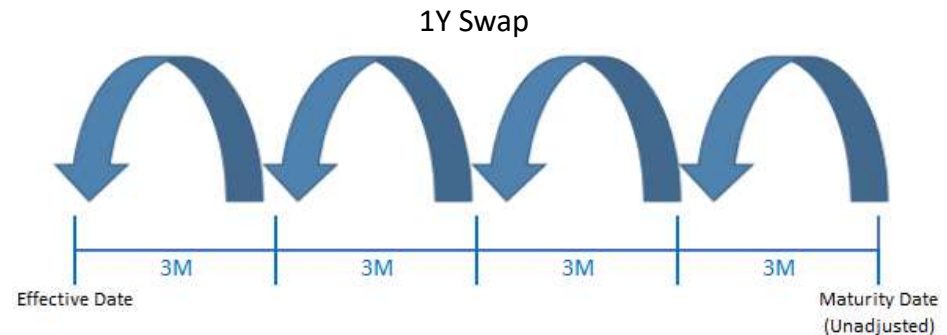
# Interest Rate Swaps - Schedules & Stubs

## Swap Schedules

- Backwards vs Forward Rolling Schedules
- Unadjusted to Preserve Roll Day
- Holiday Adjustments Ex-Ante
- Accrual Day Count Conventions

## Broken-Dated Swaps

- Stubs & Stub Rates (Linear Interp)
- Short Start/End, Long Start/End
- Market Default: **Short Start**



# IR Products – Tenor & Xccy Basis Swaps

## Tenor Basis Swaps

- Float vs Float (Same Currency)
- Exchange USD3M for USD6M say
- Match Project Cash Flow Frequency

Tenor Basis Swap Formulae (December 30, 2015).

Available at SSRN: <https://ssrn.com/abstract=2959605>

## Xccy Basis Swaps

- Float vs Float (Different Currencies)
- Exchange USD3M for EUR3M say
- Marked-to-Market / FX Notional Resets
- Reduces XVA Costs

An Illustrated Step-by-Step Guide of How to Price Cross Currency Swaps (November 11, 2018).

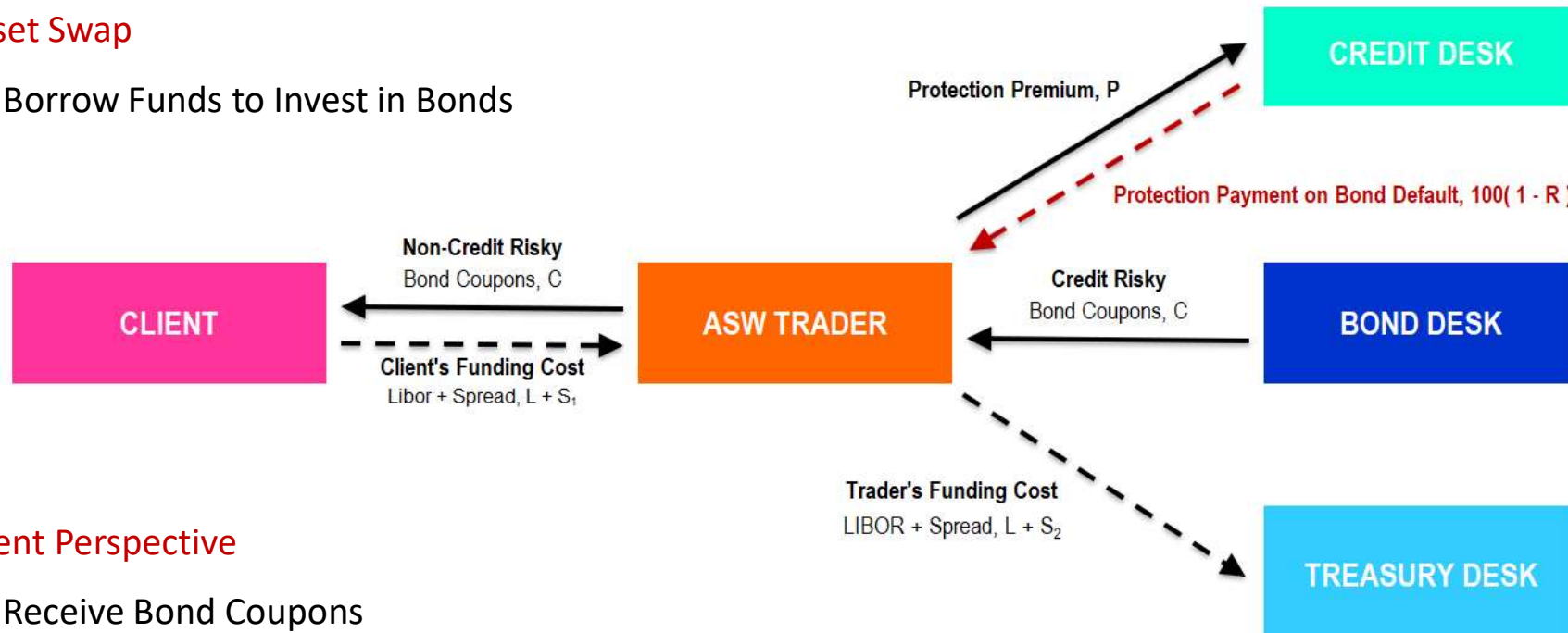
Available at SSRN: <https://ssrn.com/abstract=3278907>

91) Actions ▾	92) Products ▾	93) Views ▾	94) Info ▾	95) Settings ▾	Swap Manager
Solver (Premium) ▾	Load	Save	Trade ▾	CCP ▾	
3) Main	4) Details	5) Curves	6) Cashflow	7) Resets	9) Scenario
10) Risk	12) Matrix				
Deal	MTM XCCY Swap	Counterparty	SWAP CNTRPARTY ▾	Ticker / SWAP	20) Properties
Swap	*Notional Reset b...	3 Month Euribor	Valuation Settings		
Leg 1:Float	Receive ▾	Leg 2:Float	Pay ▾	Curve Date	03/22/2019
Notional	1MM	Notional	884,799.15	Valuation	03/26/2019
Currency	USD ▾	Currency	EUR ▾	CSA Coll Ccy	USD ▾
Effective	0D 03/26/2019	Effective	0D 03/26/2019	Valuation Ccy	USD ▾
Maturity	1Y 03/26/2020	Maturity	1Y 03/26/2020	FX Rate	1.130200
Index	3M US0003M	Index	3M EUR003M	<input checked="" type="checkbox"/> OIS DC Stripping	
Spread	0.000 bp	Spread	-12.625 bp		
Leverage	1.00000	Leverage	1.00000		
Latest Index	2.60988	Latest Index	-0.30900		
Reset Freq	Quarterly ▾	Reset Freq	Quarterly ▾		
Pay Freq	Quarterly ▾	Pay Freq	Quarterly ▾		
Day Count	ACT/360 ▾	Day Count	ACT/360 ▾		
Market					
Leg 1: NPV	1,002,566.12	Leg 2: NPV	-1,002,566.12		
Accrued	0.00	Accrued	0.00		
Premium	100.26	Premium	-100.26		
DV01	22.74	DV01	-22.74		
Valuation Results				22) Calculators ▾	
Principal	0.00	Premium	0.00000	BR01 92:EUR vs.	-102.10
Accrued	0.00	BP Value	0.00000	DV01	0.00
NPV	0.00			Gamma (1bp)	0.00

# IR Products – Asset Swaps

## Asset Swap

- Borrow Funds to Invest in Bonds



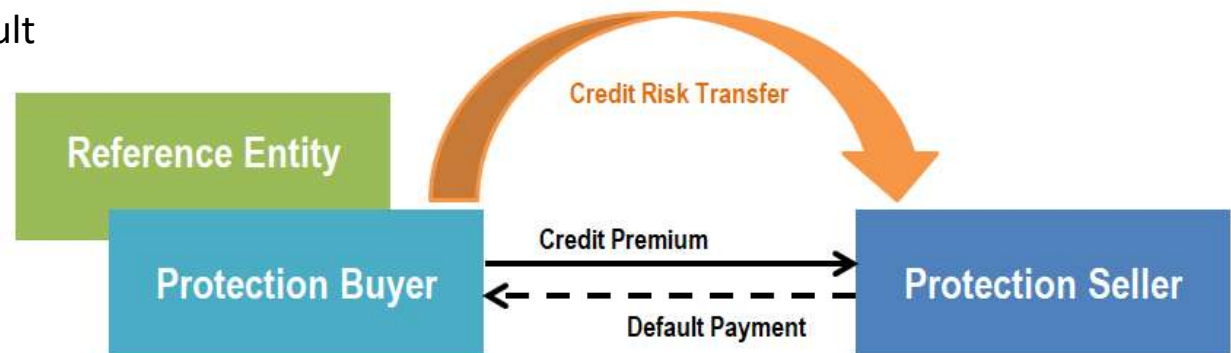
## Client Perspective

- Receive Bond Coupons
- Pay LIBOR + Spread
- Spread Includes Finance + Credit Costs

# IR Products – Credit Default Swaps (CDS)

## Insurance Against Counterparty Default

- Insuring Bond Notional Invested
- Pay Fixed Insurance Premium
- Receive Protection Payment on Default



## Credit Crisis & ISDA Big Bang (2008)

- Standardized & Cleared Contracts (IMM Dates<sup>1</sup>)
- Increased Liquidity
- Accrued Interest, Clean & Dirty Prices

<sup>1</sup> Third Wednesday of Mar, June, Sep and Dec

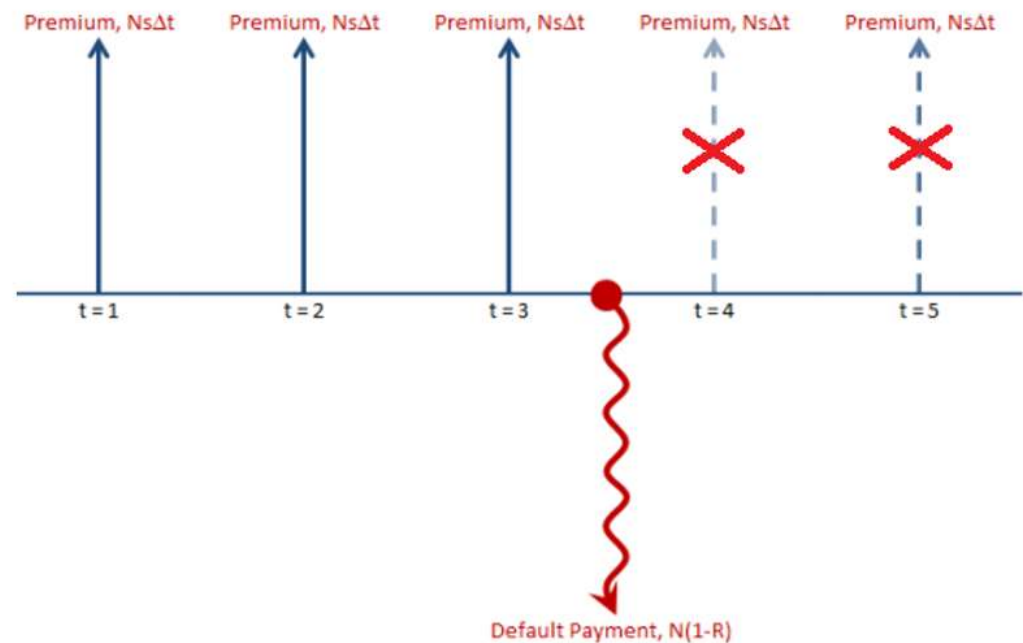


# IR Products – CDS Pricing

(Reference)

## Pricing

- Similar to Interest Rate Swap Pricing
- With Additional Survival Probability Term,  $Q(t,T)$
- $Q(t,T) = \exp\left(-\int_t^T \lambda(t,u)du\right)$
- $\lambda$  is the 'Hazard Rate' (instantaneous prob of default)



## Buying Credit Protection

$$PV = PV(\text{Protection Leg}) - PV(\text{Premium Leg})$$

$$PV(\text{Premium Leg}) = \sum_{i=1}^n \underbrace{N s \Delta(t_{i-1}, t_i)}_{\text{Coupon}} \underbrace{Q(t_i)}_{P(\text{Survive})} \underbrace{P(t_0, t_i)}_{\text{Discount Factor}}$$

$$PV(\text{Protection Leg}) = \sum_{i=1}^n \underbrace{N(1-R)}_{\text{Loss Given Default}} \underbrace{[Q(t_{i-1}) - Q(t_i)]}_{\text{Default within Premium Period}} \underbrace{P(t_0, t_i)}_{\text{Discount Factor}}$$

# IR Risk

What are the main IR risks?

- Discount Risk (DF01)
- Forward Risk (PV01)
- Discount + Forward Risk (DV01)

Risk Calculation Methods

- Analytical
- Numerical Risk (Benchmark)
- Using Yield Curve Jacobian
- Automatic Adjoint Differentiation (AAD)

USD SOFR YIELD CURVE - CALIBRATION INSTRUMENTS

Instrument	Term	Rate
USD SOFR Swap	ON	2.37000%
USD SOFR Swap	1W	2.36510%
USD SOFR Swap	2W	2.34960%
USD SOFR Swap	3W	2.35200%
USD SOFR Swap	1M	2.34550%
USD SOFR Swap	2M	2.30320%
USD SOFR Swap	3M	2.25590%
USD SOFR Swap	4M	2.19610%
USD SOFR Swap	5M	2.14750%
USD SOFR Swap	6M	2.10350%
USD SOFR Swap	1Y	1.89350%
USD SOFR Swap	2Y	1.68360%
USD SOFR Swap	3Y	1.62600%
USD SOFR Swap	4Y	1.61700%
USD SOFR Swap	5Y	1.64200%
USD SOFR Swap	6Y	1.67900%
USD SOFR Swap	7Y	1.71600%
USD SOFR Swap	8Y	1.75700%
USD SOFR Swap	9Y	1.79800%
USD SOFR Swap	10Y	1.83200%
USD SOFR Swap	15Y	1.96800%
USD SOFR Swap	20Y	2.03300%
USD SOFR Swap	25Y	2.04100%
USD SOFR Swap	30Y	2.04900%

Bucketed DV01, USD

Instrument	Tenor	DV01
USD SOFR Swap	ON	8
USD SOFR Swap	1W	0
USD SOFR Swap	2W	0
USD SOFR Swap	3W	0
USD SOFR Swap	1M	0
USD SOFR Swap	2M	0
USD SOFR Swap	3M	0
USD SOFR Swap	4M	0
USD SOFR Swap	5M	-1
USD SOFR Swap	6M	1
USD SOFR Swap	1Y	92
USD SOFR Swap	2Y	213
USD SOFR Swap	3Y	294
USD SOFR Swap	4Y	409
USD SOFR Swap	5Y	453
USD SOFR Swap	6Y	541
USD SOFR Swap	7Y	723
USD SOFR Swap	8Y	736
USD SOFR Swap	9Y	852
USD SOFR Swap	10Y	892
USD SOFR Swap	15Y	1,320
USD SOFR Swap	20Y	1,662
USD SOFR Swap	25Y	1,979
USD SOFR Swap	30Y	2,252
<b>Total Risk</b>		<b>12,428</b>

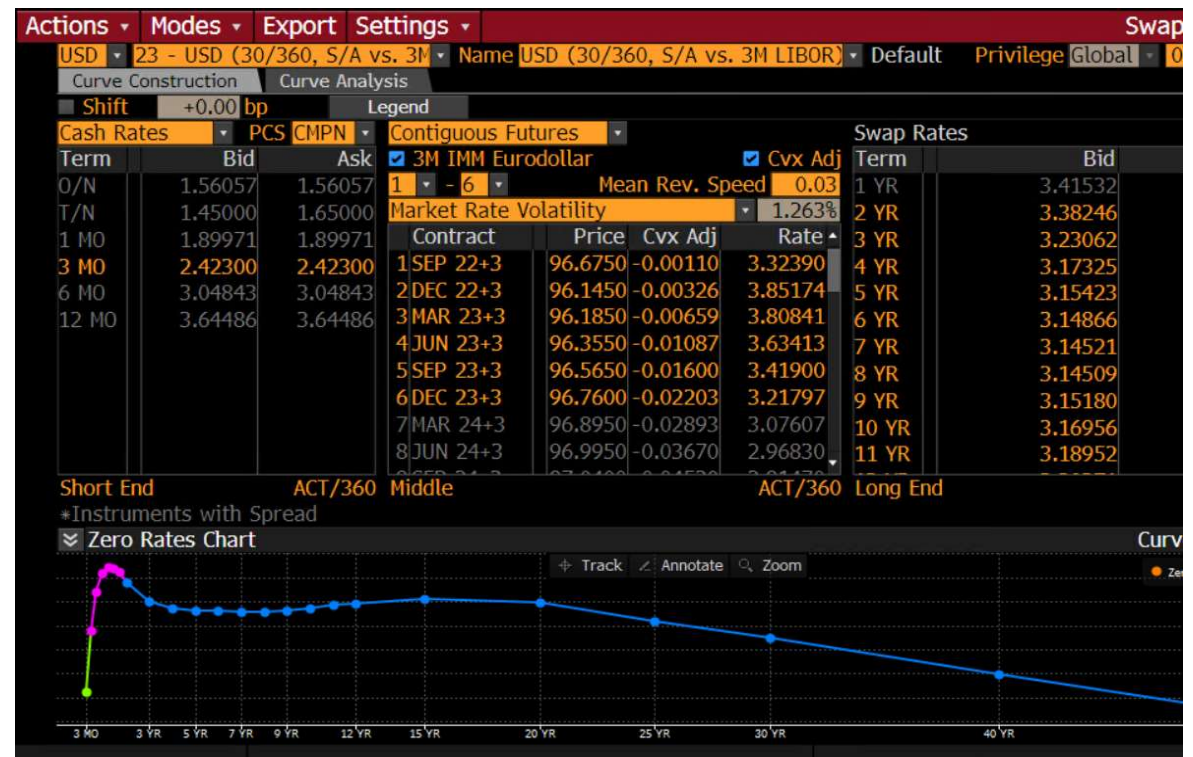
# Yield Curves - Calibration

## Model Inputs & Outputs

- Liquid Market Instrument Quotes [IN]
- Forward Rates [OUT]
- Discount Factors [OUT]

## Calibration Process

- Choose State Variable<sup>1</sup>
- Choose Interpolator (Functional Form)
- Solve and Imply Forwards & Disc Factors<sup>2</sup>



<sup>1</sup> Popular choices: forward rate, disc factor, logDF, zero rate etc.

<sup>2</sup> May need to differentiate and/or integrate state variable,  $P(t, T) = \exp\left(-\int_t^T f(t, u) du\right)$

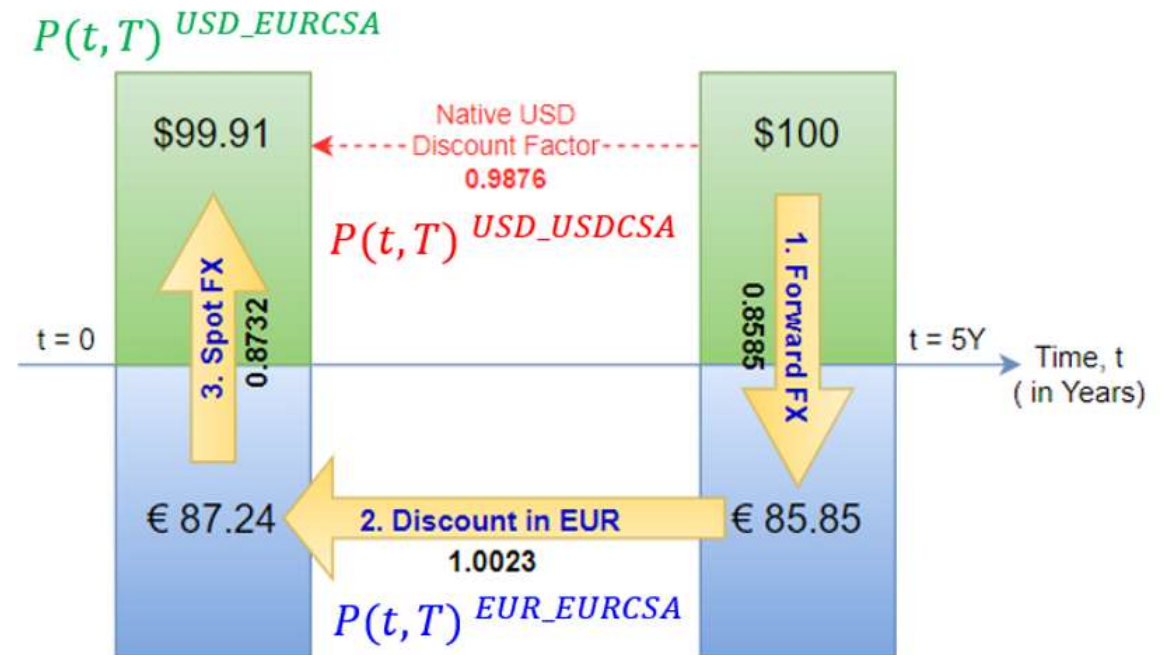
# Yield Curves – Collateral & CSA Curves

## Collateral & CSA Curves

- Calibrate to FX Forwards & Xccy Swaps
- **FX Forward Invariance** (FX Carry Trade)
- Impacts Discount Factors Only
- No Impact on Forward Rates

## Advanced CSA Topics

- Cheapest to Deliver (Multiple CSAs)
- Collateral Switch Options



$$f(t, T)^{USD/EUR} = s(t)^{USD/EUR} \underbrace{\left( \frac{P(t, T)^{EUR\_USDCSA}}{P(t, T)^{USD\_USDCSA}} \right)}_{USD \text{ CSA}} = s(t)^{USD/EUR} \underbrace{\left( \frac{P(t, T)^{EUR\_EURCSA}}{P(t, T)^{USD\_EURCSA}} \right)}_{EUR \text{ CSA}}$$

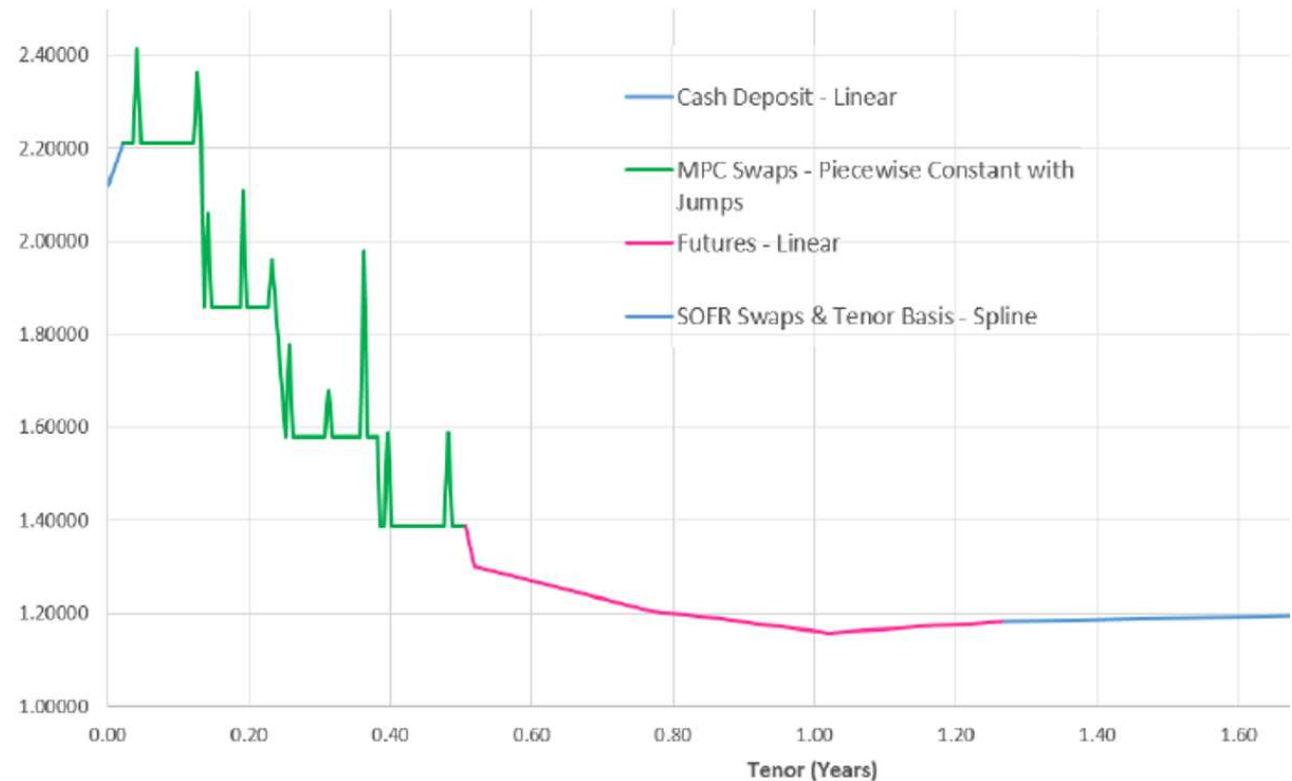
# Yield Curves - Features

## Curve Features & Considerations

- Underlying Instrument Behaviour
- Mixed Interpolation Schemes
- Turn-of-Year Effects (ToYs)

## Advanced Features for Electronic Markets

- Curve Jacobian
- Ultra-Fast Curves & Analytical Risk
- Automatic Adjoint Differentiation (AAD)





# Yield Curves – Curve Jacobian

## Electronic HFT Usage

- Ultra-Fast Rebuilds
- Real-Time Risk
- Auto-Hedging

## By-Product of Calibration Process

- Measures Changes in Market Instrument Quotes (P) on Forward Rates (L)
- First Order Derivative Matrix,  $dP/dL$  (Inverse Required)
- Controls Hedge and Risk Buckets (Same as Numerical Bumping)
- Use **Implicit Function Theorem** (IFT) to modify Risk Buckets (see Appendix)

Inverse Curve Jacobian,  $dL/dP$

Forward Pillars	Curve Calibration Instruments									
	$dP_{1Y}^{OIS}$	$dP_{2Y}^{OIS}$	$dP_{3Y}^{OIS}$	$dP_{4Y}^{OIS}$	$dP_{5Y}^{OIS}$	$dP_{1Y}^{IRS}$	$dP_{2Y}^{IRS}$	$dP_{3Y}^{IRS}$	$dP_{4Y}^{IRS}$	$dP_{5Y}^{IRS}$
$dO_{1Y}$	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$dO_{2Y}$	-1.01	2.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$dO_{3Y}$	0.00	-2.04	3.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$dO_{4Y}$	0.00	0.00	-3.08	4.08	0.00	0.00	0.00	0.00	0.00	0.00
$dO_{5Y}$	0.00	0.00	0.00	-4.13	5.13	0.00	0.00	0.00	0.00	0.00
$dL_{1Y}$	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
$dL_{2Y}$	0.00	0.00	0.00	0.00	0.00	-1.01	2.01	0.00	0.00	0.00
$dL_{3Y}$	0.00	0.00	0.00	0.00	0.00	0.00	-2.04	3.04	0.00	0.00
$dL_{4Y}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-3.08	4.08	0.00
$dL_{5Y}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-4.13	5.13

# Yield Curves – Ultra-Fast Rebuilds

New Forwards

$$L_{New} = L_{Old} + dL$$

$$= L_{Old} + (dL/dP) \cdot dP$$

New Forwards		Original Forwards		Inverse Jacobian, dL/dP											Change in Mkt Data	
L <sub>NEW</sub>		L <sub>OLD</sub>													dP	
L <sub>1Y</sub> <sup>OIS</sup>	1.44591%	L <sub>1Y</sub> <sup>OIS</sup>	1.43591%	L <sub>1Y</sub> <sup>OIS</sup>	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	L <sub>1Y</sub> <sup>OIS</sup>	0.01%
L <sub>2Y</sub> <sup>OIS</sup>	1.24323%	L <sub>2Y</sub> <sup>OIS</sup>	1.23323%	L <sub>2Y</sub> <sup>OIS</sup>	-1.01	2.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	L <sub>2Y</sub> <sup>OIS</sup>	0.01%
L <sub>3Y</sub> <sup>OIS</sup>	1.26107%	L <sub>3Y</sub> <sup>OIS</sup>	1.25107%	L <sub>3Y</sub> <sup>OIS</sup>	0.00	-2.04	3.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	L <sub>3Y</sub> <sup>OIS</sup>	0.01%
L <sub>4Y</sub> <sup>OIS</sup>	1.30130%	L <sub>4Y</sub> <sup>OIS</sup>	1.29130%	L <sub>4Y</sub> <sup>OIS</sup>	0.00	0.00	-3.08	4.08	0.00	0.00	0.00	0.00	0.00	0.00	L <sub>4Y</sub> <sup>OIS</sup>	0.01%
L <sub>5Y</sub> <sup>OIS</sup>	1.40782%	L <sub>5Y</sub> <sup>OIS</sup>	1.39782%	L <sub>5Y</sub> <sup>OIS</sup>	0.00	0.00	0.00	-4.13	5.13	0.00	0.00	0.00	0.00	0.00	L <sub>5Y</sub> <sup>OIS</sup>	0.01%
L <sub>1Y</sub> <sup>IRS</sup>	1.71896%	L <sub>1Y</sub> <sup>IRS</sup>	1.70896%	L <sub>1Y</sub> <sup>IRS</sup>	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	L <sub>1Y</sub> <sup>IRS</sup>	0.01%
L <sub>2Y</sub> <sup>IRS</sup>	1.48359%	L <sub>2Y</sub> <sup>IRS</sup>	1.47359%	L <sub>2Y</sub> <sup>IRS</sup>	0.00	0.00	0.00	0.00	0.00	-1.01	2.01	0.00	0.00	0.00	L <sub>2Y</sub> <sup>IRS</sup>	0.01%
L <sub>3Y</sub> <sup>IRS</sup>	1.50531%	L <sub>3Y</sub> <sup>IRS</sup>	1.49531%	L <sub>3Y</sub> <sup>IRS</sup>	0.00	0.00	0.00	0.00	0.00	0.00	-2.04	3.04	0.00	0.00	L <sub>3Y</sub> <sup>IRS</sup>	0.01%
L <sub>4Y</sub> <sup>IRS</sup>	1.56934%	L <sub>4Y</sub> <sup>IRS</sup>	1.55934%	L <sub>4Y</sub> <sup>IRS</sup>	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-3.08	4.08	0.00	L <sub>4Y</sub> <sup>IRS</sup>	0.01%
L <sub>5Y</sub> <sup>IRS</sup>	1.63999%	L <sub>5Y</sub> <sup>IRS</sup>	1.62999%	L <sub>5Y</sub> <sup>IRS</sup>	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-4.13	5.13	L <sub>5Y</sub> <sup>IRS</sup>	0.01%

## Implementation

- Slow Curve (Full-Rebuild) Ticks in Background (ca. 10ms)
- Fast Curve (Jacobian Method) Used Between Refreshes (Real-Time)

# Yield Curves – Real-Time Bucketed Risk

## Requirements

- Curve Jacobian
- Trade or Portfolio Jacobian

$$DV01(Analytical) = \underbrace{1bps \times \frac{dPV}{dL}}_{\text{Pricing Jacobian}} \times \underbrace{\frac{dL}{dP}}_{\text{Curve Jacobian}}$$

## Risk as a Matrix Operation

- Can be Parallelized / Vectorized
- Matrix Dimensions Must Agree
- Interpolation & Forward Mapping
- Barycentric Weights,  $w_j(t)$

$$p(t) = \sum_{j=0}^n w_j(t) f(t_j), \quad w_j(t) = \frac{\prod_{k=0, k \neq j}^n (t - t_k)}{\prod_{k=0, k \neq j}^n (t_j - t_k)}$$

Inverse Curve Jacobian, dL/dP

Forward Pillars	Curve Calibration Instruments									
	dP <sub>1Y</sub> <sup>OIS</sup>	dP <sub>2Y</sub> <sup>OIS</sup>	dP <sub>3Y</sub> <sup>OIS</sup>	dP <sub>4Y</sub> <sup>OIS</sup>	dP <sub>5Y</sub> <sup>OIS</sup>	dP <sub>1Y</sub> <sup>IRS</sup>	dP <sub>2Y</sub> <sup>IRS</sup>	dP <sub>3Y</sub> <sup>IRS</sup>	dP <sub>4Y</sub> <sup>IRS</sup>	dP <sub>5Y</sub> <sup>IRS</sup>
dO <sub>1Y</sub>	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
dO <sub>2Y</sub>	-1.01	2.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
dO <sub>3Y</sub>	0.00	-2.04	3.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00
dO <sub>4Y</sub>	0.00	0.00	-3.08	4.08	0.00	0.00	0.00	0.00	0.00	0.00
dO <sub>5Y</sub>	0.00	0.00	0.00	-4.13	5.13	0.00	0.00	0.00	0.00	0.00
dL <sub>1Y</sub>	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
dL <sub>2Y</sub>	0.00	0.00	0.00	0.00	0.00	-1.01	2.01	0.00	0.00	0.00
dL <sub>3Y</sub>	0.00	0.00	0.00	0.00	0.00	0.00	-2.04	3.04	0.00	0.00
dL <sub>4Y</sub>	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-3.08	4.08	0.00
dL <sub>5Y</sub>	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-4.13	5.13

=

Trade	
Risk Bucket	IRS 3Y
OIS 1Y	0
OIS 2Y	0
OIS 3Y	0
OIS 4Y	-1
OIS 5Y	1
IRS 1Y	0
IRS 2Y	0
IRS 3Y	291
IRS 4Y	0
IRS 5Y	0

Trade Jacobian, dPV/dL  
3Y Par Swap

Trade	OIS Curve (Discount Risk)					Swap Curve (Forward Risk)				
	dO <sub>1Y</sub>	dO <sub>2Y</sub>	dO <sub>3Y</sub>	dO <sub>4Y</sub>	dO <sub>5Y</sub>	dL <sub>1Y</sub>	dL <sub>2Y</sub>	dL <sub>3Y</sub>	dL <sub>4Y</sub>	dL <sub>5Y</sub>
dS <sub>3Y</sub> <sup>IRS</sup>	0	0	0	0	0	98	97	96	0	0

Total Trade DV01

IRS 3Y
291

# Yield Curves – Automatic Adjoint Differentiation (AAD)

## Trade Jacobian

- AAD Can Compute Instrument Price & Risk Simultaneously
- Direct Differentiation of Code + Implicit Function Theorem (IFT)
- Exact & Fast (X4 Pricing Time)

## Tangent & Adjoint Modes

- Tangent Mode (dot) : **Forward** Mode - **One Risk at a Time**
- Adjoint Mode (bar) : **Backward** Mode - **All Risks Simultaneously**
- Activation Inputs Control Risk Outputs

## Implementation Methods

- By Hand (See Appendix for Swap DV01 Risk Example)
- Derivative Code by Overloading, DCO/C++
- Professional Tools: Adept, NAG

## Pricing Calculations

$$x \rightarrow f(x) \rightarrow g(f) \rightarrow h(g) \rightarrow y$$

## Chain Rule: Forwards

$$\frac{df}{dx} \cdot \frac{dg}{df} \cdot \frac{dh}{dg} \cdot \frac{dy}{dh} = \frac{dy}{dx}$$

## Chain Rule: Backwards

$$\frac{dy}{dh} \cdot \frac{dh}{dg} \cdot \frac{dg}{df} \cdot \frac{df}{dx} = \frac{dy}{dx}$$

# Yield Curves – AD Tangent Mode Example

## Tangent Mode

- Differentiate Forwards using 'Dot' Notation
- One Risk at a Time, Controlled by Dot **Input Activation Variables** 1 or 0
- For  $\frac{df}{dx_1}$  and  $\frac{df}{dx_2}$  must call tangent method twice

```

01 double function( double x1, double x2 )
02 {
03     double a = x1*x1;           // Step 1:    a = x12
04     double b = 2*a;             // Step 2:    b = 2x12
05     double c = x2;              // Step 3:    c = x2
06     double d = 3*c;             // Step 4:    d = 3x2
07     double f = b + d;           // Step 5:    f = 2x12 + 3x2
08     return f;
09 }

```

Simple Function:  $f(x_1, x_2) = 2x_1^2 + 3x_2$

Source Code: <https://onlinegdb.com/kKqaS6hJT>

```

01 tangent(2.0, 3.0, 1.0, 0.0); // Input: x1 = 2, x2 = 3, x1_d = 1, x2_d = 0   Output: 8
02 tangent(2.0, 3.0, 0.0, 1.0); // Input: x1 = 2, x2 = 3, x1_d = 0, x2_d = 1   Output: 3

```

## Function Derivatives using Tangent Mode

```

01 double tangent( double x1, double x2, double x1_dot, double x2_dot )
02 {
03     double a = x1*x1;           // Step 1:    a = x12
04     double a_dot = 2*x1*x1_dot; // Tangent:   $\dot{a} = 2x_1 \cdot \dot{x}_1$        $\dot{a} = 2x_1$ 
05     double b = 2*a;             // Step 2:    b = a
06     double b_dot = 2*a_dot;     // Tangent:   $\dot{b} = 2 \cdot \dot{a}$            $\dot{b} = 4x_1$ 
07     double c = x2;              // Step 3:    c = x2
08     double c_dot = x2_dot;      // Tangent:   $\dot{c} = \dot{x}_2$            $\dot{c} = 1$ 
09     double d = 3*c;             // Step 4:    d = 3c
10     double d_dot = 3*c_dot;     // Tangent:   $\dot{d} = 3 \cdot \dot{c}$          $\dot{d} = 3$ 
11     double f = b + d;           // Step 5:    f = 2x12 + 3x2
12     double f_dot = b_dot + d_dot; // Tangent:   $\dot{f} = \dot{b} + \dot{d}$ 
13     return f_dot;              // Result:     $\dot{f} = 4x_1 + 3$ 
14 }

```

Simple Function  $f(x_1, x_2) = 2x_1^2 + 3x_2$  with Tangent Derivatives



# Yield Curves – AD Adjoint Mode Example

## Adjoint Mode (Reverse Mode)

- Backwards Differentiation with 'Bar' Notation
- Forward Sweep then Back Propagate Risk
- Computes All Risks at Same Time
- Risk Controlled By Bar Input Activation Variable 1 or 0
- Adjoint Method Calculates Both  $\frac{df}{dx_1}$  and  $\frac{df}{dx_2}$

```

01 double function( double x1, double x2 )
02 {
03     double a = x1*x1;           // Step 1:  a = x12
04     double b = 2*a;             // Step 2:  b = 2x12
05     double c = x2;             // Step 3:  c = x2
06     double d = 3*c;            // Step 4:  d = 3x2
07     double f = b + d;          // Step 5:  f = 2x12 + 3x2
08     return f;
09 }

```

Simple Function:  $f(x_1, x_2) = 2x_1^2 + 3x_2$

```

01 adjoint(2.0, 3.0, 1.0); // Input: x1 = 3, x2 = 2, f_bar Output: df/dx1 = 8 and df/dx2 = 3

```

## Function Derivatives using Adjoint Mode

```

01 void adjoint( double x1, double x2, double f_bar )
02 {
03     // Forward Sweep
04     double a = x1*x1;           // Step 1:  a = x12
05     double b = 2*a;             // Step 2:  b = 2x12
06     double c = x2;             // Step 3:  c = x2
07     double d = 3*c;            // Step 4:  d = 3x2
10     double f = b + d;          // Step 5:  f = 2x12 + 3x2
08
09     // Back Propagation
10     double b_bar = f_bar;       // Step 5:  b_bar = 1    from input variable
11     double d_bar = f_bar;       // Step 5:  d_bar = 1    from input variable
12     double c_bar = 3*d_bar;     // Step 4:  c_bar = 3
13     double x2_bar = c_bar;      // Step 3:  x2_bar = 3    df/dx2 = 3
14     double a_bar = 2*b_bar;     // Step 2:  a_bar = 2
15     double x1_bar = 2*x1*a_bar; // Step 1:  x1_bar = 4x1  df/dx1 = 4x1
16
17     // Display Results
18     std::cout << "df/dx1: " << x1_bar << std::endl; // x̄1 = df/dx1 = 4x1
19     std::cout << "df/dx2: " << x2_bar << std::endl; // x̄2 = df/dx2 = 3
20 }

```

Simple Function  $f(x_1, x_2) = 2x_1^2 + 3x_2$  with Adjoint Derivatives

# Credit Models – Hazard Rates & Survival Probabilities

## Calibration Summary

- Yield Curve is an Input
- Calibrate to Bonds or CDS
- Imply Hazard Rates,  $\lambda$
- Used for Survival Prob,  $Q(t,T)$

## Common Assumptions

- Piecewise Constant<sup>1</sup>
- Deterministic Hazard Rates

## Rule of Thumb

$$\lambda = \frac{S}{(1 - R)}$$

<sup>1</sup> As often there is only a single calibration instrument



$$Q(t, T) = \exp\left(-\int_t^T \lambda(t, u) du\right)$$

$$P(t, T) = \exp\left(-\int_t^T f(t, u) du\right)$$

## PART TWO – PRICING & PRACTICE

### Case Studies Interest Rate Swaps & Asset Swaps

# Interest Rate Swap – Annuity is the Key Pricing & Risk Factor

## It's All About Annuity

- Pricing & Risk Expressed in Terms of Annuity
- Similarly Float Legs Expressed in Annuity Terms
- Can Be Used to Convert a Float Leg to Fixed Leg
- Useful for Low Latency Pricing

## Key Formulae:

- $PV = (r - p) \text{Annuity(Fixed)}$
- $\text{Par Rate} = PV(\text{Float}) / \text{Annuity(Fixed)}$
- $PV01 = \text{Annuity(Fixed)} \times 0.01\%$
- $DV01 = PV01 + DF01 = PV01$  for Par Swaps

$$A_N = N \sum_{i=1}^n \tau_i P(t_0, t_i)$$

## Low Latency Interest Rate Swap Pricing

Electronic Rates Markets & Low Latency Interest Rate Swap Calculations (May 31, 2022).

Available at SSRN: <https://ssrn.com/abstract=4125565>

$$\text{Swap PV} = PV^{\text{Fixed Leg}} - PV^{\text{Float Leg}}$$

$$\begin{aligned} &= r \sum_{i=1}^n N_i \tau_i P(t_0, t_i) - \sum_{j=1}^m N_j l_{j-1} \tau_j P(t_0, t_j) \\ &= (r - p) A_{\text{Fixed}} \end{aligned}$$



# Interest Rate Swap – Pricing & Risk Example

Compute Annuity  $A_N$

= USD 4,863,971.74

$$A_N = N \sum_{i=1}^n \tau_i P(t_0, t_i)$$

$PV = (r - p) A_N$

= (5.00% - 1.59%)  $A_N$

= USD 167,892.11

The screenshot displays a financial software interface for swap pricing. The main window is titled 'Swap Manager' and contains several tabs: 3) Main, 4) Details, 5) Curves, 6) Cashflow, 7) Resets, 9) Scenario, 10) Risk, 11) CVA, 12) Matrix, and 20) Properties. The 'Main' tab is active, showing a 'Fixed Float Swap' with 'OTC' as the deal type. The 'Counterparty' is set to 'SWAP CNTRPARTY'. The 'Swap' section shows two legs: 'Leg 1: Fixed' (Receive) and 'Leg 2: Float' (Pay). Both legs have a notional of 1MM, currency of USD, and maturity of 5Y (08/25/2020). The fixed leg has a coupon of 5.000000% and a semi-annual payment frequency. The floating leg has a 3M index (US0003M) and a quarterly payment frequency. The 'Valuation Settings' section shows a curve date of 08/21/2015, valuation date of 08/25/2015, OIS DC Strip set to ON, and CSA Coll Ccy set to USD. The 'Market' section shows a discount rate of 42 M and a forward rate of 23 M, both using the USD Bloomberg Curve. The 'Valuation Results' section shows a par coupon of 1.548250, a premium of 16.78921, a principal of 167,892.11, an accrued amount of 0.00, and a net present value (NPV) of 167,892.11. The 'Calculators' section shows PV01 of 486.40, DV01 of 532.42, and Gamma (1bp) of 0.29.

Par Rate =  $PV(\text{Float}) / A_N$

= 75,306 /  $A_N$

= 1.5482%

PV01

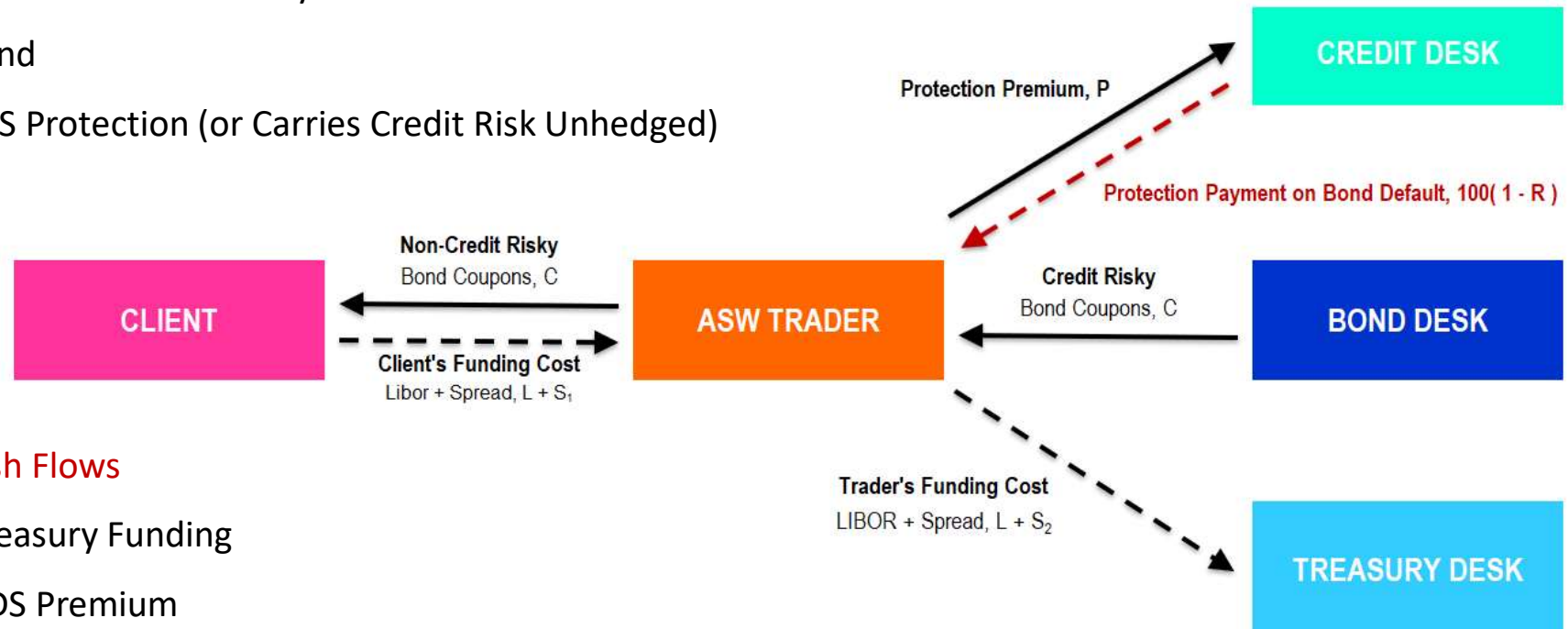
=  $A_N \times 0.01\%$

= USD 486.40

# Asset Swap – Structuring the Asset Swap Spread

## Trader Creates Synthetic Asset Swap

- Borrow Cash from Treasury to Purchase Bond
- Buy Bond
- Buy CDS Protection (or Carries Credit Risk Unhedged)



## Trader Cash Flows

- Pays Treasury Funding
- Pays CDS Premium
- Receives Bond Coupons and Passes on to Client
- Client Pays All Costs + Commission as a **Spread over LIBOR** (or RFR)



## Asset Swap – Pricing as a Spread Over LIBOR (or RFR)

DBR 0 12/15/26		5 Actions		6 Settings		Asset Swap Calculator	
1 Pricing		2 Cashflow		3 Relative Value		4 Deal Summary	
<b>Asset Swap Analysis</b>				Price	104.5800		
Calculate				Z-Spread	-40.9	ASW Spread	-40.6
Price -> ASW Spread				Yield(%)	0.02595	MMS Spread	-41.2
<b>Bond</b> JV503423		<b>Swap</b> <input checked="" type="radio"/> Par-Par <input type="radio"/> Matched Maturity		<b>B) Swap Detail   SWPM »</b>			
Par Amount	1MM	Leg 1: Fixed	Pay	Leg 2: Float	Receive		
Workout	02/15/2026	Notional	1MM	Notional	1MM		
Workout Price	100.0000	Currency	EUR	Currency	EUR		
Pay Freq	Annual	Effective Date	01/15/2016	Effective Date	01/15/2016		
Day Count	ACT/ACT	Maturity Date	02/15/2026	Maturity Date	02/15/2026		
		Coupon	0.5	Latest Index	-0.112		
		Pay/Reset Freq	Annual	Index	EUR006M		
		Day Count	ACT/ACT	Pay/Reset Freq	SemiAnnual		
				Day Count	ACT/360		
Implied Value	100.5736	<input checked="" type="checkbox"/> Include Accrued		<input checked="" type="checkbox"/> Include Accrued			
<b>Market</b>							
Curve Date	06/09/2016	Discount Curve	133 Mid	Discount Curve	133 Mid		
Settle Date	06/13/2016			Forward Curve	45 Mid		
<b>Swapped Spread Detail</b>							
Clean Price	104.5800			Money		Spread(bp)	
Swap Price	100.0000	Cash Out	4.5800		-45,800.0		-46.9
Swap Rate(%)	0.44104	Bond Cpn(%)	0.5000		5,736.5		5.8
Redemption(%)	0.0000				0.0		0.0
Funding	Spread(bp)		0.0		0.0		0.0
Swapped Spread					-40,063.5		-40.6

- **ASW Spread** - Par-Par Spread
- **MMS Spread** - Yield-Yield Spread<sup>1</sup>

<sup>1</sup> Y/Y Spread Between Swap Rate and Benchmark Gov't Bond Yield

# Asset Swap – Pricing using Par-Par Method

## Pricing as a PV

- Valuation Method for Existing Swaps, Unwinds and Novations (trade transfers)
- Again Present Value is Simply the Sum of Incoming and Outgoing Cash Flows
- An Upfront Par-Adjustment is Made if the Underlying Bond not Trading at Par, i.e., 100

$$PV^{Asset\ Swap} = \underbrace{\phi r^{Fixed} \sum_{i=1}^n N_i \tau_i P(t_0, t_i)}_{Fixed\ Leg} - \underbrace{\phi \sum_{j=1}^m N_j (l_{j-1} + s) \tau_j P(t_0, t_j)}_{Float\ Leg} + \underbrace{\phi N_1 \left( \frac{100 - B}{100} \right)}_{Par\ Adjustment}$$

## Pricing as a Par Spread

- New Asset Swaps Price to Par i.e., zero
- Instead Quote as a Par Spread  $s$
- Rearrangement of PV formula with  $PV=0$

$$s = \left( \frac{(r^{Fixed} - p^{Market}) A^{Fixed} + \left( \frac{100 - B}{100} \right)}{A^{Float}} \right)$$

# Pricing Tricks & Rules of Thumb

## Annuity Assumption

- Need to know market par rates for standard swap maturities
- Assume Annual Coupons and Discount Factors = 1.0
- This means **Annuity = Time to Maturity**
- Used to Gain Intuition when Pricing IR Products & CDS

### Interest Rate Swap PV

➤  $PV = N ( r - p ) A(\text{Fixed})$



### Approximate PV

➤  $PV = N \Delta r T$

This gives PV as USD 100 per Million per Year per  $\Delta r$  in bps

## IRS Rule of Thumb – Multiples of a Base Case

- $PV = 100 \times \Delta N \times \Delta r \times \Delta T$
- $DV01 = PV01 = 100 \times \Delta N \times \Delta T$

# Pricing Tricks – Interest Rate Swap

## IRS – Rule of Thumb

- $PV = 100 \times \Delta N \times \Delta r \text{ in bps} \times \Delta T$
- $DV01 = PV01 = 100 \times \Delta N \times \Delta T$

## Market Par Rate

- 5Y Par Rate = 150 bps
- $\Delta r = (r - p) = (500 - 150) = 350 \text{ bps}$

## Present Value

- Here  $\Delta N = 1$ ,  $\Delta r = 350$ ,  $\Delta T = 5$
- $PV = \text{USD } 175,000$
- $DV01 = PV01 = \text{USD } 500$

PV as USD 100 per Million per Year per  $\Delta r$  in bps

The screenshot shows a financial software interface with a menu bar at the top (91) Actions, 92) Products, 93) Views, 94) Data & Settings, 95) Info, Swap Manager) and a toolbar with buttons like 30) Solver (Premium), 31) Load, 32) Save, 35) Trade, 38) CCP, 43) Send to TR. Below the toolbar are tabs for 3) Main, 4) Details, 5) Curves, 6) Cashflow, 7) Resets, 9) Scenario, 10) Risk, 11) CVA, 12) Matrix. The main window is titled 'Fixed Float Swap' and contains two columns of fields for 'Leg 1:Fixed' and 'Leg 2:Float'. The 'Leg 1:Fixed' column has fields for Notional (1MM), Currency (USD), Effective (0D), Maturity (5Y), Coupon (5.000000%), Pay Freq (SemiAnnual), Day Count (30I/360), and Calc Basis (Money Mkt). The 'Leg 2:Float' column has fields for Notional (1MM), Currency (USD), Effective (0D), Maturity (5Y), Index (3M), Spread (0.000), Latest Index (0.32910), Day Count (ACT/360), Reset Freq (Quarterly), and Pay Freq (Quarterly). To the right of these fields is a 'Valuation Settings' section with fields for Curve Date (08/21/2015), Valuation (08/25/2015), OIS DC Strip (ON), and CSA Coll Ccy (USD). At the bottom of the window is a 'Valuation Results' section with a table showing various metrics.

Valuation Results			
Par Cpn	1.548250	Premium	16.78921
Principal	167,892.11	BP Value	1678.92112
Accrued	0.00		
NPV	167,892.11		

At the bottom right of the window, there are two more sections: 'Calculators' and 'More Greeks'. The 'Calculators' section shows PV01 (486.40), DV01 (532.42), and Gamma (1bp) (0.29). The 'More Greeks' section is currently empty.

$$PV = 100 \times 1 \times 350 \times 5 = \text{USD } 170K$$

$$DV01 = 100 \times 1 \times 5 = \text{USD } 500$$

# Pricing Tricks – Asset Swaps

We Can Make the Same Annuity Assumption to Price Asset Swaps

## Par-Par Spread

$$S = \left( \frac{(r^{Fixed} - p^{Market})A^{Fixed} + \left(\frac{100 - B}{100}\right)}{A^{Float}} \right)$$

## IRS Rule of Thumb

$$s = (r - p) + (100 - B/100) / T$$

$$= \Delta r - (\Delta B / T)$$

where  $\Delta r = (r - p)$  in bps

and  $\Delta B = (B\% - 100\%)$  in bps

Asset Swap Calculator			
DBR 0 2/15/26	5 Actions	0 Settings	
Pricing	Cashflow	Relative Value	Deal Summary
Asset Swap Analysis	Calculate	Price	104.5800
Price -> ASW Spread	Z-Spread	-40.9	ASW Spread -40.6
	Yield(%)	0.02595	MMS Spread -41.2
Bond JV503423	Swap	Par-Par	Matched Maturity
Par Amount IMM	Leg 1: Fixed	Pay	Leg 2: Float Receive
Workout 02/15/2026	Notional	IMM	Notional IMM
Workout Price 100.0000	Currency	EUR	Currency EUR
Pay Freq Annual	Effective Date	01/15/2016	Effective Date 01/15/2016
Day Count ACT/ACT	Maturity Date	02/15/2026	Maturity Date 02/15/2026
	Coupon	0.5	Latest Index -0.112
	Pay/Reset Freq	Annual	Index EUR006M
	Day Count	ACT/ACT	Pay/Reset Freq SemiAnnual
			Day Count ACT/360
Implied Value 100.5736	Include Accrued		Include Accrued
Market	Discount Curve	133 Mid	Discount Curve 133 Mid
Curve Date 06/09/2016			Forward Curve 45 Mid
Settle Date 06/13/2016			
Swapped Spread Detail			
Clean Price	104.5800	Cash Out	4.5800
Swap Price	100.0000	Bond Cpn(%)	0.5000
Swap Rate(%)	0.44104		
Redemption(%)	0.0000		
Funding Spread(bp)	0.0		
Swapped Spread			-40.063.5

## Par-Par Spread

Here  $\Delta r = 0.50\% - 0.44\% = 6$  bps,  $\Delta B = 458$  bps and  $T = 10$

$$S = 6 - (458/10) \approx 6 - 46 = -40 \text{ bps}$$

# References

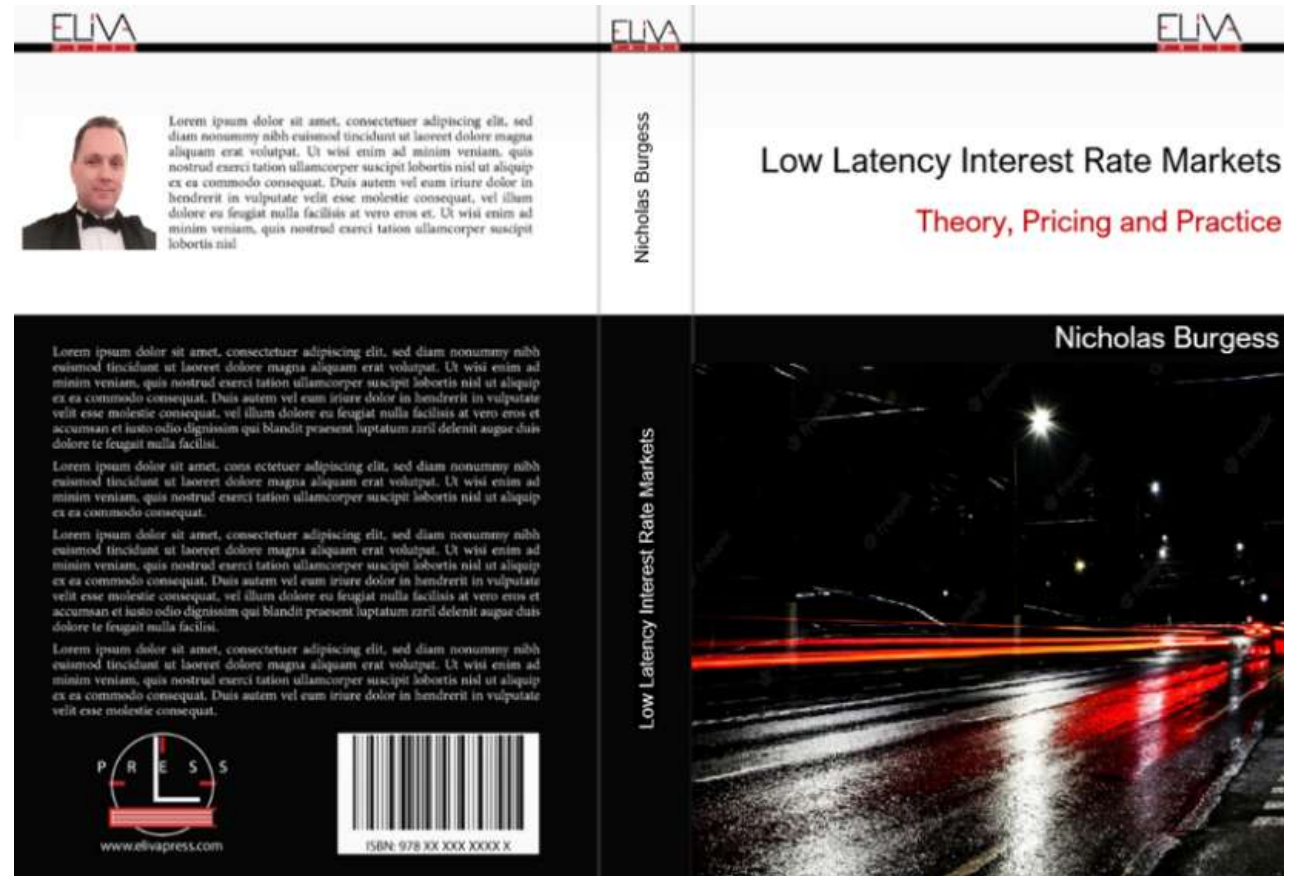


Quant Research Papers

<https://ssrn.com/author=1728976>

Support Materials, C++ & Excel Examples

<https://github.com/nburgessx/SwapsBook>





# Appendix – Implicit Function Theorem (IFT)

## IFT Theorem

To gain some intuition consider the following function  $f(x, y) = 0$  for which we have a solution  $(a, b)$ . Near the solution we can express  $y$  as function of  $x$  namely  $f(x, y(x)) = 0$ . Using this expression, we can compute the derivative in terms of  $x$  only by differentiating with respect to  $x$  as follows,

$$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial x} = 0$$

which gives,

$$\frac{\partial y}{\partial x} = - \left( \frac{\partial f / \partial x}{\partial f / \partial y} \right)$$

We have a solution under the condition,  $\partial f / \partial y \neq 0$ , since we cannot divide by zero.

## Yield Curve Application

In the context of a yield curve calibration, we solve for the solution of a helper target function,  $H(L, P) = 0$ , where  $L$  is the LIBOR forward rate state variable (model output) and  $P$  the yield curve par rate (model input). The helper target function computes the difference between model par rates as a function of the forward state variable  $L$  and a market instrument par rate quote,

$$H(L, P) = \text{Model Par Rate}(L) - \text{Market Par Rate}$$

## How does this Help with Sensitivity Calculations?

The IFT theorem says that having found a solution to the continuously differentiable function  $H(L, P) = 0$  in two variables we can express the solution solely in terms of the model output  $L$  namely  $H(L, P(L)) = 0$  and that the Jacobian derivative can be computed independent of model inputs i.e., the yield curve instruments and par rates as,

$$\frac{\partial P}{\partial L} = - \left( \frac{\partial H / \partial L}{\partial H / \partial P} \right)$$

Now, from the definition of the function  $H(L, P)$  we can easily determine  $dH/dP = -1$  which leads to,

$$\begin{aligned} \frac{\partial P}{\partial L} &= - \left( \frac{\partial H / \partial L}{\partial H / \partial P} \right) = \frac{\partial H}{\partial L} \\ &= \frac{d}{dL} (\text{Model Par Rate}) \end{aligned}$$

## For an Interest Rate Swap

$$\text{Par Rate}, p = \frac{PV(\text{Float Leg})}{\sum_{i=1}^n N \tau_i P(t_0, t_i)} = \frac{\sum_{j=1}^m N (l_{j-1} + s) \tau_j P(t_0, t_j)}{\text{Annuity}(\text{Fixed})}$$

- The derivative with respect to  $L$  is trivial to calculate
- We can calculate for any set of calibration instruments
- This allows us to modify and select any risk & hedge buckets

## Appendix – Swap DV01 Risk Example using AAD (Part I)

### IRS Present Value Code

- Swap Price Implementation
- Simplified for Demo Purposes
- For Full Example See

<https://bit.ly/SwapCodeAAD>

```
01 // Swap Inputs
02 // phi    Pay or Receive Fixed: Pay = 1, Receive = -1
03 // n      Swap Notional
04 // r      Fixed rate
05 // tau    Accrual year fraction
06 // t      Coupon Payment Time
07 // f      Floating Forward Rate
08 // s      Floating Spread
09 // z      Discounting Zero Rate for Discount Factor, where df = exp(-z*t)
10
11 double swap_pv(double phi, double n, double r, double tau, double t, double f, double s,
12 double z)
13 {
14     double df = exp(-z*t); // Step 1. Discount Factor using zero rate, z
15     double pv_fixed = phi*n*r*tau*df; // Step 2. Fixed PV =  $\phi N r \tau_1 P(0, t_1)$ 
16     double pv_float = -phi*n*(f+s)*tau*df; // Step 3. Float PV =  $\phi N (l_1 + s) \tau_1 P(0, t_1)$ 
17     double pv_swap = pv_fixed+pv_float; // Step 4. Swap PV = Fixed PV + Float PV
18     return pv_swap;
19 }
```

**Swap Price**

## Appendix – Swap DV01 Risk Example using AAD (Part II)

### Analytical DV01 Risk

- Using Adjoint Mode (AAD)
- Forward Sweep for Price
- Back Propagation for Risk
- Simultaneous Forward and Discount Risk

```

01 double adjoint(double phi, double n, double r, double tau, double t, double f, double s, double z,
double pv_bar)
02 {
03     // Forward Sweep
04     double df          = exp(-z*t);           // Step 1. Discount Factor using zero rate, z
05     double pv_fixed    = phi*n*r*tau*df;      // Step 2. Fixed PV =  $\phi N r \tau_1 P(0, t_1)$ 
06     double pv_float    = -phi*n*(f+s)*tau*df; // Step 3. Float PV =  $\phi N (l_1 + s) \tau_1 P(0, t_1)$ 
07     double pv_swap     = pv_fixed + pv_float;  // Step 4. Swap PV = Fixed PV + Float PV
08
09     // Backward Propagation
10     double pv_fixed_bar = pv_bar;              // Step 4.
11     double pv_float_bar = pv_bar;              // Step 4.
12     double f_bar        = -phi*n*tau*df*pv_float_bar*shift_size_f; // Step 3. *
13     double df_bar       = -phi*n*f*tau*pv_float_bar*shift_size_df; // Step 3. *
14     df_bar              += phi*n*r*tau*pv_fixed_bar*shift_size_df;  // Step 2. *
15     double z_bar        = -t*exp(-z*t)*df_bar; // Step 1.
16
17     // DV01 Result
18     return f_bar + df_bar; // Sensitivity to 1 bps change in forwards and discount factors
19 }

```

Swap DV01 using AD in Adjoint Mode

Source Code: <https://www.onlinegdb.com/edit/al8aNASJnQ>

```

01 // inputs( phi, n, r, tau, t, f, s, z, pv_bar )
02 adjoint( 1, 1000000, 0.02, 1, 1, 0.01, 0, 0.02, 1 ); // Output DV01 Risk

```

Swap DV01 Risk using Adjoint Mode

Have questions or want further info?

## Contact

LinkedIn: [www.linkedin.com/in/nburgessx](https://www.linkedin.com/in/nburgessx)