ISMA CENTRE - THE BUSINESS SCHOOL OF THE FINANCIAL MARKETS

UNIVERSITY OF READING
ENGLAND

IFID Certificate Programme

Structured Securities

Callable Bonds

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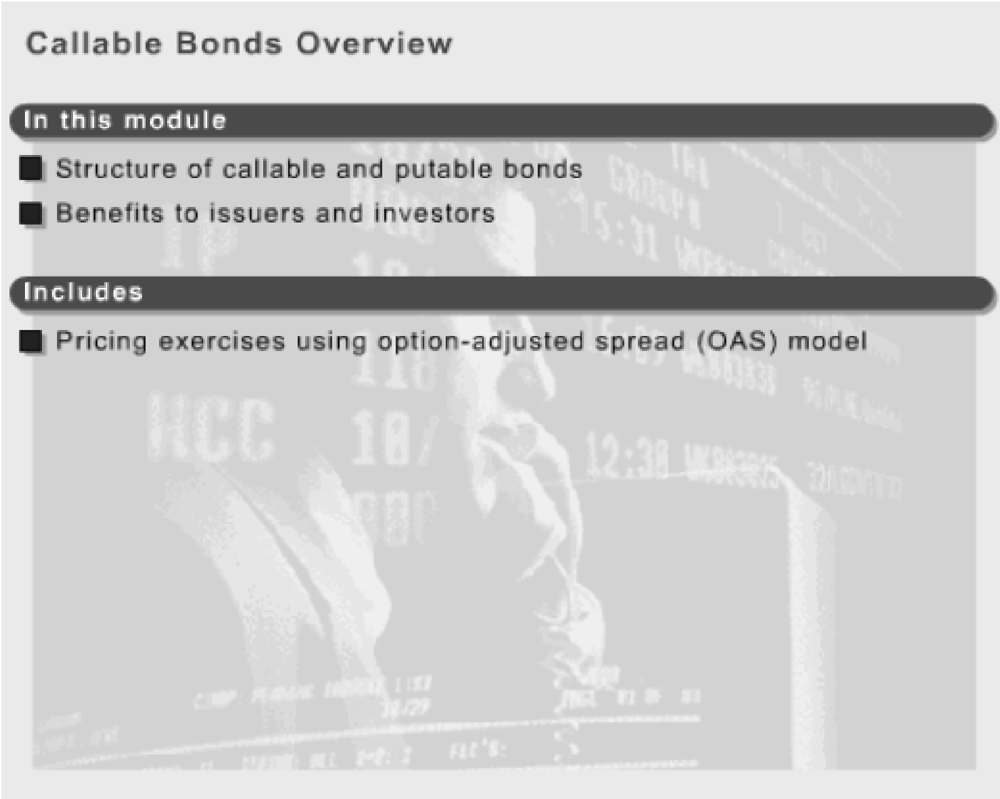
1. Overview



In this module we define the structure of callable and putable bonds, and we outline their key benefits to both issuers and investors.

In the exercises you will have a chance to explore the pricing behaviour of these bonds using an option-adjusted spread model.

Learning objectives

By the end of this module, you will be able to:

1. - Explain the rationale for issuing putable or callable bonds, including step-up callables

2. - Explain the limitations of yield to call and yield to worst as measures of the investment value of a callable bond

3. - Define what is meant by the price compression of a callable bond and describe the relationship between:
    - The price of a callable bond
    - The price of an equivalent option-free bond
    - The price of the embedded call option

4. - Explain how changes in yield volatility affect the price of callable and putable bonds

5. - Describe in outline each of the steps involved in pricing a callable bond using an option-adjusted spread (OAS) model:
    - Modelling the binomial tree for the short rate
    - Calculating the theoretical option-free bond price
    - Calibrating the rates tree
    - Calculating the theoretical option-adjusted bond price
    - Calculating the OAS implied in the actual bond price

6. - Estimate the likelihood that a callable or a putable bond will be redeemed before its scheduled maturity and therefore estimate its approximate duration.

7. - Describe the process of stripping the call features out of a callable bond using a Bermudan swaption

2. Structure

Callable bond:
A bond that gives the issuer the option to repay the principal on the debt:

- On a specified future date or set of dates (the **call dates**) ahead of its scheduled final maturity

- At a pre-determined price or set of prices (the **call prices**)

Putable bond:
A bond that gives the *investor* the right to demand principal repayment:

- On a specified future date or set of dates (the **put dates**)
- At a pre-determined price or set of prices (the **put prices**)

Example

Security: Eastman Kodak 8½% maturing 8 may 2009
Type: USD Eurobond, annual 30/360
Issue date: 8 may 2002
Issue price: Par
Rating AA / Aa2

Call schedule

Call Date	Call Price
8 may 2006	101.50
8 may 2007	101.00
8 may 2008	100.50

As is typical in most callable issues, this bond is callable only on an anniversary date at a progressively smaller premium to par.

The possibility of earlier redemption is a potential benefit to the issuer: if interest rates fall in the future the issuer has the ability to repay the bond early and refinance more cheaply.

On the other hand, the issuer's ability to redeem the bond early (even if at a slight premium to par) comes at a price: callable bonds typically pay higher coupons than equivalent straight bonds.

Question 1

Let's compare the yield on this bond with the yield on equivalent straight bonds with varying maturities.

a) Using the bond pricing model spreadsheet, calculate the yield to call on the Kodak bond following the procedure explained in Bond Yield - Yield to Call / Put. Type your answer in each box, in percentage to 2 decimal places, and hit validate.

Date	Yield on AA/Aa2 Straight	Yield on Kodak Bond
8 May 2006	6.16%	
8 May 2007	6.33%	
8 May 2008	6.50%	
8 May 2009	6.58%	

The yield on the Kodak bond is higher than that on comparable straight bonds and this is designed to compensate the investor for the pre-payment risk. The investor risks having to reinvest her capital, as early as 2006, at possibly lower rates if the bond was called - which it will be if rates fell!

Call features are also present in most convertible bonds. The option can be straightforward, as described here, or it can have an additional link to the performance of the company's share price (see Convertible Bonds - Structure).

2.1. Putable Bonds

Investor Protection

For the investor the advantage of a putable structure is that if market rates rise the investor can force early repayment and reinvest the proceeds elsewhere. This is also valuable insurance, should the creditworthiness of the issuer come into doubt.

Putable bonds are relatively uncommon and tend to be offered by companies with weaker credit ratings as a way of reducing the issue's credit risk. This may be a cost-effective funding strategy, but of course if rates rise the issuer faces the risk of being forced to refinance at the higher rates.

Question 2

In the case of putable bonds the shoe is on the other foot: the option to retire the bond rests with the investor.

a) Which one or more of the following do you think apply?

- ☐ The issuer suffers pre-payment risk
- ☐ The investors earns a higher coupon than on a comparable straight
- ☐ The investors suffers pre-payment risk
- ☐ The investor earns a lower coupon than on a comparable straight

3. Issuing Rationale

The issuer of a callable bond buys an option from the investor and pays for it by offering a higher coupon than they would on a straight bond. However, in most cases callable bond issues are swapped onto a floating rate basis and simultaneously stripped of their call features, so the issuer ends up paying below LIBOR. This is typically the rationale for issuing these securities.

Example

Security:	6% Eurobond maturing in 7 years
Issue price:	Par
Rating:	AA/Aa2
YTM on comparable AA/Aa2 straight bond (= 7 year swap rate):	4.50%

Call schedule

Call Date	Call Price
Year 4	100.00
Year 5	100.00
Year 6	100.00
Year 7	100.00

In this example, the issuer holds a Bermudan call option on the bond for which it pays a coupon, which is 1.50% higher than the rate on a comparable 7 year straight bond. On the issue date the issuer will:

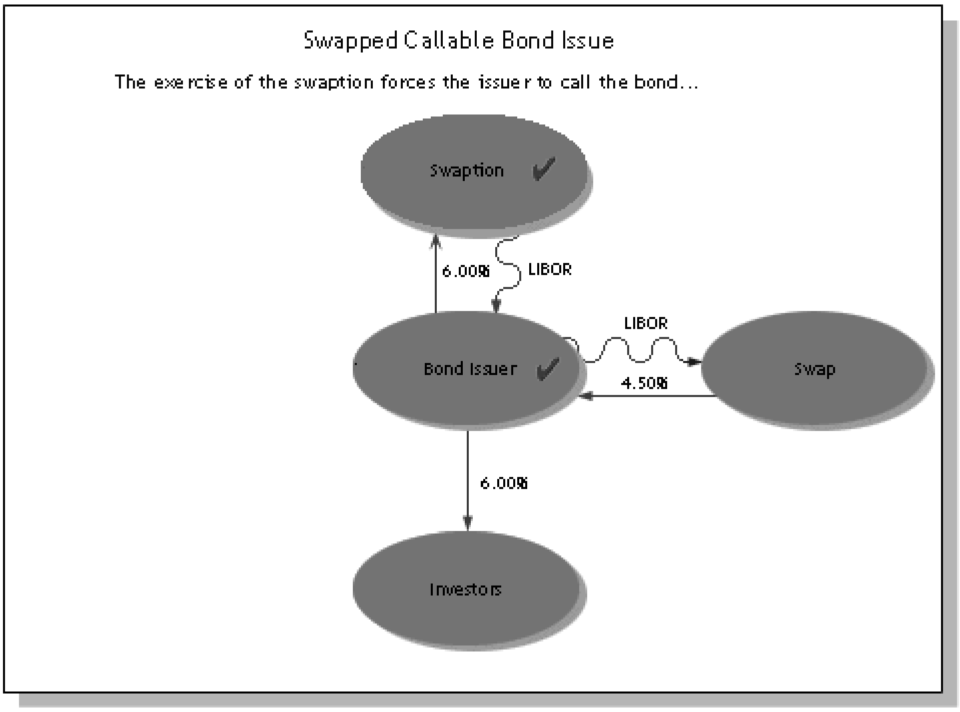
- Enter into a 7 year swap on which it receives 4.50% fixed (the market rate) against LIBOR flat
- On-sell a 3 year Bermudan swaption with an expiry of 4 years on which it pays 6.00% fixed against LIBOR flat - i.e. a **4 into 3 years** payer swaption (see Interest Rate Options - Swaptions).

The terms of the swaption used to strip the callable bond are identical to those in the embedded bond option. Typically, it is the exercise of the swaption which triggers the issuer into calling the bond.

In other words, the same market conditions that would prompt the issuer to call the bond would also prompt the buyer of the swaption to exercise.

3.1. Swapped Callable Bond Issue

The figure below illustrates this mechanism and also shows how the issuer would refinance the bond if the swaption were exercised.



The net result of this structure is that, regardless of whether the swaption is called or not (and hence the bond), the issuer achieves LIBOR-based financing for the full 7 years. Suppose the premium on the swaption, amortised over 7 years, is worth p% per annum to the issuer and consider two scenarios:

Scenario 1: the bond is not called

Net cash flows to the bond issuer:
= Premium - Bond coupon + Net payments on swap
= p - 6.00 + (4.50 - LIBOR)
= **- LIBOR + p - 1.50**

Scenario 2: the bond is called in year 4

Net cash flows to the issuer during the first 4 years are the same as in scenario 1. However, in this scenario, the issuer repays the principal on the bond in year 4 by refinancing itself at LIBOR flat.

Net cash flows during the final 3 years:
= Premium - Rate on new loan + Net swap payment + Net swaption payment
= p - LIBOR + (4.50 - LIBOR) + (LIBOR - 6.00)
= **- LIBOR + p - 1.50**

! The net cost to the issuer is the same, regardless of whether the bond is called or not!

In all scenarios, the issuer ends up paying below LIBOR for 7 years if p > 1.50%
- i.e. if the investors are prepared to sell the embedded call feature in the bond for less than its market value as a standalone swaption.

In most cases, investors accept a lower coupon than they should because they overestimate the likelihood that the bond will be called.

3.2. Step-up Callable Bonds

A step-up callable bond has a provision that its coupon rate will be increased by a predetermined amount if the bond is not called.

Often callable bonds have a step-up coupon structure to enhance the investors' perception that it will be called.

Example

Security:	4% - 8.5% Eurobond maturing in 7 years
Issue price:	Par
Rating:	AA/Aa2
YTM on a comparable AA/Aa2 straight bond:	3.50%

Call schedule

Call Date	Call Price
Year 1	100.00
Year 7	100.00

Step-up provisions

Coupon period	Coupon rate
Year 1	4%
Years 2- 7	8.5%

In this structure, the issuer will raise the coupon rate on the bond from 4% to 8.5% if the bond is not called. This increases the chances that the bond will in fact be called, so investors are encouraged to compare the bond's yield to call with the yield on 1 year paper with comparable credit quality.

4. Pricing

Conceptually, the investor in a callable bond has sold an option to the issuer, where the option premium is reflected in a higher coupon rate. With a putable bond, the investor buys the option and earns a lower coupon. In this section, we examine one approach to pricing such bonds. For illustration we shall focus on callable bonds (which are much more common), but of course the same issues arise in the case of putable bonds.

Yield to Worst

A callable bond has an uncertain maturity and therefore its yield to maturity may give a totally misleading indication of the return the investor is likely to make:

- If the bond currently trades at a price which is higher than the call price - so reinvestment rates are low - the issue is likely to be called, so the investor is likely to earn the yield to call.

- If the bond trades below the call price - so reinvestment rates are high - the bond is unlikely to be called, so the investor is likely to earn the yield to maturity.

In Bond Yield - Yield to Call / Put, we described one approach to valuing a callable bond: the investor compares the yield she would make in the worst case scenario (the **yield to worst**) against the yield that could be earned on a comparable straight bond with the same effective maturity.

4.1. Pricing the Embedded Option

Yield to worst ignores the possibility that the bond may or may not be called and the investor could end up earning more than the yield to worst.

A more realistic pricing approach would take into account the probability that the bond will be called - i.e. it would price the option embedded in the structure.

Example

Security: 12% Eurobond maturing 8 May 2005
Settlement date: 8 May 2002

Call Date	Call Price
8 May 2003	100.00
8 May 2004	100.00

For pricing purposes, we can look at this bond as a combination of two distinct instruments:

1. The investor is long a 3-year straight bond with a 12% coupon ('the bond')

   And at the same time

2. The investor is short a call option on a 12% bond ('the option').

Callable bond = Straight 3 year bond - Bond option

If the issue is called, the exercise of 'the option' neutralises the investor's position in 'the bond'. Valuing the callable bond is therefore a matter of pricing each of these two components separately - the straight bond and the bond option. But there are two important technical problems:

- In Spot Yields, we saw that zero coupon (**spot**) yields are not necessarily the same as coupon (**par**) yields. This implies that the yield on a bond depends on its coupon rate. Since coupon rates on callable bonds are significantly higher than rates on comparable straight bonds, strictly speaking we should value the underlying 'bond' by discounting each of its cash flows using a corresponding spot yield.

- The option itself is a Bermudan-style option. Like American-style options, Bermudan options must be priced using computational techniques such as the binomial model. These are more flexible but also more cumbersome to use than European option pricing models.

4.2. The OAS Model

The valuation approach explained below is based on a model developed in 1990 by Fisher Black, Emmanuel Derman and William Toy (the BDT model). After the Black 1976 model (see Option Pricing - Analytic Models), BDT is probably the most widely-used pricing model in the interest rate options market.

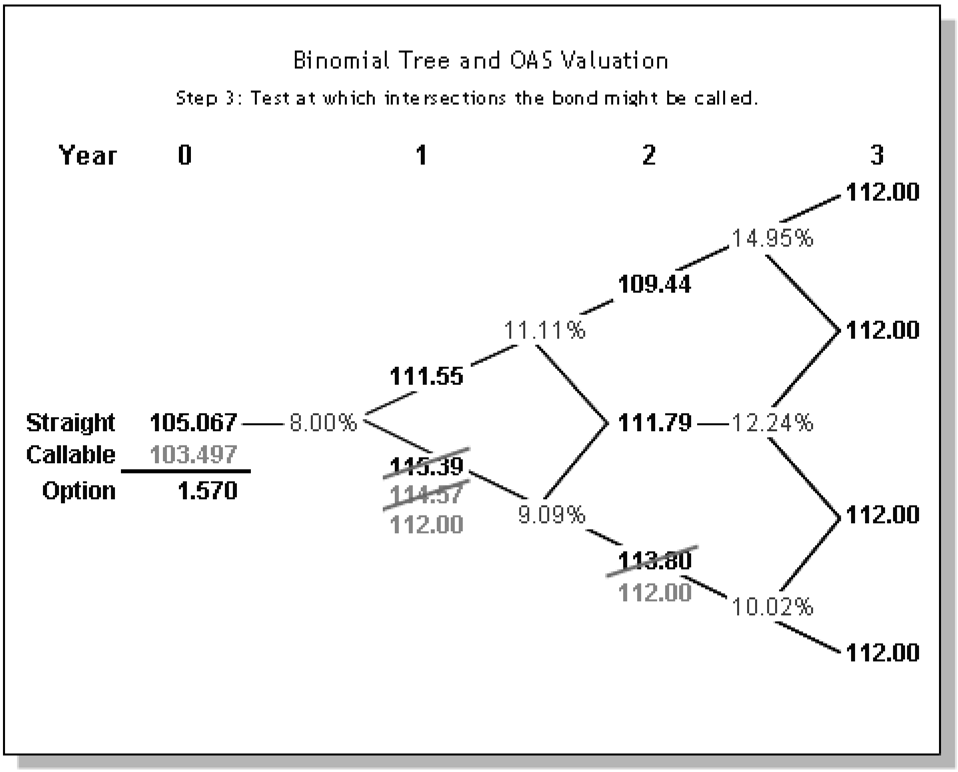
Current Market Rates

The starting point is a set of observed par yields from which we derive the corresponding spot and one-period forward yields using the techniques explained in Spot Yields - Bootstrapping and in Forward Yields.

The relevant yield curves for this market are:

Term	Par Yield	Spot Yield	Forward Yield
1 year (8 May 2003)	8.00	8.00	8.00
2 years (8 May 2004)	9.00	9.05	10.10
3 years (8 May 2005)	10.00	10.14	12.36

We shall use the spot and forward yields (instead of the 3-year yield to maturity) to present-value the expected future cash flows on the bond in the example on the previous page. These cash flows will depend on the probability of the bond being called, which in turn will depend on the volatility of the yields. The pricing method is a 3-step process, which is summarised in the figure below and explained in more detail underneath it.



For the IFID Certificate exam you will not be required to build a binomial tree or calculate the OAS on a callable bond.

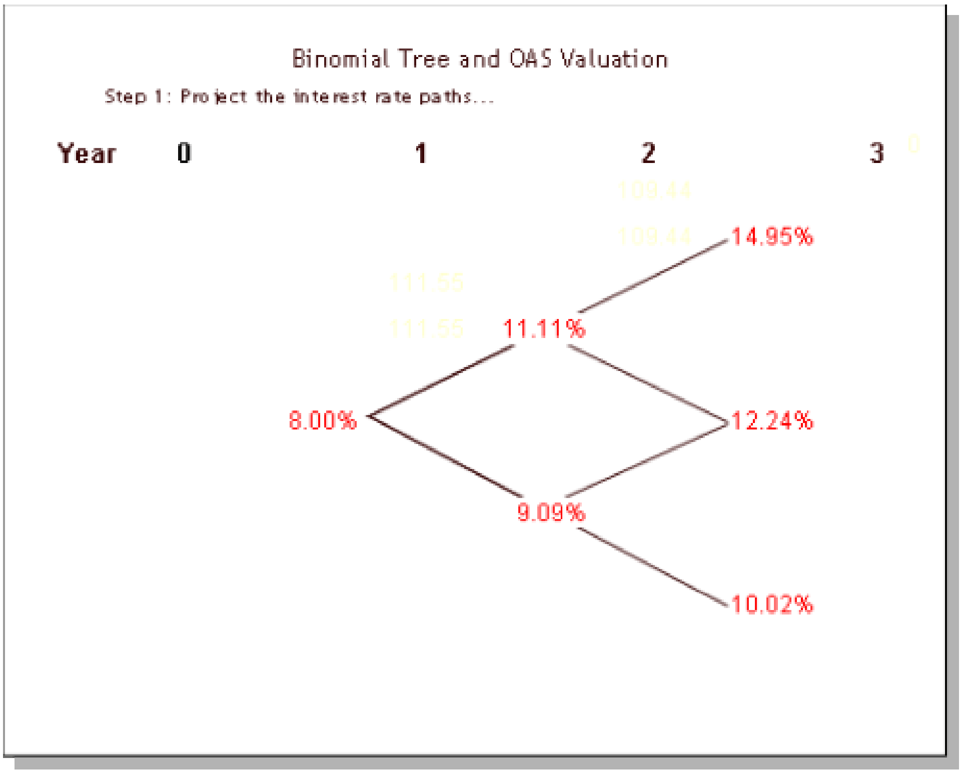
You are only required to describe in outline each of the steps involved in calculating the price of a callable bond using the OAS pricing. These steps are briefly explained below.

Step 1: project the future interest rate paths

Modelling the Rates

Underpinning BDT is a so-called **one-factor model** of the yield curve, where the future shape of the entire curve is driven by the evolution of just one yield - the short term rate. In our case the short term rate is the 1 year yield, so the next step in the valuation process is to model a set of future 1 year yield scenarios consistent with the assumed volatility.

The figure below shows a binomial tree of possible interest rate paths, which is similar to the trees we developed in Options Pricing - Binomial Model. Here the tree is made up of just three nodes; a more realistic valuation would of course subdivide the 3-year period into a much larger number of nodes.



The first node in the tree is the current 1 year yield of 8.00%. *Assuming 10% annual volatility* for this rate, then after 12 months it will either rise to 9.09% ($Y0_{1x2}$) or to 11.11% ($Y1_{1x2}$). These future rates are derived from two key assumptions.

Assumption 1: the 1 year rate follows a log normal stochastic process

This implies that:

$$\mathbf{Y1_{1x2} = Y0_{1x2} \times e^{2\sigma\sqrt{\delta T}}}$$

Where:

e = Exponential coefficient
(see Time Value of Money – Periodic and Continuous Compounding)

σ = Volatility of the underlying - the annualised percentage standard deviation of the 1 year yield, expressed in decimal (i.e. 0.10 in this example)

$\delta T = \dfrac{\text{Time to maturity, in years or fraction}}{\text{Number of nodes in the tree}}$ (i.e. 1 in this example)

Assumption 2: the market is random and the expected value of the 1 year yield is equal to the current 1x2 year forward yield

This means that taking positions in the forward rate would not, on average, generate any profit (the so-called **no-arbitrage condition**):

Expected[Y_{1x2}] = 0.5 x $Y1_{1x2}$ + 0.5 x $Y0_{1x2}$

= Forward $rate_{1x2}$

= 10.10% in our case (the forward rate)

From these assumptions we have two equations from which we can solve for the two unknowns ($Y0_{1x2}$ and $Y1_{1x2}$). Given the same volatility and the model's assumptions it can then be shown that:

- If the short rate rises to 9.09% in year 1 then in year 2 it will either rise again to 10.02% or to 12.24%
- If the short rate rises to 11.11% in year 1 then in year 2 it will either rise again to 12.24% or to 14.95%
- …and so on

Modelling Volatility

So far we have kept the volatility of the short rate constant, but this may be unrealistic. Market prices for interest rate caps and floors typically display a term structure of volatility that implies declining volatilities for the more distant forward rates. In contrast to most other market variables, whose uncertainty (hence volatility) increases as you gaze deeper into the future, interest rates appear to follow a **mean-reverting** process.

Adding a mean-reverting process has the effect of narrowing the distribution of future rates in the binomial tree. We will not show this effect here, so as not to obscure the basic working of the model, but you will have a chance to explore mean-reversion in the *Exercise* section.

More on mean reversion

Mean reversion: the tendency of short term rate to converge towards a long term average level.

Mean reversion in the BDT model can be modelled as follows:

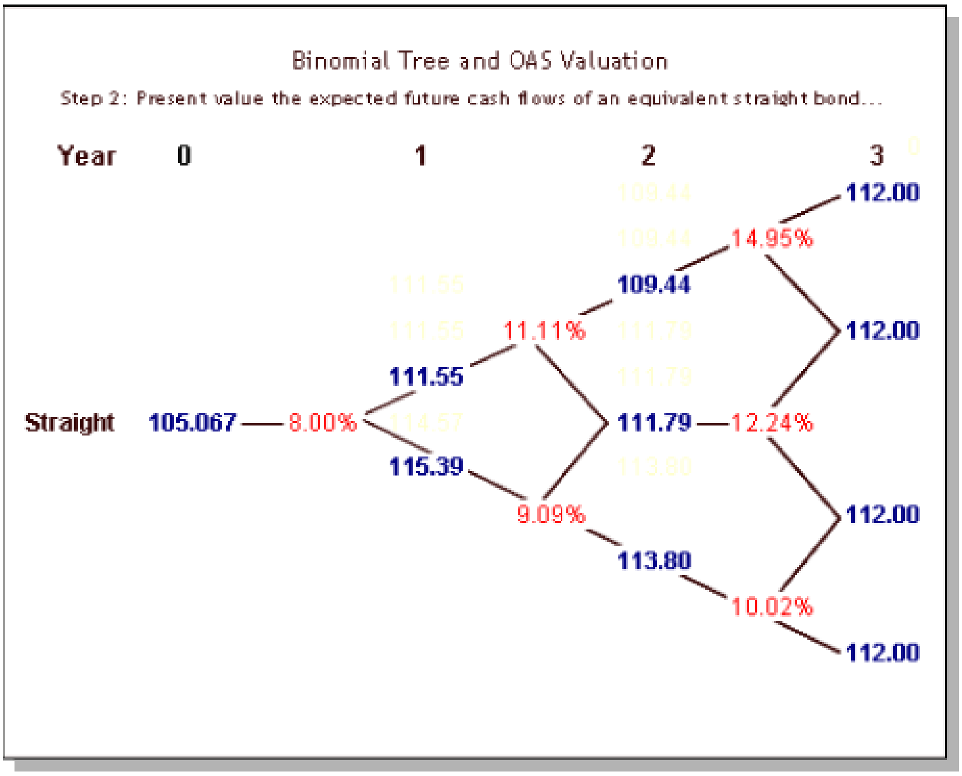
$\sigma_t = \sigma_0 \times e^{-V.t}$

Where:
σ_t = Volatility of the short rate t years after the effective date of the option
σ_0 = Initial volatility of the short rate
V = Reversion speed, in decimal (i.e. 1.8% is 0.018)

The BDT model makes the volatility of the short rate a function of time: the higher the reversion speed, V, the quicker the volatility declines. The reversion speed is calibrated using the market implied volatilities for interest rate caps and is typically of the order of 0.018 - 0.040. In other words, the volatility of the short rate typically declines by between 1.8% and 4% per year.

Step 2: calculate the bond's option-free price

Having generated the tree of future interest rates, the next step is to calculate the **option-free price** of this bond by assuming that it is not callable. Starting from year 3 and working backwards, at each intersection we calculate the dirty price of the bond at each intersection by discounting its future cash flows at the prevailing interest rates.



At the end of year 3 the bond will pay 112.00 (principal plus final coupon) in all scenarios. At the end of year 2 the present value of this bond will depend on the 1-year rate at the time:

- If the rate is 10.02% then, including the current coupon, the bond will be worth:

  $$12 + \frac{112.00}{(1 + 0.1002)} = \mathbf{113.80}$$

- If the rate is 12.24% then you can check by a similar calculation that the bond will be worth **111.79**

- If the yield is 14.95% it will be worth **109.44**, as shown in the figure above

At the end of year 1 the present value of this bond will also depend on the 1-year rate at the time. If the rate is 9.09% then we know that one year later it may rise:

- To 10.02%, in which case the bond will be worth 113.80
- Or to 12.24% in which case the bond will be worth 111.79

Since the probability of each scenario is 50%, **expected future value** of the bond at the end of year 2 will be:

0.5 x 111.79 + 0.5 x 113.80 = 112.79

And the present value of the bond at end of year 1 is:

$$12 + \frac{112.79}{(1 + 0.0909)} = \mathbf{115.39}$$

Using the same procedure we can calculate the other possible values of this bond at the end of year 1 as **111.55**, in the event that the short rate rises to 11.11%. Therefore the value of this bond today will again be the present value of its expected future value at the end of the year:

$$\left(\frac{0.5 \times 111.55 + 0.5 \times 115.39}{(1 + 0.08)} \right)$$

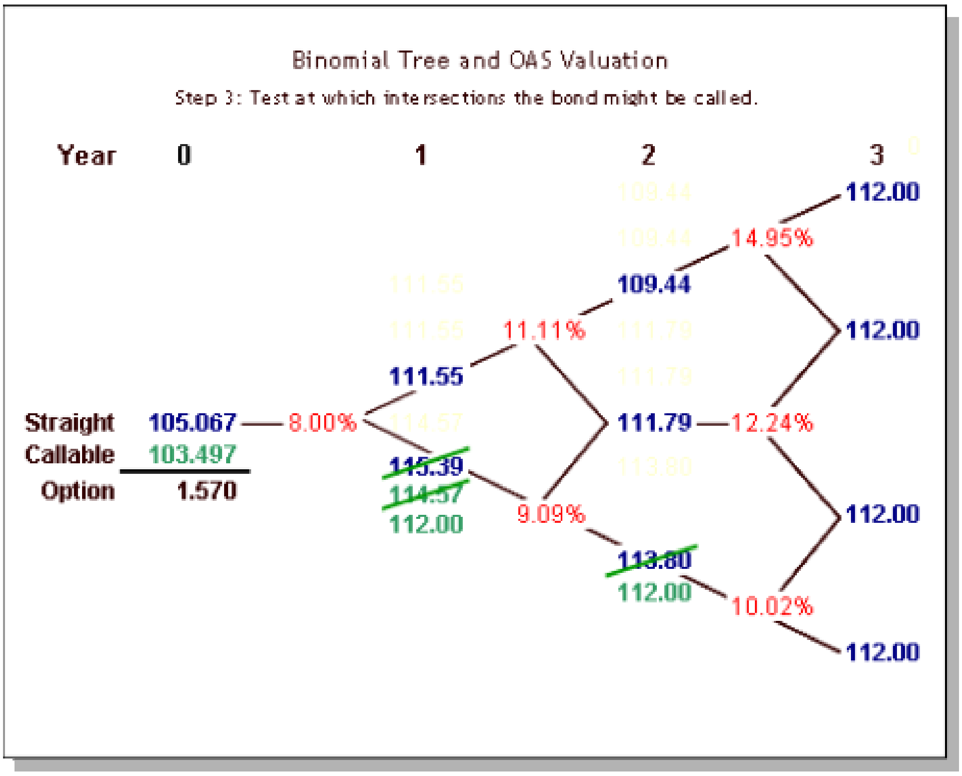
= **105.067**

This would be the fair price of the bond if it had no call features, according to our modelled path of market yields. Using a simple bond pricing calculator, you can check that its **option-free** yield to maturity would be 9.96%.

Calibrating the rates tree: at this point in the process the analyst will also compare the obtained bond price with the market price of equivalent 3-year straight bonds, and if necessary shift the entire rates tree up or down in parallel to ensure that the calculated prices match the observed market prices.

Step 3: calculate the bond's option-adjusted price

The call features raise the possibility that the bond may be called in years 1 or 2 if its present value at those times turns out to be higher than 112.00. This will be the case in year 2, if the yield is 10.02%, so at that intersection (or **yield vertex**) in the model we assume that the bond will be called and replace its previously-calculated present value of 113.80 with 112.00. Since the bond is called at that vertex, we are effectively discarding all subsequent branches of the tree emanating from that point.



The fact that the bond may be called in year 2 also reduces its value at the end of year 1, should rates rise to 9.09%. In fact, the present value of the bond at that vertex is still higher than 112.00, so again we assume the bond will be called and we place a value of 112.00. This reduces the present value of the callable bond today to 103.497. Comparing this option-adjusted price with the bond's option-free price, this implies that the call feature is worth **1.570%**.

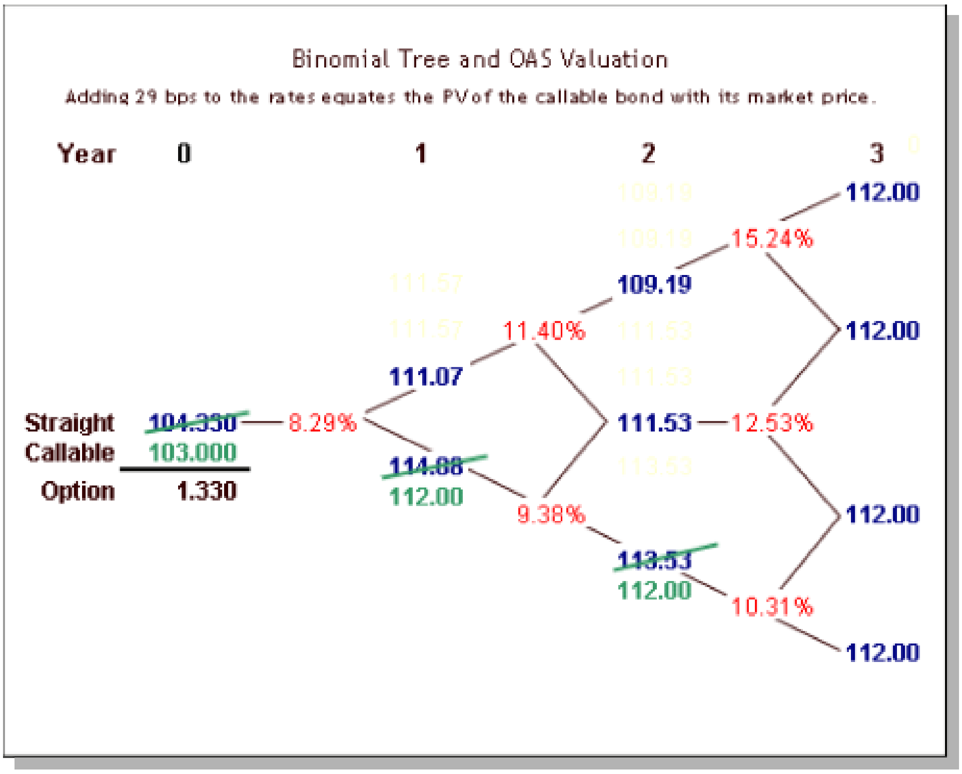
4.3. The Option-adjusted Spread

The OAS is the yield spread that must be added to (or subtracted from) all short term rates in the binomial tree in order to equate the option-adjusted present value of a bond with its market price.

The OAS is used by some market practitioners to assess the extent to which callable (or putable) trade rich or cheap to fair value. To calculate the OAS, we compare the calculated present value of the callable bond (103.497) with its actual market price.

Example

Suppose the bond were trading at 103.00. To reduce the option-adjusted price from 103.497 to 103.00, we need to add 29 basis points to the spot 1 year yield and to each of the future 1 year yields in the binomial tree, as shown below. The bond is said to trade with an OAS of +29 basis points.



The OAS signals potentially profitable trading opportunities:

- A bond that trades with a positive OAS may be considered cheap to fair value
- A bond that trades with a negative OAS may be considered rich to fair value.

5. Convexity

In Bond Market Risk - Convexity, we showed that the higher the yield on a straight bond the lower is its price risk: straight bonds have **positive convexity** and this is a Good Thing. In contrast, callable bonds have **negative convexity**, as the example below illustrates.

Example

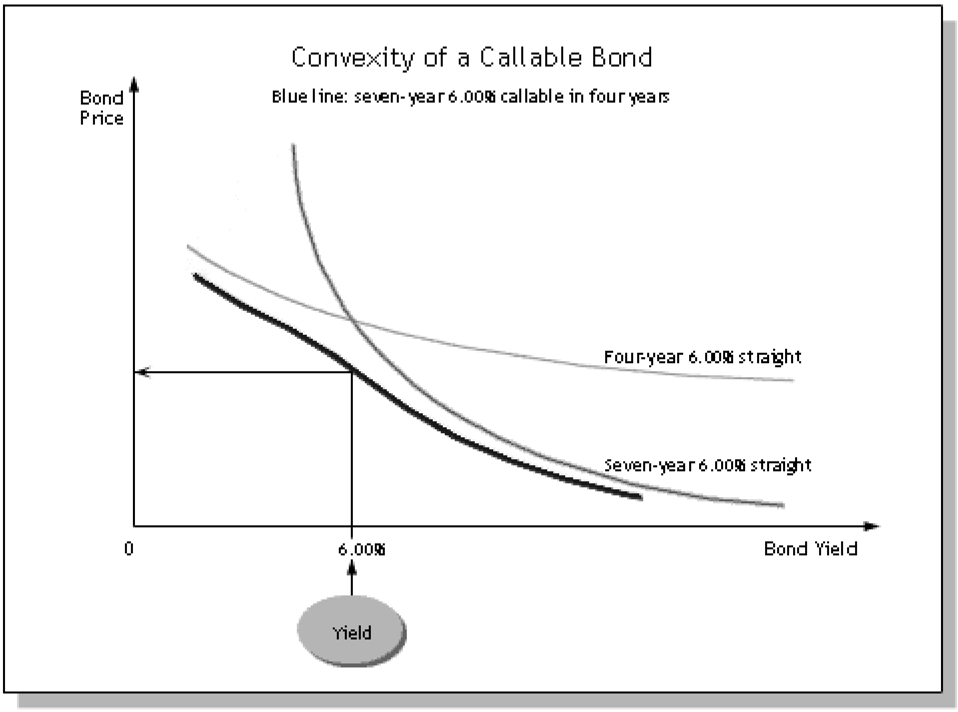
Security: 6% Eurobond maturing in 7 years

Call Schedule

Call Date	Call Price
Year 4	100.00
Year 7	100.00

In the figure below, we compare the price behaviour of this callable bond under different yield curve levels with those of:

- An equivalent straight bond maturing in 4 years
- An equivalent straight bond maturing in 7 years



Interpretation

- As yields rise the callable bond is less likely to be called, so its effective maturity - hence its price behaviour - approximates that of a 7 year bond.

- As yields fall the bond is more likely to be called, so its effective maturity shortens to that of a 4 year bond.

As indicated in the figure, we can interpret the price of this bond as the net of two components:

- The price of a 7-year straight bond with a 6% coupon (the red curve on the chart)

  Minus

- The price of a 4-year call option on a 3-year 6% bond (the shaded area).

As yields fall the value of the first component increases, but this is to some extent offset by the value of the embedded option, which goes ITM.

Implication for Investors

Callable bonds tend to trade with higher yields than equivalent straights, because for a bond holder negative convexity is a Bad Thing: the market risk on the bond increases in a bear market but its leverage decreases in a bull market.

Callable bonds are attractive to yield-seeking investors, but unattractive to fixed income fund managers who need to **duration-match** their portfolios (see *Bond Market Risk - Using Duration*).

The duration of the callable bond tends to be most unpredictable when market yields are close to its coupon rate. At such levels there is an even chance that the bond may be called, so the gamma of the embedded option is at its highest and the bond's duration is most sensitive to changes in market yields.

As a result, we often find such bonds trading significantly cheaper to fair value and this opens profitable opportunities for **asset swappers**, who will:

- Buy these 'unwanted' bonds
- Strip their embedded options using swaptions (see section *Issuing Rationale*)
- Re-offer them as synthetic floating rate notes for attractive yield spreads to LIBOR.

6. Exercise

Question 3

This exercise is designed to consolidate your understanding of the price behaviour of callable and putable bonds using a simple Excel-based OAS bond pricing model similar to the one described in the previous section.

You can safely skip this exercise if you are already comfortable with this topic and with the broad structure of the OAS pricing model.

We shall work with a 4 year bond that is callable at par in years 2 and 3. Please ensure the following data is correctly set in the model:

Cash flows and Yields	1	2	3	4
Call price(s)	(blank)	100.00	100.00	100.00
Base yield curve	8.25%	9.00%	9.50%	9.80%
Volatility of short rate	17.00%			

The bond

Type	Callable
Coupon rate	12.00%
Reversion speed	1.40%
Option-adjusted spread	0.00%

a) Complete the table below. Type your answer in each box and validate.

Calculated clean price:

Straight bond [ ]

Callable bond [ ]

Call option [ ]

Please check the calculator settings, above, if your answers don't match.

b) If the bond trades at 104, what is its OAS (%)?

Instructions

You may try different OAS levels by trial-and-error until you arrive at a calculated price of 104.00 for the callable bond. Alternatively, let Excel® find it for you.

Select **Tools | Goal Seek** in the Excel menu. In the dialog box enter the following (shown in **bold**):

Set cell: **Bond_price**
To value: **104.00**
By changing cell: **OAS**

[ ]

c) A positive OAS means:

- [ ] The bond trades cheap to fair value
- [ ] The bond is likely to be called
- [ ] The bond trades rich to fair value
- [ ] The bond trades at fair value

d) Keeping the OAS at the level calculated in (b), what would you expect the bond price to be for the following levels of volatility?

Volatility of 1x2 year forward rate	Bond price
14.00%	
17.00%	
20.00%	

e) Restore the volatility of the 1x2 year rate back to 17.00% and now change the call price in year 2 to 102.50. Which (one or more) of the following statements are true?

- ☐ The bond is more likely to be called early
- ☐ The embedded option is less valuable
- ☐ The embedded option is more valuable
- ☐ The bond is less likely to be called early

f) Restore the call price in year 2 back to 100.00. We shall now explore how shifts in the yield curve affect the price of this callable bond, compared with the price of an otherwise identical straight bond. Entering the indicated values in the OAS cell, complete the table below and validate.

OAS	Straight bond	Callable bond	Difference
-1.00%			
0.00%			
+1.00%			
+3.00%			

g) Which (one or more) of the following statements are true?

- ☐ The callable bond has lower convexity than the straight bond
- ☐ As yields fall the bond is more likely to be called, and vice versa
- ☐ As yields fall the callable bond behaves more like a 2 year straight bond
- ☐ As yields rise the callable bond behaves more like a 4 year straight bond

h) Restore the OAS back to 0.00% and now make the bond *putable*, instead of callable. Which (one or more) of the following statements are true?

- ☐ As yields rise the putable bond is more likely to be put
- ☐ An increase in volatility reduces the price of the putable bond
- ☐ As yields fall the putable bond behaves more like a 2 year straight bond
- ☐ The putable bond has higher convexity than an otherwise equivalent straight