ISMA CENTRE - THE BUSINESS SCHOOL
OF THE FINANCIAL MARKETS

UNIVERSITY OF READING
ENGLAND

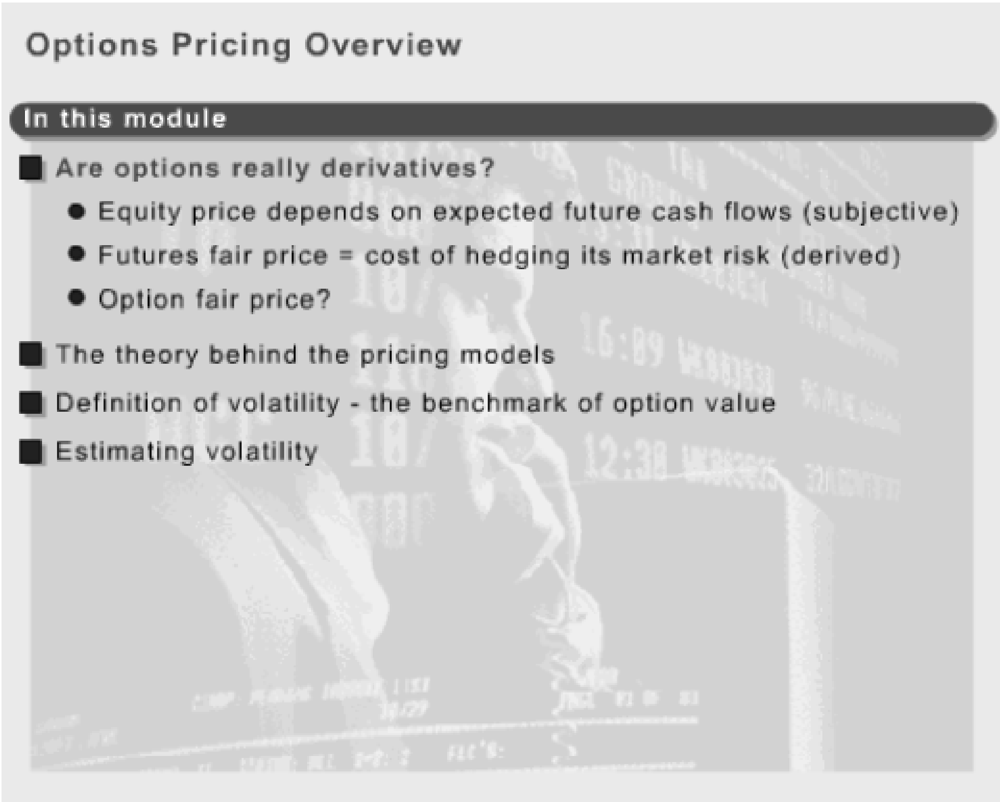
IFID Certificate Programme

Rates Trading and Hedging

Options Pricing and Risks

Table of contents

1. Overview



Are Options Really Derivatives?

Options have been traded for many hundreds - even thousands - of years, but it was not until the early 1970s that the theory behind their pricing has been properly understood. The central issue is whether options are really derivatives, or whether they are just another class of financial instruments, like bonds or equities.

The price of a corporate bond or of an equity is the present value of its expected future cash flows. Since nobody knows with certainty what those future cash flows will be, there is little sense in talking about fair prices: what might be a fair price for a bull may be considered far too expensive for a bear.

On the other hand, a futures (or forward) contract is a derivative because its price does not depend on expectations about the future but rather on the cost of hedging its market risk (see the pricing sections in *Futures*). So in this context it makes sense to talk of fair pricing.

Since the value of an option depends on the probability of the option being exercised - and probabilities are subjective - is the option really a derivative? And can we establish a fair price?

In this module...

- We discuss the theory behind options pricing models
- We present a rigorous definition of **volatility** - the benchmark of option value
- We examine different ways of estimating volatility

Option pricing models are complex and you can skip some of the 'technical' sections if you like. However, if you really want to gain a good feel for the issues involved it is important that you work through the *Exercises* using the pricing software supplied.

You are not expected to derive or apply option pricing models for the IFID Certificate programme. The models included in the IFID Study Pack are there only for your future reference and to help you understand conceptually the factors that drive option prices (which you are required to know for the IFID exam).

Having explored the key variables that drive an option price, we will also show how we can use the models to assess the sensitivity of an option's price to changes in the model inputs. These sensitivity measures, known collectively as **the Greeks**, are the tools with which the option trader manages her exposure to each dimension of option risk.

Exercise 2 below illustrates how the options trader uses the Greeks to manage potentially catastrophic risks.

Learning Objectives

By the end of this module, you will be able to:

1. - Describe the risk-neutral backward induction method of valuing an option using a binomial tree of the underlying asset price
2. - State the key theoretical assumptions underpinning standard analytical option pricing models
3. - Apply put-call parity principles to derive the implied synthetic positions generated by combinations of futures and options
4. - Define and interpret:
    - Historic volatility
    - Implied volatility
    - The volatility curve
5. - Explain the square root of time rule and apply it to the calculation of annual historic volatility from sample data
6. - Outline different approaches to forecasting future volatility
7. - Analyse the exposure of an options trading book in terms of:
    - Delta
    - Gamma
    - Vega
    - Theta
    - Rho and Psi
8. - Calculate the net delta of an options position and implement a delta hedge
9. - Distinguish between an option's nominal gearing (or leverage) and its effective gearing
10. - Interpret gamma as a measure of exposure to actual volatility
11. - Define the volatility smile and the volatility skew and suggest reasons for its existence

2. Binomial Model

2.1. Key to Pricing is the Hedge

The key to pricing an option as a derivative is to consider how its risks might be hedged by carrying a position in the underlying instrument. First, we illustrate the binomial pricing approach with a simple example. Later, we shall refine this approach to handle more realistic conditions.

The Problem

A trader is asked to quote a price for the following:

Trade:	Sell 3 ATM calls
Underlying:	A non income generating asset
Underlying price:	$100
Option Style:	European
Expiry:	12 months
12 month LIBOR:	25%

There are three related issues for the trader:

What should the trader quote for the calls, just to break even?

? How much of the underlying asset should he buy in order to hedge the market risk on the options?

What are the funding implications?

To begin with, assume a simple world in which there are only two possible scenarios. By the options expiry, the underlying asset may:

- Either trade down to $50
- Or trade up to $200

The Solution

In this simple world there is a simple mathematical answer to the trader's questions. We shall demonstrate that the only possible solution is as follows:

1. The trader should charge $40 per contract, so for three contracts the total premium income would be $120.

2. Having sold the three calls the trader should hedge by going long 2 of the underlying securities, at a total cost of $200. The **hedge ratio** in this case is 2/3 or 67%. (In fact this is the options' **delta**, see Option Risks - Delta).

3. Net of the premium received, the trader is left with a funding shortfall of $80 which he borrows. At 25% interest, this means he has to repay $100 ($80 plus 0.25 x 80) next year, irrespective of the outcome.

! Having implemented this strategy the position is fully covered against market risk: the hedged portfolio is a risk-free package!

The Outcome

Let's examine what might happen at the expiry of the options:

- **Scenario 1: the stock price falls to $50**
  The options expire worthless and the trader unwinds the hedge, recovering $100 (2 x $50) on the shares sold. This generates exactly the cash required to repay the loan.

- **Scenario 2: the stock price rises to $200**
  The options expire in the money and the counterparty pays a total of $300 to buy the 3 securities (3 x the strike of $100).

  But the trader only has two securities to deliver, so he purchases the third one in the current market at $200. This leaves a net cash surplus of $100, which again is just sufficient to repay the bank loan!

The figure below summarises the **binomial** (i.e. two-state) evolution of the underlying security price (S) and the trader's position in each case.

Spot		12 Months	
		S = $200	
		Calls are exercised: @ $100 ea.	+300
		Buy the 3rd security @ $200	-200
		Repay loan + Interest @25%	-100
		Net cashflow	**0**
S = $100			
Sell 3 ATM calls @ $40 ea.	+120		
Buy 2 securities @ $100 ea.	-200		
Fund the shortfall	+ 80		
Net cashflow	**0**		
		S = $50	
		Calls expire OTM	0
		Unwind the hedge @ $50 ea.	+100
		Repay loan + interest @25%	-100
		Net cashflow	**0**

Arbitrage pricing: the fair price of any derivative equals the cost of hedging its market risk.

Probability is Not a Factor

The trader has a hedge that works equally well in either future scenario. If he quoted $41 for each option, then he would make an arbitrage profit of $1 per contract; if he quoted $39 he would be sure to lose $1.

The example illustrates this important principle in derivatives pricing: if you can hedge it (and you know the cost of the hedge) then you can price it!

Notice that at no point does the trader assess the *probability* of the underlying price going up or down, or of the option expiring ITM or OTM. Having specified the volatility of S, both the option's fair price and its hedge ratio can be established irrespective of whether the trader believes there is a 99% chance that S will rise, or a 99% chance that it will fall. Whether you are a bull or a bear the fair price will be the same!

2.2. Alternative Solution

Since probabilities do not come into the pricing (or hedging) calculation, we could assume anything we like about the probability distribution of the underlying price - it makes no difference to the option's price.

We could, for example, assume that the probability distribution of the underlying price is normal (or log-normal), with its mean centred around the current market level (see *Fundamental Statistics - Normal Distribution*). This is what we would expect if the price action follows a **random walk**.

Random Walk
A market follows a random walk if at any time there is an equal probability that the price may rise or fall.

Assuming a random walk, a probabilistic approach would calculate the fair price of the option in the example above as the present value (PV) of its expected (i.e. probability-weighted) future cash flows.

- There is a 50% chance that the option will expire ITM, in which case the investor earns the option's intrinsic value of $100 (i.e. spot price of $200 - strike of $100)

- There is a 50% chance that the option will expire OTM, in which case the investor receives nothing

Option fair price = PV of expected future cash flows

$$= \left[\frac{(0.50 \times 100) + (0.50 \times 0)}{(1 + 0.25)} \right]$$

=**$40**

…the same price we calculated using the binomial model!

Risk-neutral Valuation

The numerator in this equation is the option's expected (i.e. probability-weighted) future cash flow, and the denominator is a discount factor.

Notice that we present-value the future cash flow using a risk-free rate - without adding a risk-premium for the uncertainty of the option's payoff. Although the option itself is risky, this risk may be fully hedged so a risk-free rate is appropriate.

Remarkably, the probabilistic model gives the same result as the binomial model - only by a much quicker route. The random walk assumption is just a mathematical convenience.

2.3. Adding Some Realism

Of course, the notion that the underlying price can only double or halve over one year is unrealistic. If we divide the time to expiry into a number of smaller sub-periods, or **time nodes**, we can break down the evolution of the underlying price into a series of more realistic price changes.

Example

Option type:	Call
Spot:	100.00
Strike:	100.00
Expiry:	1.00 year
LIBOR:	25.00%
Yield:	20.20%
Volatility (σ):	31.00%

As before, we assume the same price range for the underlying at the option's expiry - between $50 and $200 - except the **binomial tree** is now broken down into 5 time nodes, each one lasting 0.2 of a year. We also keep the same risk-free rate of 25% (a different rate would result in a different option price).

Nodes

		1	2	3	4	5
						200
						100
					174.11	
					72.17	
				151.57	74.11	**151.57**
				49.43		51.57
			131.95	51.57	**131.95**	
			32.31		31.64	
		114.87	33.4	**114.87**	31.95	**114.87**
		20.33		18.43		14.87
		20.88	**100**	18.58	**100**	
Call price:-	**100**		10.38		7.08	
European	12.4		10.45	**87.06**	7.08	**87.06**
American	12.68	**87.06**		3.37		0
		5.71	**75.79**	3.37	**75.79**	
		5.74	1.6		0	
			1.6	**65.98**	0	**65.98**
				0		0
				0	**57.43**	
					0	
					0	**50**
						0

Modelling the Asset Price
The figures in white above show the price behaviour of the underlying asset as we move forward in time, starting from today (node 0) where the price is known to be $100. Thus at node 1:

- If the price rose to $114.87, then at node 2 it will:
  - Either rise again to $131.95
  - Or fall back to $100

- If the price fell to $87.06, then at node 2 it will:
  - Either fall again to $75.79
  - Or rise back to $100.00

And so on for nodes 3-5. At each intersection in the binomial tree we assume that the price either rises or falls from that point by a given fixed percentage amount (for an explanation of the math behind these calculations, see *Math of the Binomial Model*).

Pricing the European Option
Having modelled the asset price, the next step is to price the option at each intersection, this time starting from the last time node and going backwards. Thus, at node 5 (the option's expiry):

- If the underlying trades at $200, the option is worth $100 (its intrinsic value)
- If the underlying trades at $151.57, the option is worth $51.57
- And so on, for all the other intersections at node 5

Going back one period, at node 4 if the underlying trades at $174.11 then a European option would be worth $72.17. This is the PV of the option's expected future cash flow at that point, given that from that point there is a 50% chance that it might be worth either $100 or $51.57 at expiry.

Similarly, at node 4 if the underlying trades at $131.95 then the European option would be worth $31.64 at that point, given that from that point there is a 50% chance that it might be worth either $14.87 or $51.57 at expiry.

Having calculated the option price at all the intersections in node 4, we then repeat the same calculations for all the intersections at node 3, and so on until we reach the present time.

Pricing the American Option
The binomial model is suitable for pricing European options (figures in black) or for American options (figures in red). To price an American option we also need to test the following condition at each intersection:

American call option price = MAX { Spot - Strike, PV(European option) }

In other words, if the calculated PV of the option's expected cash flow at any intersection is lower than its intrinsic value at that point, then we assume that the rational investor will exercise the option immediately and therefore its price will reflect its intrinsic value. For example, at node 4:

- If the underlying is $174.11, the American option's price is $74.11 (its intrinsic value) and not $72.12 as calculated earlier

- If the underlying is $131.95, the American option's price is $31.95 and not $31.64

This implies that at node 3, if the underlying is $151.57 the American call is worth more than an equivalent European option, because its expected future value will be higher. In fact, at that intersection the PV of the American option's cash flows is still lower than its intrinsic value at that point, so again we replace its PV by its intrinsic value. The cumulative effect of these adjustments means that the American option in the given market conditions is 28 cents more expensive than the equivalent European option.

What Makes the Models Realistic?

Of course even a 5-node binomial tree is far too crude; nowadays practitioners work with at least 10,000 nodes. In the limit, as the number of nodes is increased, the binomial tree approximates the **normal probability distribution** and the binomial pricing model converges to the analytic pricing models discussed in the next section

Although for convenience the models assume that the underlying price is normally (or log-normally) distributed, it is important to remember that this is not the critical assumption.

The critical assumption of options pricing models is that the trader should be able at all times to dynamically hedge the risks on the option position by trading varying amounts of the underlying, as she travels from one time node to the next.

If that were not the case - for example, if there was insufficient market liquidity - then *all* option pricing models would be in serious trouble! We shall explore the art of hedging options risks in practice in Option Risks - Exercises.

3. Analytic Models

3.1. Closed-form Solution

In section *Binomial Model* we saw that pricing an option on the *working assumption* that the underlying price follows a log-normal probability distribution, yields the same result as pricing it from the cost of hedging its risks.

The models described in this section are analytic in the sense that the option price can be reduced to a relatively small **closed-formed** expression, whereas the binomial models require a whole series of calculation steps.

To derive the analytic models from first principles requires the use of **stochastic calculus** which lies beyond the scope of this program. Here we just give you some of the intuition behind these models, as well as the key formulas for reference purposes.

For the sake of clarity, we shall assume that the price is normally distributed, rather than strictly speaking log-normally distributed.
Using the normal distribution we can estimate precisely the probability of the underlying market price going through an option's strike, given its mean and standard deviation.

We can then calculate the option's **expected payoff**, hence its fair price, as the following simple examples illustrate.

Example 1: At the Money Call

The underlying markets:

Cash price:	$100
Volatility:	10% per annum
Dividend yield:	6.00%
12-month LIBOR:	6.00%

The option:

Type:	Call
Style:	European
Expiry:	12 months
Strike:	$100

? What is the fair price for this option?

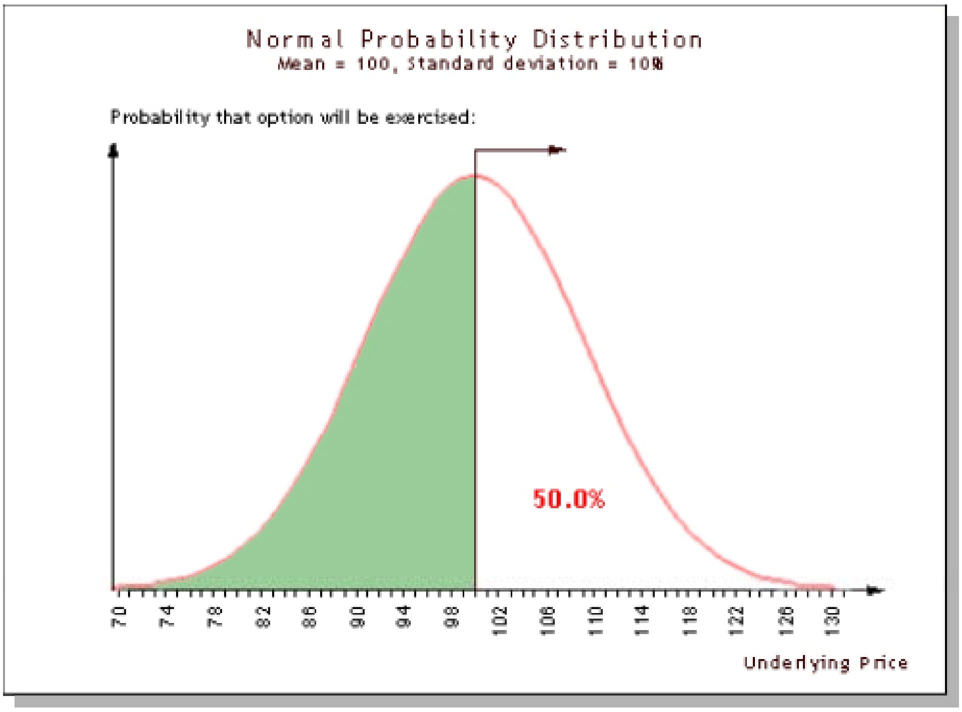
Volatility
In the options market, volatility is the standard deviation of the (assumed log-normal) price distribution of the underlying market, expressed in percentage per annum.

The option's fair price is the present value of its expected future payoff - that is, its future payoff under each possible price scenario, weighted by the probability of that scenario occurring.

With a normal distribution there are an infinite number of possible price scenarios - the normal distribution curve is a **continuous function** - so calculating the probability-weighted cash flow requires the use of calculus. However, we can get some idea of how these models work by reasoning as follows:

1. Calculate from the normal distribution the probability that the option will expire in the money - i.e. that the underlying price will be higher than $100.00.

   Because of the symmetry of the normal distribution, in this case we have a 50% chance of exercising. This means that if we repeated this experiment 100 times - if we could buy this option every year in the same conditions over 100 years - then statistically we can expect the asset price to finish above the strike 50 times out of 100.



1. Next, ask the following question: if the option does expire ITM, is it likely to be more or less than $10 in the money?

   The shape of the distribution suggests that if the option expires ITM it is more likely to be less than $10 ITM. In other words, if we repeated this experiment 100 times, of those times that the price at expiry ends up higher than $100 it is more likely to be close to $100 than a long way up.

   Just by eye-balling the curve - and this is not scientific! - of the 50 out of 100 times that we are able to exercise we could be looking to make, on average, around $8.00. Since we only have a 50% chance of making this, the probability-weighted future cash flow of the option is:

   (0.50 x 8.00) + (0.50 x 0) = $4.00

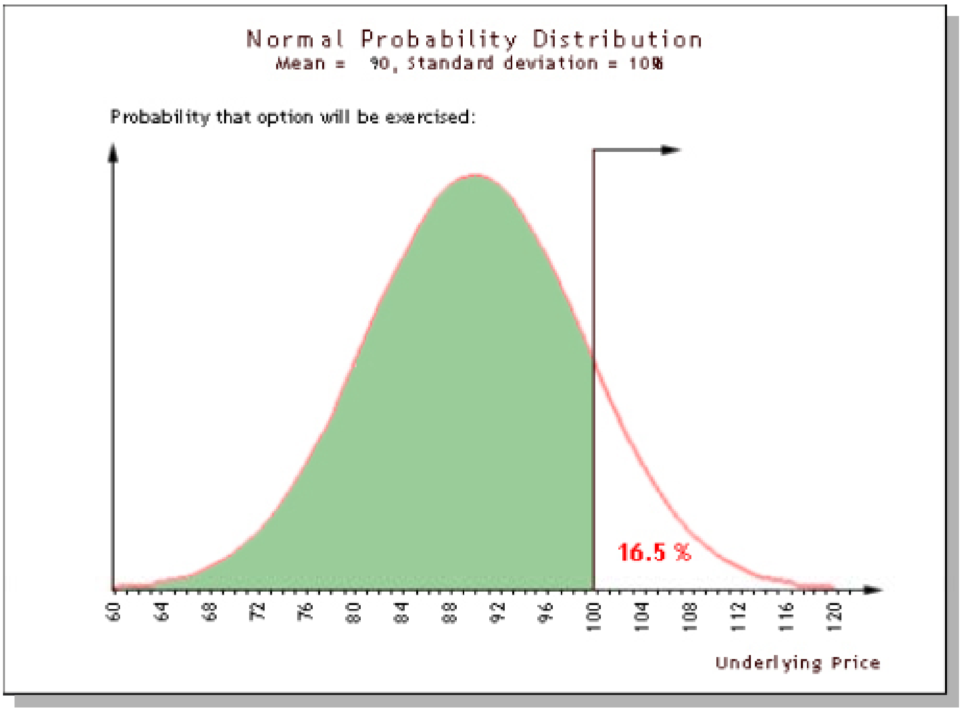
2. Finally, present-value this expected future cash flow, discounting it by the risk-free rate of 6%.

   Present value of call
   = fair price
   = 4.00 / (1 + 0.06)
   = $3.77

Even this crude estimation generates a fair price which is only 1 cent away from that calculated using the Black-Scholes-Merton model, shown in *Example 2*.

3.2. Example 2: 10% OTM Call

We use the same data as in *Example 1*, except that the underlying stock now trades at $90 instead of $100. The figure below shows the probability distribution of the underlying price, which is now centred around $90.



From the normal distribution, the probability of exercising the call is now only 16%:

- We have a 68% probability that the underlying price will be within a range of $80-$100 in 12 months' time (i.e. one standard deviation, using round figures)
- Therefore we have a 32% probability that the price will be either below $80 or above $100
- Because of the symmetry of the distribution, we therefore have a 16% chance (= 32% / 2) that the price will be below $80, and a 16% chance that it will be above $100

Again, the shape of the distribution curve suggests that, if we do exercise, we can expect to make less than $10. Just eye-balling the curve we could be looking to make about $4.50, on average. Since we have only a 16% chance of making even this:

Probability-weighted cash flow = (0.16 x 4.50) + (0.84 x 0.0)
= $0.72

And present value of call = 0.72 / (1 + 0.06)
= $0.68

The Black-Scholes model will also price this option at $0.68!

Black-Scholes-Merton Option Pricing Model
The analytic model described here is just for reference only. You are not required to know or apply option pricing models in the IFID Certificate exam!

The pricing model summarised below was first formulated by Fisher Black, Myron Scholes and Robert Merton in 1973, and later adapted by Mark Garman and E. Kohlagen for FX options.

The Black-Scholes-Merton Model (BSM)

$$C = e^{-RT} \times [S \times e^{(R-Y)T} \times N(d) - K \times N(d - \sigma \times \sqrt{T})]$$

Where:
C = Call option premium
e = Exponential coefficient
(see Fundamental Math - Powers & Exponentials)
R = Funding rate, in decimal (e.g. 6% is 0.06)
Y = Yield on the underlying asset, in decimal
S = Underlying cash price
K = Strike
σ = Volatility of the underlying price
T = Time to expiry, in years or fraction thereof

$$d = \frac{Ln\{ S/K \} + (R - Y + \frac{1}{2}\sigma^2) \times T}{\sigma \times \sqrt{T}}$$

Ln{...} = Natural logarithm
N(...) = The cumulative standard normal probability distribution function,
where $N(-\infty) = 0$, $N(0) = 0.5$ and $N(+\infty) = 1$

4. Put-Call Parity

4.1. Arbitrage Relationships

Prices of European puts and calls can be derived from each other through simple arbitrage relationships. For example, if we know the price of a call then the price of a put with the same strike can be derived from a logical parity between the put, the call and an equivalent futures (or forward) position.

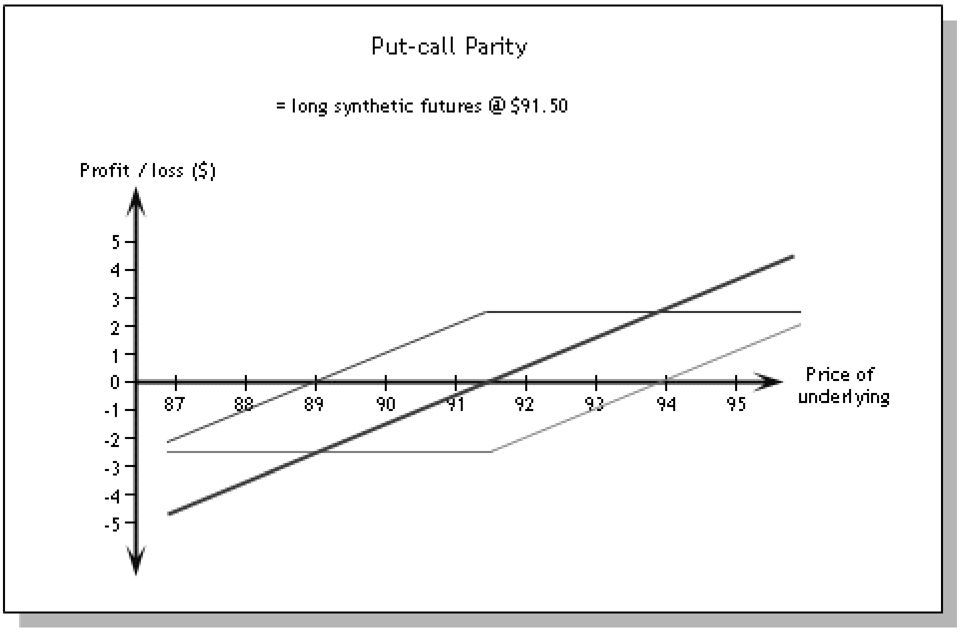
Consider the following position:

- Long an ATM forward European-style call
- Short an ATM forward European-style put

In effect, this position is a **synthetic long futures**. Since both options are European, neither one may be exercised before expiry. At the expiry of the options:

- If $S > K$ we exercise the call, thus becoming long the underlying
- If $S < K$ the puts are exercised against us, making us long the underlying

Either way we become long the underlying, which is the same as being long a futures.



Put-call parity shows that prices of European calls, European puts and their equivalent futures are related to each other by arbitrage, and you can derive the price of any one of these if you know the prices of the other two.

4.2. Put-Call Parity Relationships

Long European call	+	Short European put	=	Long synthetic futures
Short European call	+	Long European put	=	Short synthetic futures
Long European call	+	Short futures	=	Long synthetic put
Short European call	+	Long futures	=	Short synthetic put
Long European put	+	Long futures	=	Long synthetic call
Short European put	+	Short futures	=	Short synthetic call

The math behind put-call parity

Given the price of a call, we can derive the corresponding put price as follows:

P(K) = C(K) - Present value of equivalent futures position

$= C(K) - e^{-RT} \times (F - K)$

$$\mathbf{= C(K) - e^{-RT} \times (S \times e^{(R-Y)T} - K)}$$

Where:

P(K) = Premium price of a put with strike K
C(K) = Premium price of a call with strike K
e = Exponential coefficient
(see Fundamental Math - Powers & Exponentials)
R = Funding rate, in decimal (e.g. 6% is 0.06)
Y = Yield on the underlying asset, in decimal
K = Strike
F = Futures price
S = Cash price of the underlying instrument
T = Time to expiry, in years or fraction thereof

Note that if F = K (i.e. the options are ATM forward) then P(K) = C(K)

! ATM forward European-style puts and calls should trade for the same premium!

5. Historic Volatility

Of all the variables that go into option pricing, volatility is the most difficult one to determine. Indeed, trading options is ultimately about taking a view on the volatility of the underlying during the term of the option. Traders estimate future volatility from two main sources: **historic volatility** and **implied volatility** - the volatility implied in current option premiums.

Historic Volatility: the annualised standard deviation of the historic return on the underlying instrument over a selected sample period.

Limitations

Historic volatility only takes the trader so far:

- The 250-day year assumption is not universal: some markets - e.g. foreign exchange - work 7 days a week, although liquidity is much lower over week-ends. Even in markets which close for the week-end, many traders continue to reflect on conditions and very often prices gap on Mondays.

- Five days is too small a sample, but what should be the correct sample size? The volatility estimate depends very much on the historical period chosen.

- Most importantly, history is an unreliable guide to the future!

Example
Calculating historic volatility.

The following were the daily prices observed in a market over the course of one week:

1 Day	**2 Price**	**3 Change(%)**	**4 Deviation from Mean**	**5 (Deviation)^2**
1	99.40	-0.649120	-0.739410	+0.546725
2	99.60	+0.201005	+0.110717	+0.012258
3	100.10	+0.500752	+0.410464	+0.168481
4	100.00	-0.099950	-0.190240	+0.036191
5	100.50	+0.498754	+0.480847	+0.166844
Sum		**+0.451441**		**0.930498**

From the sum of column 5 we calculate the standard deviation of this series. If n is the number of observations in the sample:

Standard deviation $= \sqrt{[\ \Sigma \text{ Deviations}^2\ /\ (n-1)\ ]}$
$= \sqrt{(0.930498 / 4)}$
$=$ **0.482312%**

This is the daily historic volatility of this market estimated from a 5-day sample. Option pricing models require an annual volatility, so the final step is to annualise the estimate using the square root of time rule:

Annualised volatility = Observed volatility x √ Days in year

Since most markets are closed on week-ends and public holidays, analysts typically use a 250-day working year, rather than 365.

Annualised volatility $= 0.482312 \times \sqrt{250}$
$=$ **7.63%**

Description of the columns

Column 3:
Calculate the daily percentage changes in the price, and their sum. The convention is to use the approximating formula introduced in module Time Value of Money – Periodic and Continuous Compounding:

Daily return (%) = Ln (P_t / P_{t-1}) x 100

Where
P_i = Price at time i
Ln(..) = Natural logarithm

We use this formula in preference to the conventional percentage change calculation because in the option pricing models the expiry of the option is broken down into a very large (in the analytic models infinite) number of time nodes, so the price change from one node to the next is assumed to be very small.

The **mean** of this series is the arithmetic average of all the observations.

Mean = + 0.451141 / 5
= + 0.090288%

Column 4:
Calculate the difference between each daily percentage price change and the mean.

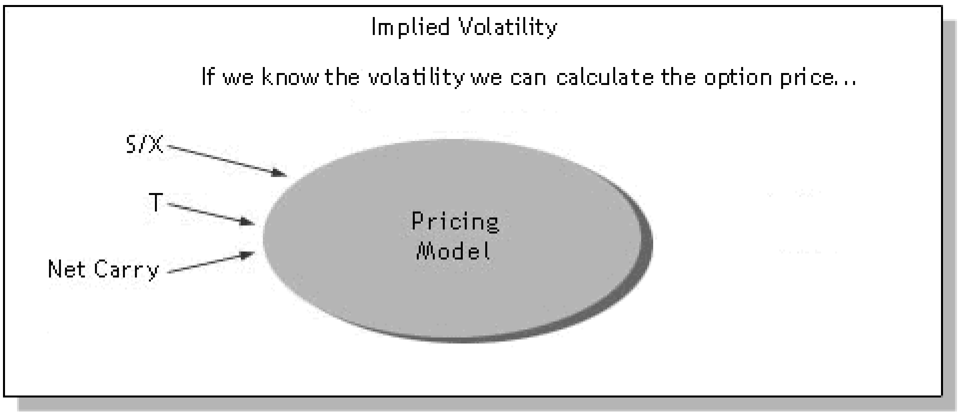
Column 5:
Calculate the square of the differences, and then their sum.

6. Implied Volatility

Implied Volatility:
The volatility level that is consistent with a quoted option premium, taking into account all other market factors.

We know where the underlying price is at any time, and we can be reasonably certain about the cost of carrying it, but nobody really knows what the volatility of the market actually is, or will be. This is the one factor in options pricing where you have to take a view and, ultimately, this is what you trade.

If you want to know where the market *believes* volatility currently is, take a quoted option price as given and solve the pricing model 'in reverse', as illustrated in the figure below.

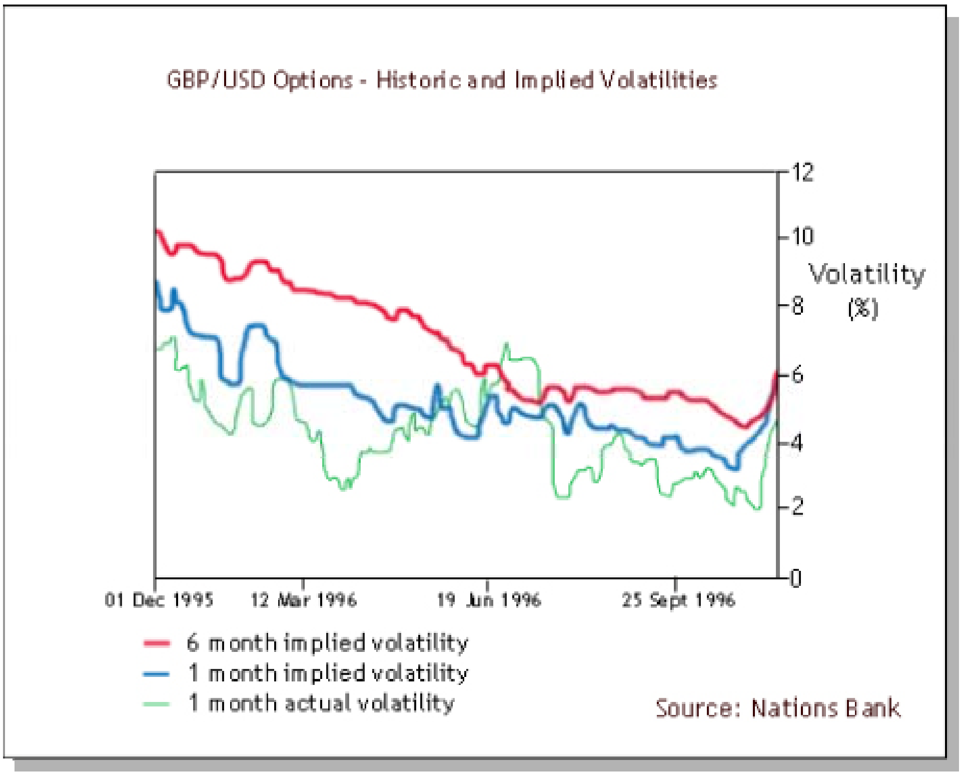


Implied volatility is to the options market what yield is to the fixed income market: both are benchmarks of value. Nowadays, in the more established OTC markets, traders and brokers quote options prices in volatility terms, rather than in cash terms (see Interest Rate Options and FX Options a). An option could be considered cheap if it trades with low implied volatility compared with:

- Other options on the same or similar underlying
- Previous implied volatility levels for the same option
- The estimated historic volatility of the underlying market

6.1. The Volatility Curve

The figure below tracks an estimate of historic volatility in spot GBP/USD against the implied volatilities of 1 month and 6 month FX options.



Options traders buy volatility (i.e. buy options) when they believe it's too low and sell it when they believe it's too high, just like any other commodity. If you had been able to buy the GBP 1 month volatility at 3% in mid-October 1996 and sold it just a few weeks later at 6%, you would have doubled your money! We shall explore the relationship between volatility and premium price in the *Exercises*, and in Options Strategy we examine various volatility trading strategies.

Volatility curve:
A graphical display of the relationship between implied volatility and time to expiry

Notice in the figure above how the 6 month implied volatility is different from the 1 month implied, and how the two do not necessarily move in step. The market perceives different risks for different time periods. The volatility curve is to the options market what the yield curve is to the fixed income markets: both can be analysed for clues about market sentiment and future prospects (see Bond Yield - The Yield Curve).

7. Exercise 1

7.1. Question 1

Question 1

In these exercises we explore the pricing behaviour of options using the standard European option pricing model described in section *Analytic Models*. Please launch the spreadsheet and begin by setting the **Market data** as per the table below.

Market data

Spot	100.00
Strike	100.00
Expiry (yrs)	1.00
Funding rate	6.00%
Yield	6.00%
Volatility	10.00%

a) Complete the table below, showing the premium price for a call with different strikes. Type your answer into each box and then validate.

Strike	Call Price
$102.00	
$100.00	
$98.00	

b) What is the relationship between the strike and the premium price

For a call if the strike is higher the premium is:

- ☐ Higher
- ☐ Lower

c) For a put if the strike is higher the premium is:

- ☐ Higher
- ☐ Lower

7.2. Question 2

Question 2

Premium and Time Value

Set the **Market data** in the pricing model as per *Question 1*.

a) Complete the table below for the $100 strike call.

Underlying Price	Option Price	Change	Intrinsic Value	Time Value
85.00		---		
90.00				
95.00				
100.00				
105.00				
110.00				

Instructions

- Since the option holder cannot be forced to exercise, intrinsic value can only be either positive or zero; it cannot be negative
- For a European option:
  Intrinsic value = Present value of { Forward – Strike }

  You can read this value from the **Mark-to-market** cell in the spreadsheet

b) Where is time value highest?

- When the option is ATM
- When the option is ITM
- Indeterminate
- When the option is OTM

7.3. Question 3

Question 3

Premium and Time

Set the **Market data** in the pricing model as per *Question 1.*

a) Complete the table below for the $100 strike call.

Expiry (years)	Option Price	Change
1.0		---
0.8		
0.6		
0.4		
0.2		
0.0		

b) Describe the relationship between time value and time to expiry.

- Time value decays evenly
- Time value decays unpredictably
- Time value decays faster over time
- Time value decays more slowly over time

7.4. Question 4

Question 4

Premium and Volatility

Set the **Market data** in the pricing model as per *Question 1*.

a) For the $100 strike option with 1 year to expiry, what is the premium price for the following levels of volatility:

Volatility	Option Price	Change
0%		---
5%		
10%		
15%		
20%		
25%		

b) Describe the relationship between option price and volatility.

- ☐ Volatility has an unpredictable effect on premium
- ☐ For ITM options premium accelerates as volatility increases
- ☐ For ATM options premium is proportional to volatility
- ☐ For OTM options premium accelerates as volatility increases

c) The $100 strike call with 1 year to expiry trades at $5.25. What is its implied volatility?

Instructions

In Excel select **Tools | Goal Seek**. In the dialog box, enter the following (shown in **bold**):

Set cell: **Call_price**
To value: **5.25**
By changing cell: **Volatility**

7.5. Question 5

Question 5

Premium and Cost of Funding

Set the **Market data** in the pricing model as per *Question 1.*

a) Complete the table below.

LIBOR	Call Price	Put Price
4.00%		
6.00%		
8.00%		

b) Other things being equal, when interest rates increase calls become more expensive and puts become cheaper because:

- ☐ Buying the call instead of the underlying saves on capital
- ☐ The market believes the underlying price will rise
- ☐ The put seller hedges by shorting the underlying, so earns the interest on the proceeds
- ☐ The call seller has to carry the underlying

7.6. Question 6

Question 6

Premium and Profit/loss

Set the **Market data** in the pricing model as per *Question 1*.

a) If you bought the $100 strike 1 year call at the price calculated in *Question 1(a)* and after 3 months (0.25 years) the underlying price rose to $102.00 but volatility fell to 6%, what is the profit/loss on the trade?

b) You bought the $100 strike 1 year call at the price calculated in *Question 1(a).* Calculate your profit/loss at the option's expiry in the following scenarios:

Underlying Price	Initial Premium Paid	Expiry Value	Profit/loss
98.00	$3.76		
100.00	$3.76		
102.00	$3.76		
104.00	$3.76		

c) What does the option's price at expiry represent?

- The strike
- Time value
- Intrinsic value
- The price of the underlying

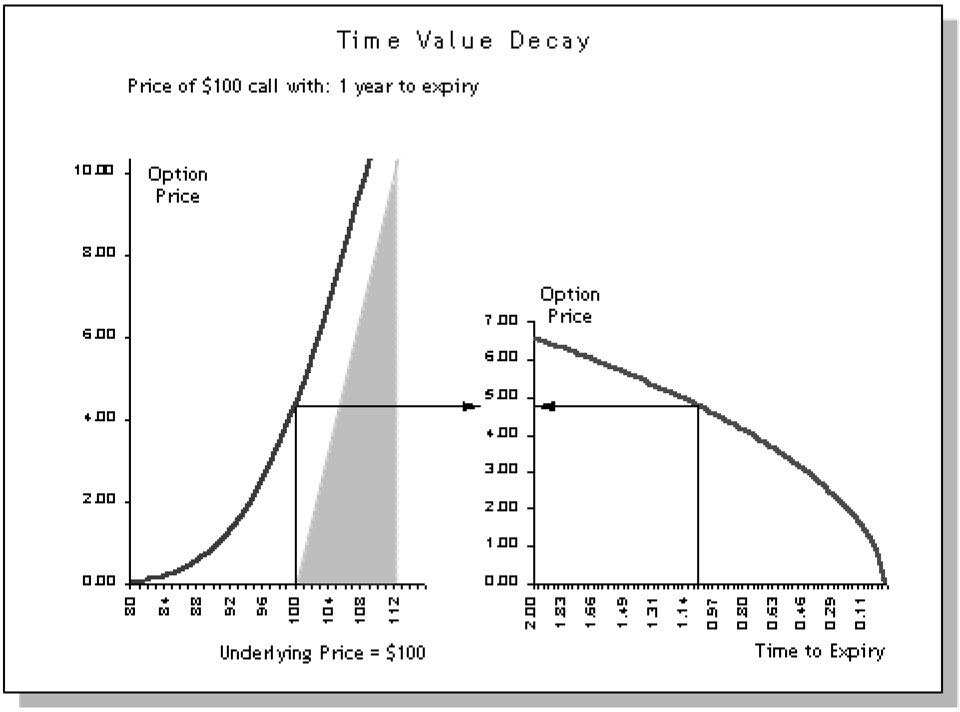
d) What is the breakeven on your trade?

8. Theta

Option Theta
The change in option premium for a given change in time to expiry (typically one day).

$$\text{Theta} = \frac{\text{Change in premium price}}{\text{Change in time to expiry}}$$

Options are wasting assets, and in Option Pricing - Exercise 3 we explored the rate of time value decay. The figure below shows how the option price declines towards its expiry value as the expiry date draws nearer.



The panel on the left shows the **option price curve** (a smooth line depicting the relationship between the option's current premium and the price of the underlying) shifting down over time towards its expiry value. The panel on the right shows the rate of time value decay accelerating as the expiry date approaches. At expiry the option has no time value at all: its price reflects pure intrinsic value, if any. Theta is measured by the slope of the curve on the right. By convention:

- If you are long an option then you are **short theta**: time is against you as the options lose value. Short-dated options may be cheaper than long-dated options, but they cost more to carry in terms of time value decay!

- If you are short an option then you are **long theta**. Time is on your side: other things being equal the options become progressively cheaper to buy back.

9. Vega

Option Vega
The change in option premium for a given increase in volatility
(typically a 1 percentage point increase).

Vega = Change in premium price / Increase in volatility

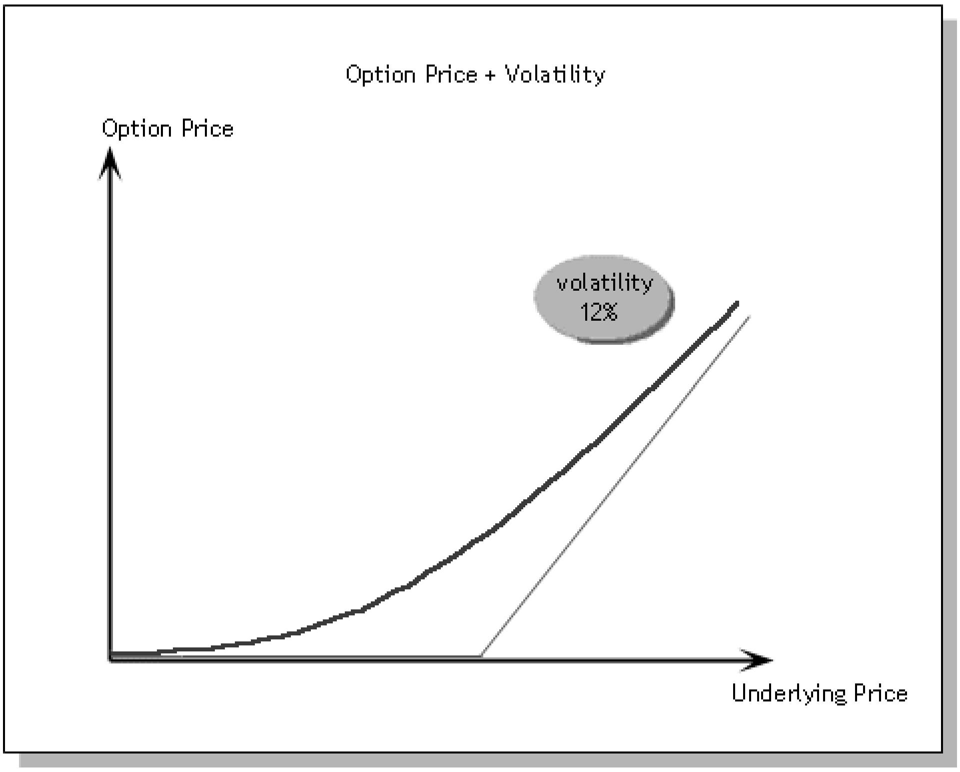
Also known as: **Epsilon**, **Kappa**.

Vega measures the sensitivity of the option's premium to a change in the underlying market's volatility. Higher volatility:

- Increases the chance that an OTM option may be exercised, therefore the option becomes more valuable

- Increases the chance that an ITM option may not be exercised, therefore the option's limited-risk feature becomes more valuable.

The figure below shows how the option price curve shifts upwards as volatility increases, and vice versa. The straight line in the figure is the option's intrinsic value, so the implication is that for any given price of the underling, the higher the volatility the higher is the option's time value (hence its price).



By convention:

- Long options positions are said to be **long vega**, or **long volatility**: the trader profits from an increase in volatility
- Short options positions are **short vega** or **short volatility**: the trader loses because the options become more expensive to buy back.

Traders can 'buy' or 'sell' volatility through options and their profit/loss is exposed to changes in this variable. In the *Exercises* below, and in Options Strategies - Volatility Trading, we explore strategies designed to trade volatility in its purest form.

10. Rho & Psi

Option Rho
The change in option premium for a given increase in the funding rate
(typically a 100 basis point increase).

$$\text{Rho} = \frac{\text{Change in premium price}}{\text{Increase in funding rate}}$$

Other things being equal, when the cost of funding increases:

- **Call prices rise**: sellers of calls hedge their positions by purchasing amounts of the underlying. They will therefore pass the higher funding costs onto the call buyers in the form of higher premiums

- **Put prices fall**: put sellers hedge by shorting the underlying, therefore they pass any interest benefit onto the put buyers

The other things being equal condition is important here. Typically, when interest rates rise equity and bond prices tend to fall and this of course reduces call prices, possibly offsetting any positive rho effects. Similarly, lower underlying prices increase put option values, again possibly offsetting the negative impact of rho. In general, the rho risk tends to be relatively small, except for long-dated options.

Option Psi
The change in option premium for a given increase in the yield on the underlying instrument (typically a 100 basis point increase).

$$\text{Psi} = \frac{\text{Change in premium price}}{\text{Increase in yield}}$$

Since the cost of carry is the difference between the funding rate and the yield on the underling instrument, psi has the opposite effect to rho. Obviously, fixed coupon securities have no psi (if the coupons are fixed), but for equity options the **dividend risk** may be significant, particularly for long-dated options.

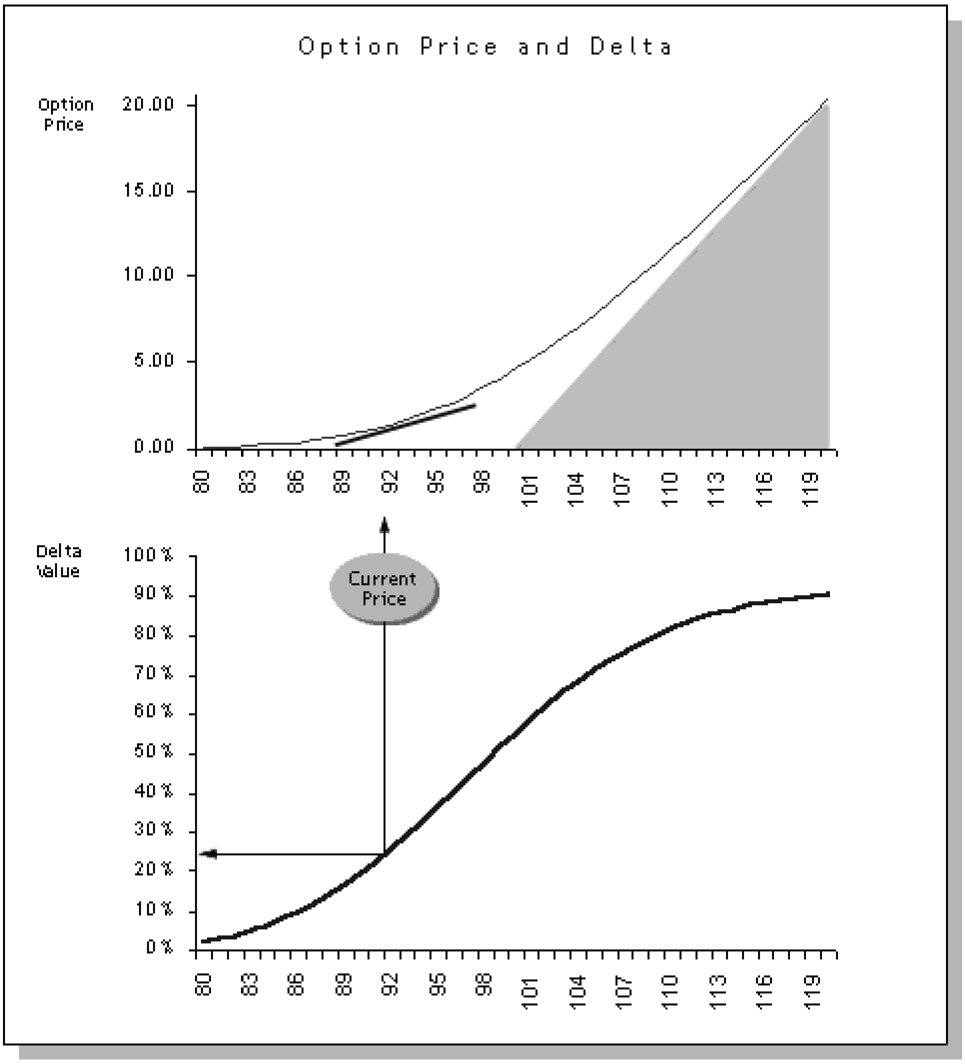
11. Delta

Option Delta
The change in option premium for a *small increase* in the underlying price.

$$\text{Delta} = \frac{\text{Change in premium price}}{\text{Increase in underlying price}}$$

Delta gives the trader a measure of his exposure to the underlying market. If a call has a delta of + 0.35 (or 35%), this means that its premium price should increase by 3 1/2 ticks if the price of the underlying were to increase by 10 ticks.

Delta is measured as the slope of a tangent to the option price curve at any point on the curve. The figure below tracks the delta of a $100 strike call at different levels of the underlying.



It is important to compute delta for *a small increase* in the price of the underlying, as for larger changes delta itself changes.

Range of Delta Values

If the option is OTM (in our example underlying price < 100) the probability of the option being exercised is low and so is its delta: an increase in the underlying price from that level adds only a small amount to the option value. Delta approaches zero as the option moves further OTM.

- If option is ATM (in our case underlying price = 100), there is an even chance that the option will expire ITM (or OTM), so delta is 0.5 or 50%. The option behaves like half the real thing!

- If the option is ITM (in our case underlying price > 100) the probability of exercise is higher than 50% and so is its delta. With a deeply ITM option, any further increase in the price of the underlying drives up the option price by virtually the same amount, so delta approaches 1.0 or 100%. It is now highly likely that the option will be exercised, so it begins to behave like the real thing!

11.1. Delta as the Hedge Ratio

The example below shows how Delta helps options traders to hedge their exposure to underlying price movements.

Example

Suppose we sold a 1.1400 EUR call / USD put for EUR 10 million. Market conditions at the time are as follows:

Spot EUR/USD:	1.1500
Option price (in USD):	0.0350
Option delta:	0.60

In effect the position gives us a market exposure equivalent to running a short position spot EUR/USD in EUR 6 million. To **delta-hedge** this position we could therefore buy EUR 6 million in spot currency.

To demonstrate that the hedge formula works, suppose EUR/USD rises to 1.1510. With a delta of 0.60, we would expect the call price to rise by 6 pips to 356. We can now calculate the profit/loss on the net position - short EUR 10 million calls and long EUR 6 million spot.

P/L on options position:	10,000,000 x (0.0350 - 0.0356) =	- USD 6,000
P/L on spot position:	6,000,000 x (1.1510 - 1.1500) =	+ USD 6,000
Net		**0**

Hedge amount for options on cash = Option delta x Contract amount
Hedge amount for futures options = Option delta x No. of options contracts

Converting options positions into spot equivalent amounts gives us a method for adding up the deltas of individual options to obtain an overall position delta.

11.2. Delta, Gearing and Leverage

This section introduces two concepts which are closely related to the option's delta: **gearing** and **leverage**. These terms are often used to describe the fantastic profit potential of derivative products such as futures and options, and it is worth stopping for a moment to consider exactly what they mean.

From the slope of the options price curve in the figure on the previous page, we can see that in absolute terms the movement in the option's price will generally be smaller than that of the underlying security: delta is numerically equal to or less than 1. In percentage terms, however, the movement in the option's price is much larger.

Example

Consider the change in value of the following call struck at $190.

	Price		Change	
	Before	**After**	**$**	**%**
Spot underlying	180.00	181.00	1.00	0.556
Call value (cents)	1.73	1.73 + (0.41 x 1.00) = 2.14	0.41	23.70
Delta	0.41			

In this example the option's nominal gearing is 104.05 times (180 / 1.73). Gearing measures the capital efficiency of an option. It shows that we would need to tie up 104 times more capital to get an exposure to this market if we purchased the underlying security rather than purchasing an option on it.

Nominal gearing does not, by itself, measure the option's return on capital. The return on capital will also depend on the option's delta. In absolute terms, the call rose by only 41 ticks, compared with the 100-tick rise in the underlying. In percentage terms, however, the gain is nearly 43 times larger (23.70 / 0.556).

This is the option's **effective gearing** and the simple formula shown below allows us to quickly calculate effective gearing from nominal gearing.

$$\text{Nominal gearing} = \frac{\text{Price of underlying}}{\text{Option price}}$$

Effective gearing: the return on capital from an option position relative to the return on capital on the equivalent position in the underlying.

Effective gearing = Nominal gearing x Delta

Also known as: **Leverage**, **Lambda**

In our example effective gearing = 104.05 x 0.41 = 43 times, rounded.

Nominal and effective gearing are commonly used terms in the warrant market.

Sign of Delta

- A long call position has positive delta - if the underlying price rises so does the option premium, which is good for the trader
- A long put position has negative delta - if the underlying price rises the option premium falls which is bad for the trader
- It follows that a short call position has negative delta - a rise in the underlying price causes the price of the call to rise as well, and since the trader is short the call this is bad news for the trader
- Finally, a short put has positive delta - if the market rises the put becomes cheaper to buy back, which is good news for the trader who is short the put.

You can see from the figure above that delta is not a constant, but varies depending on the extent to which the option is ITM or OTM. The stability of delta is measured by gamma, which is discussed in the next section.

12. Gamma

Option's Gamma
The change in delta for a given change in the price of the underlying.

$$\text{Gamma} = \frac{\text{Change in delta}}{\text{Change in underlying price}}$$

A useful analogy is to view delta as the 'speed' of the option's price at a particular point on the option price curve and gamma as its 'acceleration'.

Since delta itself changes as the underlying price moves, delta-hedged positions must be periodically rebalanced. Gamma provides the trader with an indication of how quickly she might have to rebalance her delta-hedge, if the market moved by a given amount.

The unit of gamma - the size of the 'given' change in the underlying - is arbitrary. For FX options in the major currencies, the convention is to compute it for a 1% change in the FX rate, but for more volatile currencies or markets (e.g. equities) it is prudent to compute it for larger changes.

Example

We are short a 1.5500 USD call / SGD put for USD 10 million. The position has a delta of -0.60 which we have hedged by going long USD 6 million in spot USD/SGD. Spot USD/SGD is currently 1.6000 and the table below summarises what would happen if the spot rate rose to 1.6100.

	Before	**After**	**Gamma**
Prices:			
Spot USD/SGD	1.6000	1.6100	
Option position:			
Price (in SGD)	0.0350	0.0428	
Delta	-0.60	-0.73	**-0.13**
Required hedge:			
Long spot USD/SGD	6,000,000	7,300,000	+1,300,000

The option went further into the money and its delta rose to -0.73. The position gamma, for a 100-point change in the spot rate, is -0.13 [= (-0.73) - (-0.60)]. The new delta requires the hedging book to be long USD 7.3 million, so to keep the position delta-neutral the trader should buy an additional USD 1.3 million in spot FX. USD 1.3 million is simply the position gamma expressed in spot-equivalent units (= 10,000,000 x 0.13).

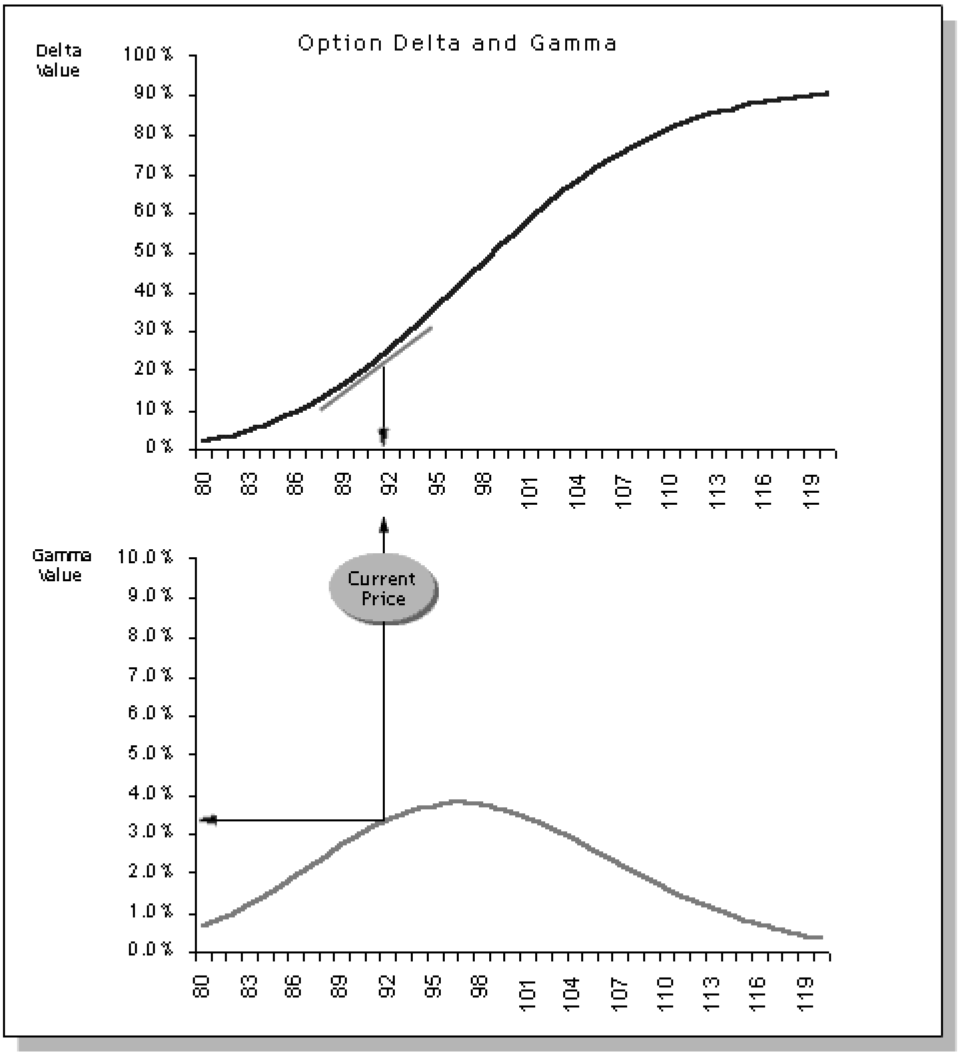
Of course, if the spot rate subsequently fell 100 points, the trader would have to sell the spot currency to maintain his book delta-flat. The higher the position gamma, the more the options trader has to trade in and out of the hedge as the spot rate moves, so the higher are the costs of running the hedge. This is why options traders are interested in gamma:

Short options positions with high gamma are potentially expensive positions to maintain delta-neutral.

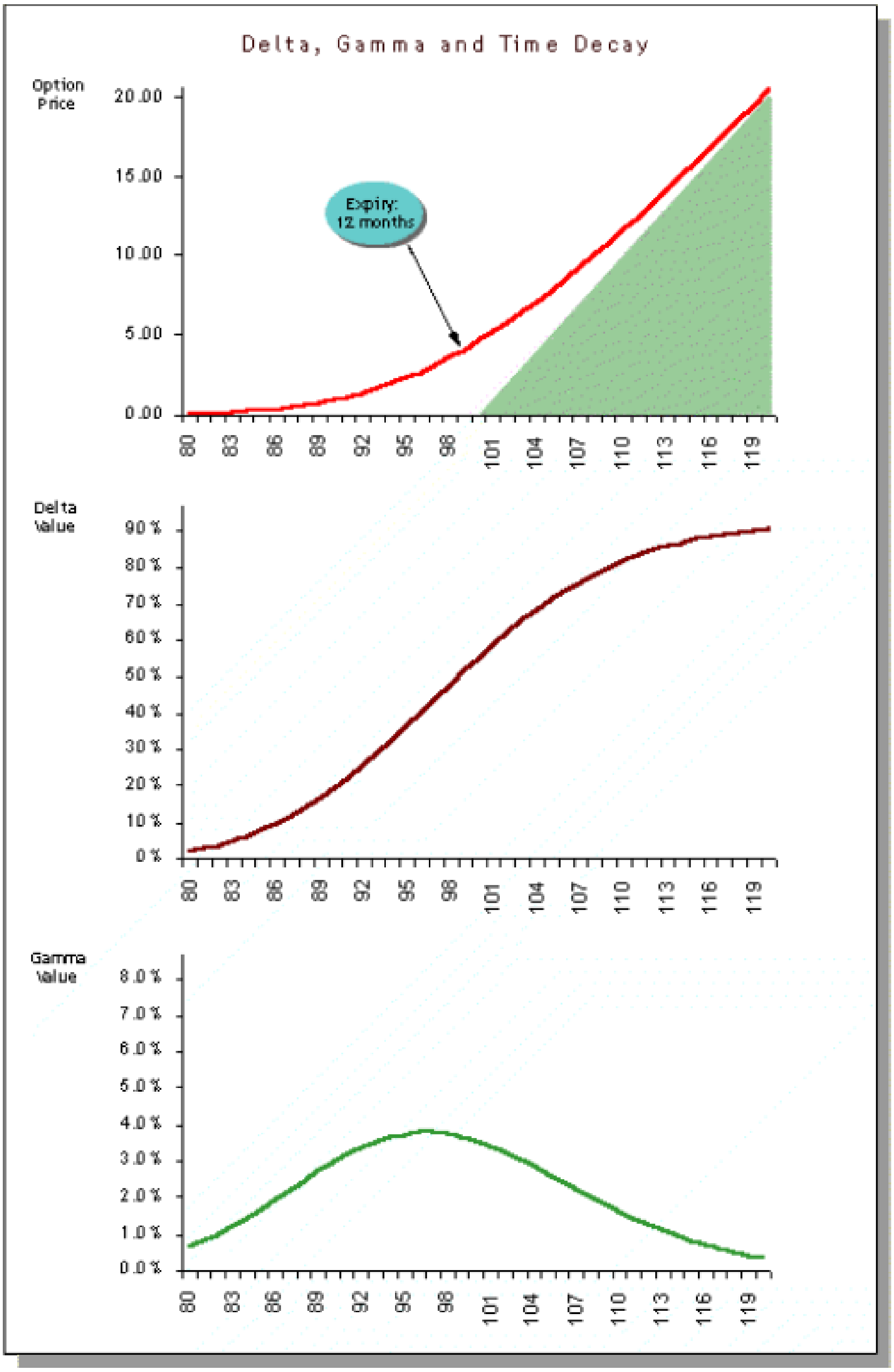
12.1. Ranges of Gamma Values

- **Gamma is highest at the steepest point on the delta curve, where the option is ATM**
- **Gamma of OTM and ITM options declines as expiry approaches**
- **Gamma of ATM options increases as expiry approaches**

The figure below plots gamma values (bottom panel) at different points on the delta curve (top panel) for a $100 strike call. The figure shows that ATM options tend to have higher gamma than ITM or OTM options.



12.2. Gamma and Time Value Decay



Explanation

- As an ITM option gets closer to expiry it is more and more certain to be exercised and so behaves more like the underlying. The option's delta will reflect this and will converge towards 1, while gamma converges to zero as the delta becomes more and more stable

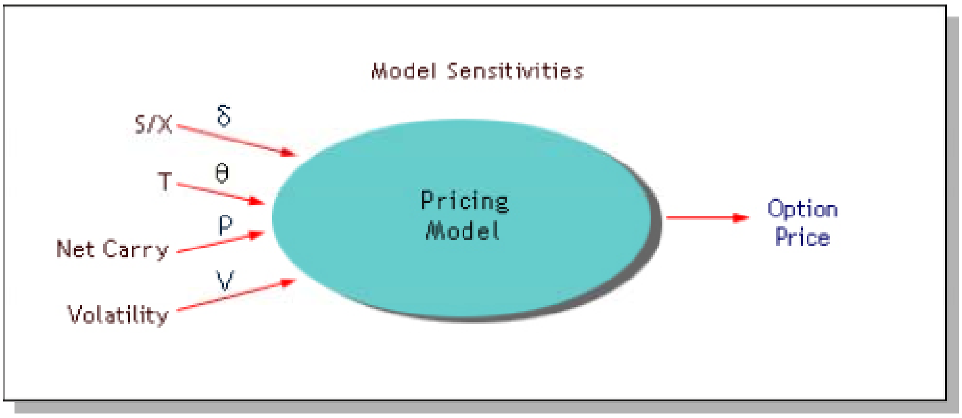
- For an OTM option, delta will converge towards 0 as expiry approaches, and so will its gamma, reflecting the increased certainty that the option will expire worthless

- For an ATM option delta remains at 0.5 as expiry approaches, because it is still uncertain whether the option will be exercised or not. However, delta becomes increasingly sensitive to changes in the price of the underlying. The option becomes progressively more 'schizophrenic' as it approaches expiry, appearing to be virtually identical to the underlying if it is just a little ITM, or virtually worthless if it is just a little OTM!

Graphically, as the option price curve 'sinks' towards its expiry value its curvature around the strike increases but it flattens at either end. At the same time, the S-shape of the delta curve becomes more pronounced and the gamma curve grows taller and thinner.

13. Summary of Greeks

As we explain in Option Pricing, an option's premium price is a function of a number of market variables. The sensitivity of the option's price to changes in each of these variables is captured by the Greeks. The figure below summarises the role of each Greek in quantifying the many dimensions of option risk.



All the Greeks, except gamma, perform essentially a similar role: they monitor the sensitivity of the option price to a change in one of the input variables. Gamma is the odd one out: it monitors the change in delta, which is another Greek.

Gamma is important because it measures the likely changes in the hedge required to maintain an options position delta-neutral. The exercises in the next section illustrate how, for an option seller, a high-gamma position may be potentially very costly to run.

13.1. Sign of the Greeks

The risk on an options book is the net sum of the risks of the individual trades. The table below summarises the direction of the exposure generated by each option or futures position. A positive sign means the profit on a position gains from an increase in the input variable; a negative sign means it loses.

	Call	Futures	Put
Buyer	Right to buy Delta > 0 Rho > 0 (Psi < 0) Vega > 0 Gamma > 0 Theta < 0	Commitment Delta > 0 Rho > 0 (Psi < 0) Vega = 0 Gamma = 0 Theta < 0 if R% > D% Theta > 0 if R% < D%	Right to sell Delta < 0 Rho < 0 (Psi > 0) Vega > 0 Gamma > 0 Theta < 0
Seller	Obligation to sell Delta < 0 Rho < 0 (Psi > 0) Vega < 0 Gamma < 0 Theta > 0	Commitment Delta < 0 Rho < 0 (Psi > 0) Vega = 0 Gamma = 0 Theta > 0 if R% > D% Theta < 0 if R% < D%	Obligation to buy Delta > 0 Rho > 0 (Psi < 0) Vega < 0 Gamma < 0 Theta > 0

Key to the table

- Vega and gamma have the same sign: vega measures exposure to expected volatility; gamma measures exposure to actual volatility
- Theta has the opposite sign to vega (and gamma): time value is the 'price of volatility' and time value decays over time
- Delta and Rho have the same sign: delta measures the exposure to the underlying; Rho measures the cost of carrying it, which is in the derivative price
- Rho and Phi have opposite signs: Phi captures the dividend changes which offset funding costs
- Futures have zero vega and gamma: futures positions are not exposed to changes in expected volatility and changes in actual volatility have a symmetric effect on the position
- The theta of a futures position depends on the net carry:
    - If the funding rate (R%) is higher than the income yield (D%), a long futures position is theta-negative: the basis is positive and its erosion represents a cost. A short position is therefore theta-positive
    - If R% is lower than D%, then a long futures position is theta-positive and a short position is theta-negative

14. Exercise 2

14.1. Question 1

Question 7

Suppose we have the following position in treasury bond options:

Position	Delta
Short $1,000,000 in 102.30 DEC calls	0.60
Long $2,000,000 in 100.27 DEC puts	0.35
Short $500,000 in 100.50 MAR puts	0.70
Long $300,000 in MAR futures	1.00

a) What would you do to make this position delta-neutral?

- ☐ Sell cash bonds
- ☐ Buy cash bonds

b) Enter the nominal amount of bonds you would trade: type your answer in the box below, to the nearest dollar, and then validate.

14.2. Question 2

Question 8

In this and the following questions we shall explore some of the risk management implications of trading options, using a specially developed spreadsheet. Launch the *Options Strategist* and set the contract **Specification** as per the table below.

Underlying instrument	UK Gilts
Currency of payment	GBP
Underlying year basis	Act/365
Funding year basis	Act/365
Contract Size	100,000
Option style	American
Underlying	Futures
Minimum price change	1/100[1]
Value of one tick	10.00 (per tick)

Case Study - May

Following the recent fiscal and political uncertainties in the UK, you observe that the LIFFE JUN UK Gilt futures options are trading with implied volatilities of around 14%, and you feel these should settle back to their normal 7.00 - 8.00% levels very soon.

[1] I.e.: minimum price change = 0.01%.

Enter the following initial and current market data in the **Trades** worksheet of the spreadsheet:

Initial market data

Spot price	95.80
GBP rate[2]	3.50%
UK Gilts yield[3]	4.80%
Volatility	14.00%

Current market data

Days elapsed	0
Spot price	95.80
Volatility	14.00%

a) What trade would you put in place if you wanted to **sell volatility** (i.e. **short vega**) but did not have a strong view on the direction of UK Gilts?

- ☐ Buy calls only
- ☐ Buy puts only
- ☐ Sell puts only
- ☐ Sell both calls and puts

b) In this case study, we shall implement a short volatility position by selling calls and delta-neutralising them with Gilt futures.

With the June futures trading at 95.68 you decide to sell 20 JUN 97.00 calls expiring in 34 days. In column 1 of the **Trades** worksheet enter:

Positions	***1***
Number of NKI contracts	-20
Contract type	Call
Strike	97.00
Initial days to expiry	34

What is the premium price of these options, in ticks?

c) What is the total premium earned on the trade, in USD?

d) What is the delta of these options? Enter your answer in percent, to 1 decimal place, *including the sign of the delta*.

e) How many Gilt futures would you need to buy or sell in order to make this position delta neutral? Enter the number of contracts, rounded to the nearest unit.

[2] The funding rate from cash settlement up to the expiry date of the JUN futures contract.

[3] The yield on the JUN CTD bond, which for simplicity in this exercise is assumed to have a conversion factor of 1.0000, so it's easier to calculate the theoretical bond futures price (see Bond Futures – Cheapest to Deliver).

f) In column 2 of the **Trades** worksheet enter the number of contracts calculated in (e) for your delta hedge:

Positions	***1***	***2***
Number of futures contracts	-20	**???**
Contract type	Call	Underlying
Strike	97.00	95.68
Initial days to expiry	34	34

Complete the table below to test the effectiveness of your futures hedge by moving the **Current spot price** up and down by 0.10%.

Spot price	**95.70**	**95.80**	**95.90**
Current futures price			
97.00 calls (in ticks)			
Profit/loss:			
Futures position			
Options position			
Net			

g) Does the delta hedge work?

- Not at all
- Pretty much

14.3. Question 3

Question 9

Same case as in *Question 2*. Make sure you restore **Current spot price** back to 95.80 and keep the long position in the 8 futures contracts as well. Now change the **Days elapsed** in the spreadsheet to 15.

a) What would be the net profit/loss on the position (options and futures) at this point if the options traded at the following current volatility levels:

Volatility	**Profit/loss ($)**
7%	
14%	
18%	
19%	

b) Explain the results.

- The position is short theta and long vega
- The position is long theta and long vega
- The position is long theta and short vega
- The position is short theta and short vega

14.4. Question 4

Question 10

Same case as in *Question 2*, and make sure you restore the **Days elapsed** back to zero and the **Current volatility** back to 14%.

a) A few hours after the trades were put on the Gilts futures jumps up one full point (100 ticks) to 96.68, on account of a very strong pound. Complete the table below.

Spot price	**95.80**	**96.80**
Current futures price		
97.00 calls (in ticks)		
Profit/loss:-		
Futures position ()		
Options position (USD)		
Net (USD)		

b) Explain the results:

- ☐ Loss on the option position is greater than profit on the futures hedge
- ☐ Profit on the option position is less than profit on the futures hedge
- ☐ The option's price fell by more than its original delta had predicted
- ☐ The option's price rose by more than its original delta had predicted

c) Which of the following is now true about the position?

- ○ The position is gamma-positive
- ○ The position is still delta-neutral
- ○ The position is short delta
- ○ The position is long delta

d) What additional risks are you now taking?

- ○ The UK Gilts market could rally further
- ○ The UK Gilts market could fall back again

e) What would you now do if you were concerned about the risk of a further rally in the UK Gilts market?

- ○ Nothing
- ○ Hope for a fall in the Gilts market, then close the position
- ○ Buy at least 2 more futures contracts
- ○ Sell at least 2 futures contracts

14.5. Question 5

Question 11

Same case as in *Question 4*. Having taken your losses, you now need to reset the *Options Strategist* to the current market level:

- Set both the **Initial spot price** and the **Current spot price** to 96.80
- Change the **Strike** of the futures position to the current futures price of 96.68
- In column 2 of your **Position** adjust your futures hedge to make the option position delta-neutral again

a) After having rebalanced your hedge, the JUN Gilts futures then gives up all the gains achieved earlier in the day, to close back at 95.68. Complete the table below and comment on the results.

Spot price	**96.80**	**95.80**
Current futures price		
97.00 calls (in ticks)		
Profit/loss:-		
Futures position ()		
Options position (USD)		
Net (USD)		

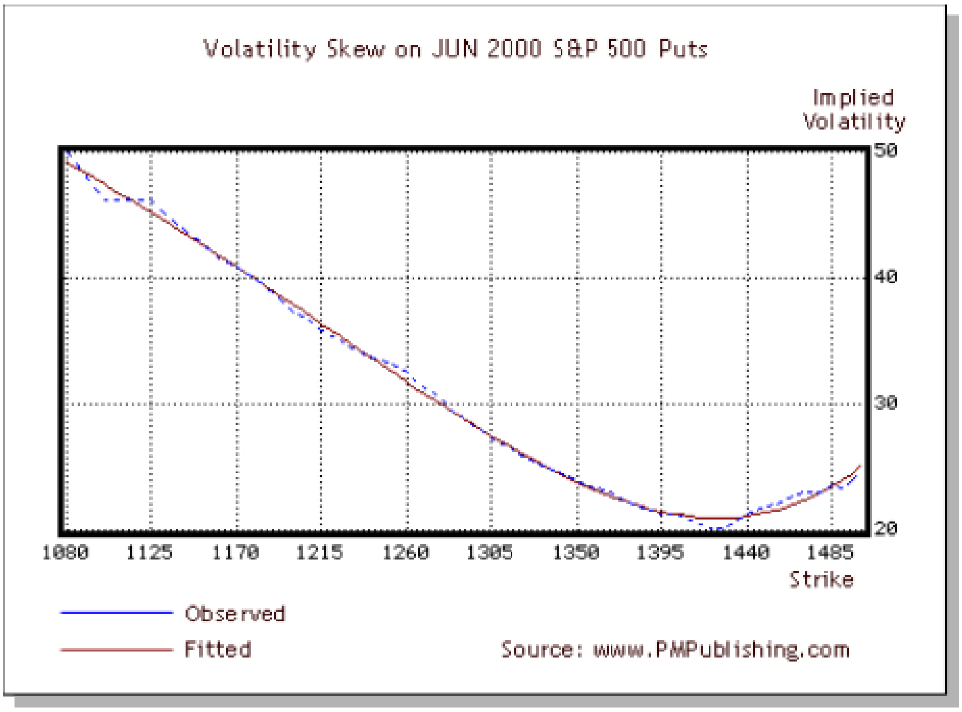
b) What is the value of your position gamma, for a 100 tick move in the Gilts futures price? Enter your answer in percent to one decimal place.

c) Compare your position with that of a trader who was long vega. To remain delta-neutral, and lock in the profits, the trader who is long vega would have to:

- ☐ Sell the underlying when the market falls
- ☐ Buy the underlying when the market falls
- ☐ Buy the underlying when the market rises
- ☐ Sell the underlying when the market rises

15. Volatility Smile

In some markets, options with the same expiry dates but different strikes often trade with different implied volatilities. OTM and ITM options often trade with higher implied volatilities than ATM options, generating a **volatility smile**, as illustrated below.



Sometimes we observe the ITM series trading with higher vols than the OTM series (as in the example above) or vice-versa, so the curve looks more like a 'smirk' than a smile, and this is termed a **volatility skew**.

15.1. Pricing Anomalies?

In Options Strategies - Question 8, we show how traders try to profit from anticipated movements in the shape of the smile or the skew. Here we explore two fundamental questions:

? If the expiry of the options is the same, why do different strikes trade with different implied volatilities?

Are these real price anomalies?

Sometimes they are ...

- OTM options are cheap in monetary terms and very popular with people who want to speculate or to protect their exposures for a small amount of premium. In fact, they pay too much for the options in terms of implied volatility.
- ITM options trade with high deltas. This means that a given change in the underlying market translates into a more or less equivalent change in the option price. Some investors therefore use ITM options as a means of taking positions in the underlying securities without tying up so much capital. Again, they often pay too much in terms of implied volatility.

...but often they are not

As we explain in Option Pricing - Analytic Models, option pricing models work on the assumption that the underlying market is liquid enough to allow traders to re-hedge their options risks at all times. Under these conditions it is legitimate to assume, for mathematical convenience, that the underlying price changes are log-normally distributed (i.e. the markets are random).

However, extreme market conditions such as major rallies or setbacks seem to occur with far greater frequency than the normal distribution would predict. In many markets the actual price distributions have fatter 'tails' than those of the normal distribution (see Market Value at Risk - Limitations). In extreme markets, liquidity tends to dry out and it becomes very difficult for traders to adjust their delta hedges. Therefore the conditions which underpin option pricing models cease to hold.

This implies that OTM or ITM options priced using a normal distribution may be too cheap: in reality there is more risk in selling these options than the models allow for and the smile is a reflection of this. In extreme markets:

- Delta-hedged OTM options may become up to 100% underhedged, and delta-hedged ITM options may become 100% overhedged

- ATM options cannot be more than 50% underhedged or overhedged

Mathematically, models based on normal distributions are much easier to work with than perhaps more realistic ones based on **non-parametric probability distributions**, so the market sticks with conventional models but compensates for the higher risks of OTM and ITM options by pricing higher implied volatility into them.

This line of argument suggests that it may well be possible, now and then, to profit from the volatility smile but in the long run the notion of arbitraging the smile may be illusory.