ISMA CENTRE - THE BUSINESS SCHOOL
OF THE FINANCIAL MARKETS

UNIVERSITY OF READING
ENGLAND

IFID Certificate Programme

Rates Trading and Hedging

Bond Futures

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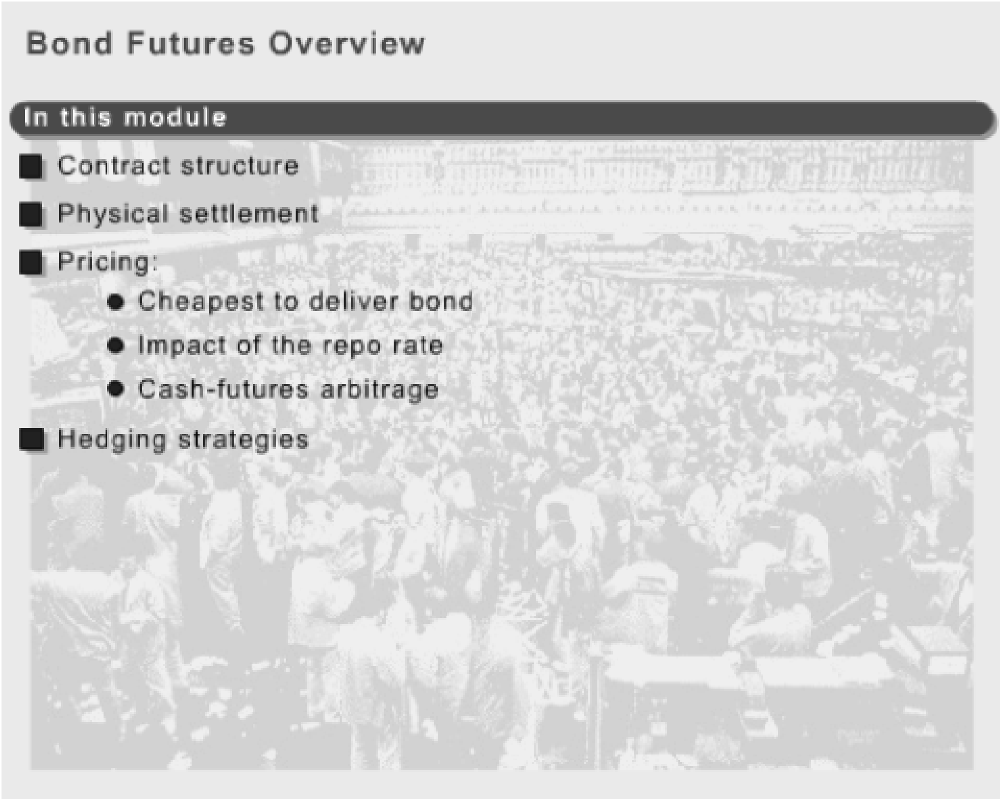
1. Overview



In this module we focus on the specific features of the bond futures contract. We describe how the contract is settled through physical delivery and we explain the profound effect this has on its pricing.

You will have an opportunity to explore the impact of changes in the **cheapest to deliver** and in the **repo rate** on cash-futures arbitrages using a specially-built pricing model, and you will also learn how to use the futures to hedge bond portfolios.

Learning Objectives

By the end of this module, you will be able to:

1. Calculate the tick value of a bond futures contract, given the contract size and the tick size
2. Calculate the invoice amount payable against physical delivery settlement
3. Calculate the theoretical (arbitrage-free) forward price of a bond
4. Explain how the conversion factors of the deliverable bonds are calculated and what is their purpose
5. Explain why one or more bonds become cheapest to deliver (CTD)
6. Describe the inflexion analysis technique for determining which bond is likely to become the CTD in different yield curve conditions

7. - Define the implied repo rate and explain how this is used in cash-futures arbitrage
8. - Define the gross basis
9. - Identify the main factors that affect the gross basis
10. - Design a risk-weighted basis trade
11. - Explain why the gross basis converges towards zero as the futures contract approaches delivery
12. - Define the net basis and explain why it exists
13. - Explain the relationship between the net basis and the implied repo rate
14. - Identify the convexity risk associated with a short basis position
15. - Construct a risk-weighted futures position to hedge the market risk on a bond portfolio
16. - Identify the residual risks that may be present in a futures hedged bond portfolio

2. Definition

2.1. Contract Terms

Below is the definition of the US Treasury bond futures contract traded at the Chicago Board of Trade (CBOT) in the US. This is by far the most active bond futures market in the world, with about 5-7 million contracts traded every month.

Contract	CBOT US Treasury Bond Futures
Unit of Trading	USD 100,000 nominal value notional US Treasury bond with 6% coupon
Delivery Months	March, June, September, December
Delivery Day	Any business day in delivery month (at seller's choice)
Last Trading Day	Seven business days preceding the last business day of the delivery month
Quotation	Percentage of face value (i.e. per $100 nominal)
Minimum Price Movement (Tick Size)	1/32%
Tick Value	$\frac{1/32}{100} \times 100{,}000 = \text{USD } 31.25$
Regular Trading Hours	07:20 - 14:00 Chicago time

Other bond futures contracts differ from this one in terms of:

- **Unit of trading**: for example the Bund futures contract traded at EUREX is for a notional EUR 100,000 bond with a 4% coupon.

- **Delivery day**: only the US Treasury bond and the UK Gilts futures contracts allow the seller to choose on which day of the delivery month they can make delivery. All other futures markets have a single delivery day - e.g. for the Bund futures contract traded at EUREX, it is the 10th calendar day of the delivery month.

- **Tick size**: in most other markets the minimum tick size is 0.01%

$$\text{Tick value} = \frac{\text{Tick size}}{100} \times \text{Contract size}$$

2.2. Contract Months

Although 8 delivery months are available for trading in this contract, in practice most of the liquidity is concentrated in the near months. Below is a price chart for the US Treasury bond futures over a number of months.

Keywords in this chart:

- Volume
- Open interest
- Moving average

3. Settlement

3.1. Delivery Settlement

In all bond futures the underlying is a notional bond. However, at expiry futures positions must be physically settled by delivering real bonds. For example, in the US Treasury bond futures contract traders who are short futures may deliver any Treasury bonds which meet the following criteria:

- They must have fixed coupons
- They must have a residual maturity of at least 15 years from the first business day of the delivery month. If the bonds are callable, the first call date must be at least 15 years away.

Physical settlement is quite a bit more complicated than cash settlement and you need to know how it works if you want to understand how bond futures prices behave.

Before a bond futures contract expires, all open positions are marked-to-market daily against the futures **settlement price** and the positions are subject to **variation margin** in the normal way (see Futures Market Structure - Trading Procedures).

On the last trading day (or, in the case of the US and UK contracts, on any day during the delivery month) all positions that are still open are settled against an **Exchange Delivery Settlement Price** (EDSP), which is the official closing price for that day. Traders who are still short bond futures may then:

- Send a delivery note to the exchange, indicating which of the deliverable bonds they intend to deliver
- Enclose an invoice for the bonds being delivered.

The invoice amount is calculated by adjusting the EDSP by a **conversion factor** (or **price factor**) which depends on the bonds being delivered. The exchange establishes a unique conversion factor for each of the deliverable bonds and this is fixed for the duration of the futures contract.

The purpose of the conversion factors is to establish a value equivalence between the EDSP, which is the same for all bonds, and the deliverable bonds which may have different coupons and maturities. We shall explain how the exchange calculates the conversion factors in section *Conversion Factor*, below. For the moment, you should note how it is used to calculate the invoice amount against futures settlement.

Futures Delivery Settlement Formula

Invoice amount

$$= \left(\frac{\text{Adjusted futures price} + \text{Accrued interest}}{100} \right) \times \text{Contract size} \times \text{Nr. of contracts}$$

Adjusted futures price = EDSP x Conversion factor

Example
Delivery settlement calculation.

You intend to settle a short position of 7 contracts in the JUN 2002 US Treasury bond futures.

June EDSP:	120-08[1]
Delivery date (i.e. settlement):	10 June 2002
Security to be delivered:	5¼% US Treasury maturing 15 November 2028
Accrued interest:	0.37092391304%
Conversion Factor:	0.9014

[1] Price is in 32^{nd}, so in decimal = 120 + 8/32 = 120.25.

Your delivery note to the exchange is for USD 700,000 face value of the 6% of 2026 plus an invoice, calculated as follows:

$$\text{Invoice amount} = \left(\frac{120.25 \times 0.9014 + 0.37092391304}{100} \right) \times 100{,}000 \times 7$$

= **USD 761,349.92**

This example shows an important feature of physically-settled contracts:

- While the futures contracts remain open there is a single futures price for all the deliverable assets - a single price that is representative of the whole market
- When the contracts are settled there is a different adjusted futures price for each deliverable asset.

4. Trading Applications

Like all futures contracts, bond futures may be used as speculative instruments or to manage the risk on cash bond portfolios.

Example – Speculation

Date: 9 July 2002
Scenario: You would like to profit from anticipated weakness in the UK bond markets.
Strategy: Short 15 DEC LIFFE long gilt futures contracts.

Contract	LIFFE Long Gilt Futures
Unit of Trading	£100,000 nominal value notional gilt with 7% coupon
Delivery Months	March, June, September, December
Delivery Day	Any business day in delivery month (at seller's choice)
Last Trading Day	11:00 Two business days prior to last business day in delivery month
Quotation	Percentage of face value (i.e. per £100 nominal)
Minimum Price Movement (Tick Size)	0.01%
Tick Value	$\frac{0.01}{100} \times 100{,}000$ = GBP 10.00
Regular Trading Hours	08:00 – 18:00

The Outcome
A few seconds later your broker confirms that your order was filled at 92.27. Below is a summary of your trading book over the next few days.

Date	Position	Profit/loss
9 July	Sell 15 DECs @ 92.27	0.00
12 July	Buy 10 DECs @ 92.13	(9227 - 9213) x 10.00 x 10 = 1,400.00
15 July	Buy 5 DECs @ 92.10	(9227 - 9210) x 10.00 x 5 = 850.00
Net (9 - 15 July):		**2,250.00**

In section *Hedging*, below, we shall see how bond futures may be used to manage the risk on a cash bond portfolio.

5. Pricing

The figure below shows a typical bond futures price page. The Explanation below the table has a definition of key terms.

US Treasury Bond Futures Prices

	Open	High	Low	Last	Chge
MAR 98	12020	12029	12016	12026	+7
JUN 98	12008	12018	12008	12016	+8
SEP 98	12007	12007	12002	12005	+8
DEC 98		11925	11925	11925	+7
MAR 99		11914	11914	11914	+7
JUN 99		11904	11904	11904	+7
SEP 99		11826	11826	11826	+7
DEC 99		11816	11816	11816	+7

Total	Volume	Open Int
02/09/98	220917	47021

Explanation

Opening Price
The first price quoted or traded during a trading session. See also: Settlement Price

Volume
In a futures or options market, the total number of contracts traded during a session. In an OTC market, the amount traded over a given period of time. Volume figures are available from futures and options exchanges, but they are seldom reported in the OTC markets. Volume is a useful indicator of the amount of speculative interest, hence the character and stability of a market. See also **Open Interest**.

Open Interest
The number of futures or options contracts outstanding at a given time, calculated as the sum of all long or all short positions in all contracts of a given class. Open interest is a useful indicator of the amount of speculative interest, hence the character and stability of a market. See also **Volume**.

All prices on this page are quoted to the nearest 1/32%. Thus, the last trading price for the MAR contract, 12026, means 120 + 26/32 = 120.8125. Although in most bond futures exchanges there are at least 8 delivery months available for trading, in practice most of the liquidity is concentrated in the near delivery months.

5.1. Cash & Carry Arbitrage

Notice an interesting pattern on the price page above: the more distant the contract delivery month, the cheaper is the futures price. This pattern is not a reflection of the market's expectations about future bond prices.

Futures prices reflect the cost of carrying a bond position through to the futures delivery date, plus the net cost of carrying that position.

It is easier to illustrate the principles involved by considering how a trader might price an OTC bond futures (or forward) contract. Later we shall explain how the same principles apply to exchange-traded bond futures.

Example

On 22 July 2002 one of your clients asks you to quote a forward price for the 8¼% US Treasury Note of 26 March 2008, for delivery on 12 August 2002. The note is currently quoted in the cash market at 109-11 (offered price) for settlement 23 July, implying a yield of 6.265%.

? What price would you quote your client, on a breakeven basis, if the 20-day repo rate is 2.75%?

Analysis
As with any derivative instrument, the breakeven forward price is calculated by considering the costs of hedging its risks. In this case the trader will hedge the position against the client by:

- Buying the deliverable bond
- Carrying it for 20 days until the client takes delivery

The breakeven price is therefore calculated as follows:

Step 1: Calculate the all-in cost of buying the bond for cash settlement (i.e. 23 July)

Number of days since last coupon (26 March to 23 July) = 119
Number of days in current coupon period (26 March - 26 September) = 184

Dirty price:
= Clean price + Accrued interest (**spot accrued**)
= 109.34375 + 8.25/2 x 119/184
= 112.01155

Step 2: calculate the cost of funding the bond for 20 days (23 July to 12 August)

= 0.0275% x 20/360 x 112.01155
= 0.17113

Step 3: add up the total cost of cash & carry
= 112.01155 + 0.17113
= 112.18268

This is the total amount that you must recover from your client in 20 days, when you deliver the bond to her, in order to break even, so this is the breakeven forward dirty price.

Step 4: calculate the amount of accrued interest payable on the bond at the forward delivery date

By 12 August the bond will have 139 days of accrued interest. Therefore, **forward accrued interest**:

= 8.25/2 x 139/184
= 3.11617

When you deliver the bonds, the client will pay you your quoted forward clean price (FP) plus this amount of accrued, and the total should be equal to the forward dirty price calculated in step 3.

Forward dirty price = FP + Forward accrued
112.18268 = FP + 3.11617

FP = 112.18268 – 3.11617
= 109.06651 or **109-02**, rounded to the nearest 32nd.

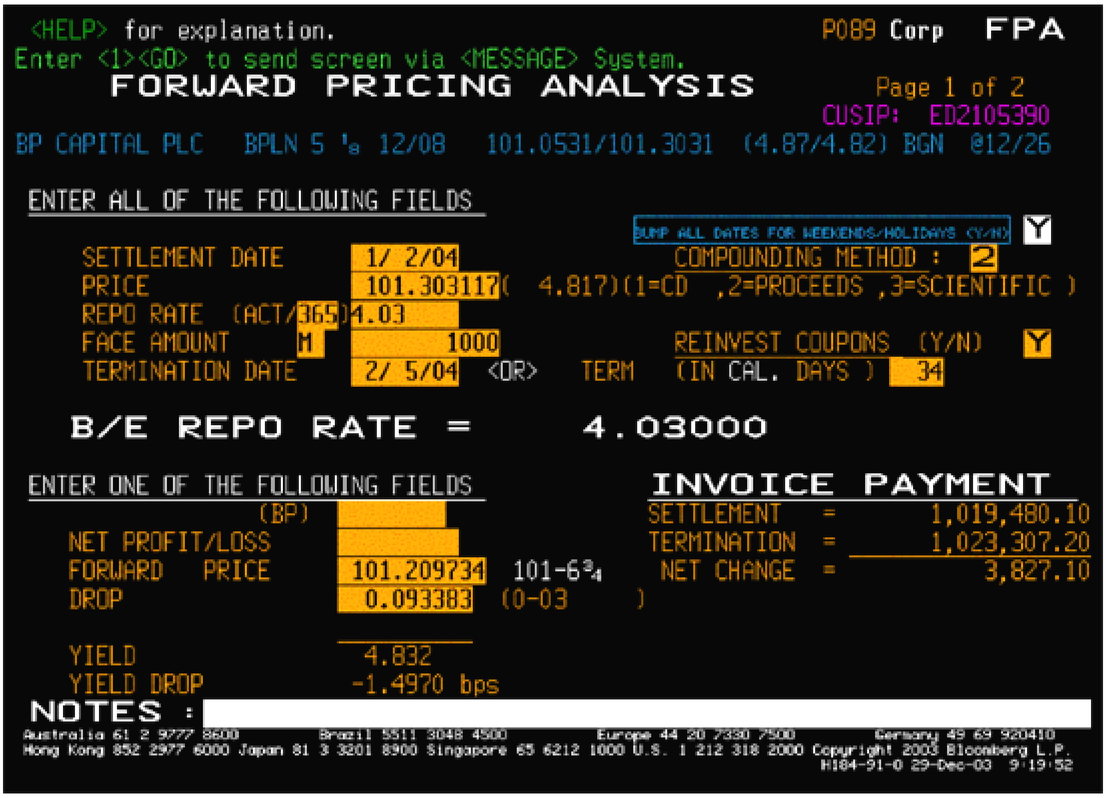
This breakeven price is known as the **fair value** forward price. Market competition should ensure that forward or futures prices normally trade around fair value, although in practice other factors will intervene to keep bond futures prices away from fair value (see section *Cheapest to Deliver*).

Analytic systems
Examples of Bloomberg and Reuters forward bond pricing functions

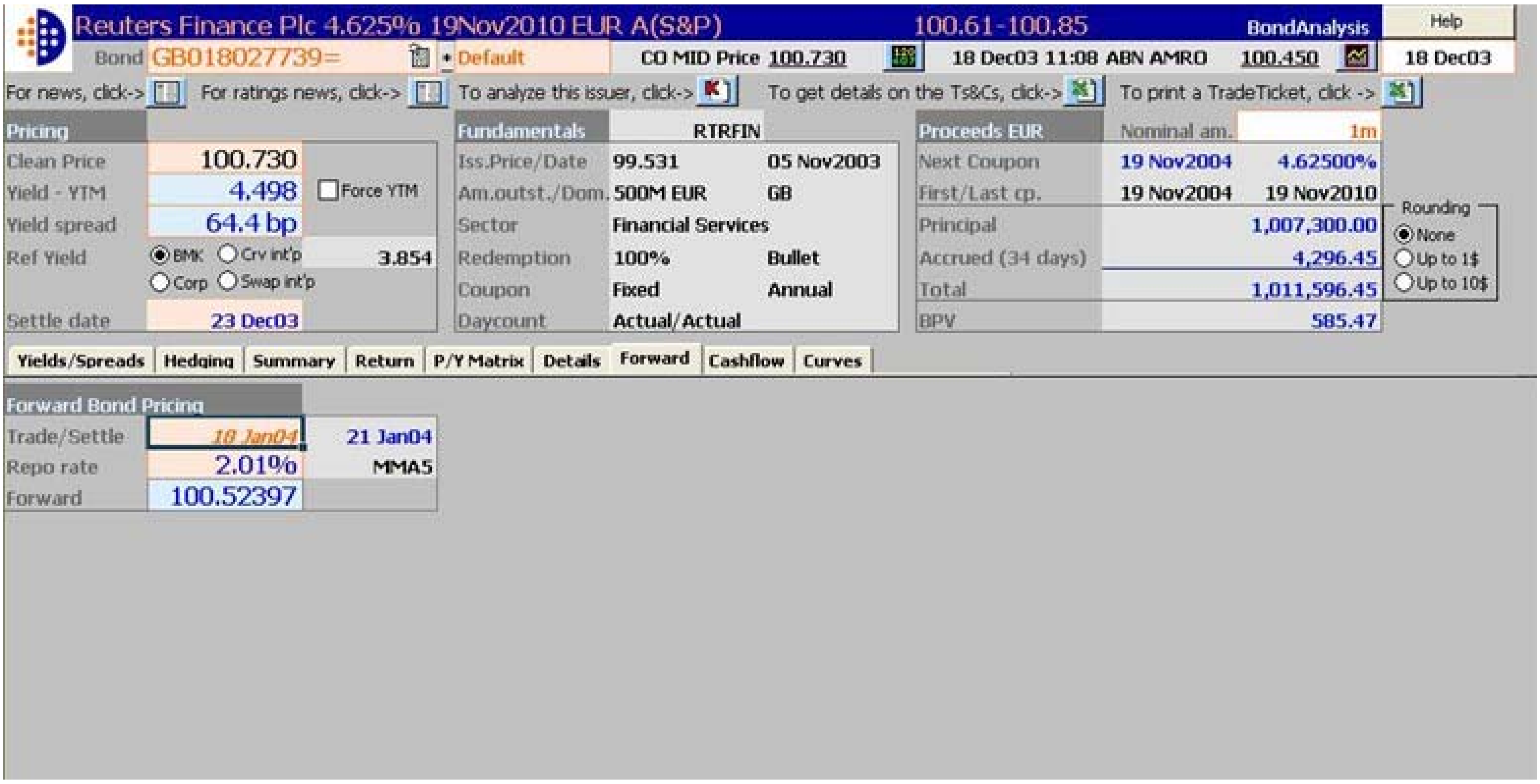
Below are sample screens from two widely-used providers of market information and analytics.

These examples are for illustration purposes only and do not form part of the IFID Certificate syllabus.

Bloomberg forward pricing analysis



Reuters bond analysis



Notes

- The function calculates:
    - Either the breakeven forward price on your bond, given a repo rate
    - Or, if you specify the forward price, the repo rate that you need to lock into this trade in order to break even (known as the **implied repo rate** in the bond futures market – see section *Implied Repo Rate*, below)

5.2. Forward Pricing Formula

Forward price = Cash cost + Net cost of carry
= Cash cost + Funding cost - Accrued interest

In symbols,

$$FP = (SP + SA) + (SP + SA) \times RP \times N / YBasis - FA$$
$$= SP + (SP + SA) \times RP \times N / YBasis - (FA - SA)$$

Where:
FP = Theoretical forward clean price
FA = Accrued interest at forward settlement date
SP = Spot clean price
SA = Accrued interest at spot settlement date
RP = Funding (i.e. repo) rate
N = Carry period
YBasis = 360 or 365, depending on repo day-count convention

The net cost of carry is the cost of funding the position, (SP + SA) x R x N/360, less the accrued income on the bond earned during the carry period, (FA - SA). The following formula is often used to approximate the breakeven forward price:

FP = SP + (RP% - C / SP x 100) x N / 365

Where C is the annual coupon rate on the bond (so C/SP is the **current yield**). Using the figures in our example above:

FP = 109.34375 + (2.75 - 8.25/109.34375 x 100) x 20 / 365
= 109.08101
= **109-03** to the nearest 32^{nd}, one tick away from the true breakeven.

Positive and Negative Carry

In this case, the bond's breakeven forward price is actually below its cash price. This is because the coupon income earned ($8.25 per $109-11 nominal for 20 days) is greater than the funding cost (2.75% for 20 days). The bond is said to have **positive carry**. If the funding cost was greater than the coupon income earned, then the bond would be said to have **negative carry** and the breakeven futures price would be above the cash price.

Positive carry:	Forward price of bond	<	Cash price of bond
	Financing costs	<	Income earned
Negative carry:	Forward price of bond	>	Cash price of bond
	Financing costs	>	Income earned

So the pattern of declining bond futures prices that we noticed at the beginning of this section is a reflection of a positive carry market, which tends to be the case when the yield curve is positive.

6. Exercise 1

6.1. Question 1

Question 1

a) By allowing a range of government bonds to qualify for delivery against the same futures contract, the futures exchange tries to:

(i) Make the derivative represent a whole section of the yield curve, rather than just a single point on it

(ii) Increase the liquidity of the futures contract

- Only (ii) is correct
- Neither is correct
- Only (i) is correct
- Both are correct

6.2. Question 2

Question 2

The conversion factor attempts to establish some value equivalence between the futures price and the price of each deliverable bond.

a) If the coupon rate on a deliverable bond is higher than that on the notional bond, its conversion factor is:

- Less than 1
- Greater than 1

6.3. Question 3

Question 3

Settlement date: 6 June 2003

Situation
Today is the last trading date of the June Bund futures and you are short 75 contracts, against which you intend to deliver the 5.50% maturing on 11 June 2012 (annual, actual/actual), which is currently the cheapest to deliver with a conversion factor of 0.965967.

Name:	EUREX Bund Futures
Unit of trading	EUR 100,000 of a notional Bund with a 6% coupon and maturity of 8½ - 10½ years
Delivery Months	March, June, September, December
Delivery Day	Tenth calendar day of delivery month
Last Trading Day	Two business days prior to Delivery Day
Quotation	Percentage of face value, to 2 decimal places
Minimum Price Movement (Tick Size)	0.01%
Tick Value	0.01% x 100,000 = EUR 10.00

a) What is the face value of the bonds that you should deliver?

EUR [ ]

b) If the EDSP is 107.08 and the CTD has 5.46986301% of accrued interest, what will be your invoice amount, rounded to the nearest Euro?

EUR [ ]

6.4. Question 4

Question 4

In this exercise you will use a forward bond pricing model based on the formula developed in section *Pricing*.

Launch the forward price and yield calculator spreadsheet and please ensure that the model contains the following data:

Bond structure		**Funding**	
Coupon rate	7.5000%	Repo rate	6.00%
Maturity	6-Apr-09	Day count: Actual/	360
Coupon period	Semi		
Year basis	Actl/Actl		

Spot date		**Forward date**	
Settlement	4-Nov-02	Settlement	4-May-03
Accrued	0.59753	Accrued	0.57377
Clean price	88.50000	Clean price	87.44453
Yield	9.965%	Yield	10.388%
Basis point value	4.3189	Risk factor	3.9957

a) Why is the forward clean price lower than the cash price of this bond?

- [ ] Because the market is bearish

- ☐ Because the yield on the bond is higher than the funding rate
- ☐ Because the market has negative carry
- ☐ Because the coupon rate on the bond is lower than the repo rate

b) What is the repo rate that would make the forward clean price of this bond equal to 89.00?

Instructions

You may try entering different repo rates until you find one that makes the forward price equal to 89.00. Alternatively, you may let Excel find it for you. Select **Tools | Goal Seek** from the Excel menu and enter the data shown in bold into the dialog box

Set cell: **Forward_price**
To value: **89.00**
By changing cell: **Reinvest**

Repo rate: ____

c) Restore the repo rate back to 6.00% and complete the table below, *entering your figures rounded to 2 decimal places*:

Spot Settlement	Forward Clean price	Basis = Cash price - Forward price
4-Nov-2002		
4-Feb-2003		

d) Which of the following is (are) true, *other things being equal*?

- ☐ A rise in the funding rate is beneficial to a long forward position
- ☐ A rise in funding rate makes a long forward position less profitable
- ☐ If carry is positive, a long forward position becomes more profitable over time
- ☐ If carry is negative, a long forward position becomes less profitable over time

7. Conversion Factors

Pricing a forward (or OTC futures) contract on a single bond is just a matter of adding the net carry on the bond to its cash price. Pricing exchange-traded bond futures is a lot more complex because there is more than one bond that is deliverable into the same contract. In order to understand what drives bond futures prices we need to know a little more about conversion factors.

Conversion factor: the clean price of a deliverable bond to yield the coupon rate on the notional bond by the first futures delivery date.

As we mentioned in section *Settlement*, the exchange fixes a conversion factor for each bond that is deliverable into a futures contract. The purpose is to establish a value equivalence - a 'level playing field' - between the deliverable bonds, so that the same futures price applies to them all.

Example

Date: 1 Sep 1998
Situation: The UK government yield curve is flat at 7.00%
EDSP: 100.00
Position: Short 1 LIFFE DEC Gilt futures contract

The table below lists all the bonds that were acceptable for delivery against this contract, together with the conversion factor for each bond.

Bond	**Coupon**	**Maturity**	**Conversion factor**
Treasury	7.25	07-Dec-07	1.0166793
Treasury	9.00	13-Oct-08	1.1431160
Treasury	8.00	23-Sep-09	1.0760380
Treasury	5.75	07-Dec-09	0.9035685
Treasury	6.25	25-Nov-10	0.9389076
Treasury	9.00	12-Jul-11	1.1676297

The coupon rate of the notional bond in the Gilt futures is 7%, so the conversion factors mean that:

- The 7.25% of 2007 would trade at 101.66793 to yield 7% on 1 September
- The 9.00% of 2008 would trade at 114.31160 to yield 7% on 1 September
- …
- The 9.00% of 2011 would trade at 116.76297 to yield 7% on 1 September

Notice that if the coupon on a bond is lower than the coupon on the notional, its conversion factor must be less than 1 and vice versa.

The conversion factors ensure that in this scenario there should be no gain (or loss) from delivering any of these bonds against the futures position.

Bond	**Cost of bond (per 100 nominal)**	**Invoice amount (per 100 nominal) = Futures price x Conv. factor + A.I.**	**Net gain**
7.25% '07	101.66793 + Accrued	100.00 x 1.0166793 + Accrued	Nil
9.00% '08	114.31160 + Accrued	100.00 x 1.1431160 + Accrued	Nil
...	...	...	...
9.00% '11	116.76297 + Accrued	100.00 x 1.1676297 + Accrued	Nil

We can also see that here if the yield curve is flat at 7%, then the EDSP must be 100.00; any other EDSP would open up an arbitrage window:

- If the EDSP is higher than 100: buy a cash bond and short the futures
- If the EDSP is lower than 100: short-sell a cash bond and buy the futures

In the next section we explore what happens when the yield curve is not flat at the same rate as the coupon on the notional bond.

8. Cheapest to Deliver

In the previous section, we showed that *if the yield curve is flat at the same rate as the coupon on the notional bond*, then the conversion factors ensure that any deliverable bond (no matter what its coupon rate or maturity) should have the same futures-equivalent value. In other words, in this scenario there should be no bond that is cheapest to deliver against the futures contract.

In this section, we explore what happens when the yield curve has a different shape or level.

Example

Date: 1 Sep 1998
Position: Short 1 LIFFE DEC Gilt futures contract

This is the same example that we used in section *Conversion Factors*. The table below lists all the bonds that were acceptable for delivery against this contract, together with the conversion factor for each bond.

Bond	Coupon	Maturity	Conversion factor
Treasury	7.25	07-Dec-07	1.0166793
Treasury	9.00	13-Oct-08	1.1431160
Treasury	8.00	23-Sep-09	1.0760380
Treasury	5.75	07-Dec-09	0.9035685
Treasury	6.25	25-Nov-10	0.9389076
Treasury	9.00	12-Jul-11	1.1676297

Scenario 1: The UK government yield curve is flat at 9.00%

In this scenario none of the bonds will be trading at their conversion factors: bonds with higher basis point value (because of their maturity or coupon) should trade relatively cheaper than bonds with lower BPVs. One of those bonds therefore becomes the **cheapest to deliver** (CTD) and the futures price will reflect the cost of delivering that bond. The table below shows the prices of these bonds in this scenario and the net loss achieved by delivering each bond into the futures, at the futures price established by the CTD.

Bond	1 Cost of bond (per 100 nominal)	2 Invoice amount (per 100 nominal) = Futures price x Conv. factor	Net loss = 1 - 2
7.25% '07	89.13710 + Accrued	85.03 x 1.0166793 + Accrued	2.69
9.00% '08	99.98242 + Accrued	85.03 x 1.1431160 + Accrued	2.78
8.00% '09	93.07632 + Accrued	85.03 x 1.0760380 + Accrued	1.58
5.75% '09	77.26835 + Accrued	85.03 x 0.9035685 + Accrued	0.44
6.25% '10	**79.83776 + Accrued**	**85.03 x 0.9389076 + Accrued**	**Nil**
9.00% '11	99.98022 + Accrued	85.03 x 1.1676297 + Accrued	0.69

In this scenario, the CTD is the 6.25% of 2010 and the adjusted futures price reflects the price of that bond; a trader delivering any other bond would make a loss shown in the last column

Scenario 2: The UK government yield curve is flat at 5.00%

Again, none of the bonds will be trading at their conversion factors: bonds with higher BPVs should trade relatively richer to bonds with lower BPVs. Now, the CTD is the 7.25% of 2007, which has a lower BPV than the 6.25% of 2010, and the futures price reflects the cost of delivering that bond.

Bond	**1 Cost of bond (per 100 nominal)**	**2 Invoice amount (per 100 nominal) = Futures price x Conv. factor**	**Net loss = 1 - 2**
7.25% '07	**116.51156 + Accrued**	**114.60 x 1.0166793 + Accrued**	**Nil**
9.00% '08	131.44438 + Accrued	114.60 x 1.1431160 + Accrued	0.44
8.00% '09	125.24565 + Accrued	114.60 x 1.0760380 + Accrued	1.93
5.75% '09	106.39149 + Accrued	114.60 x 0.9035685 + Accrued	2.84
6.25% '10	111.32529 + Accrued	114.60 x 0.9389076 + Accrued	3.73
9.00% '11	137.60099 + Accrued	114.60 x 1.1676297 + Accrued	3.79

8.1. General Pricing Formula

This example shows that normally one of the deliverable bonds becomes the CTD and this is the bond that actually drives the futures price. Whether we are looking at the futures price on the delivery date or before, all the cash & carry arbitrage is therefore between that bond and the futures contract.

Formally, the bond futures is a contract on a notional bond. In practice, it behaves as if it was a contract on one specific bond, namely the CTD.

Below is the theoretical forward pricing formula developed in section *Pricing*, adapted for the bond futures market.

Bond Futures Pricing Formula

$$FP \times CF = (SP + SA) + (SP + SA) \times RP \times N / YBasis - FA$$
$$= SP + (SP + SA) \times RP \times N / YBasis - (FA - SA)$$

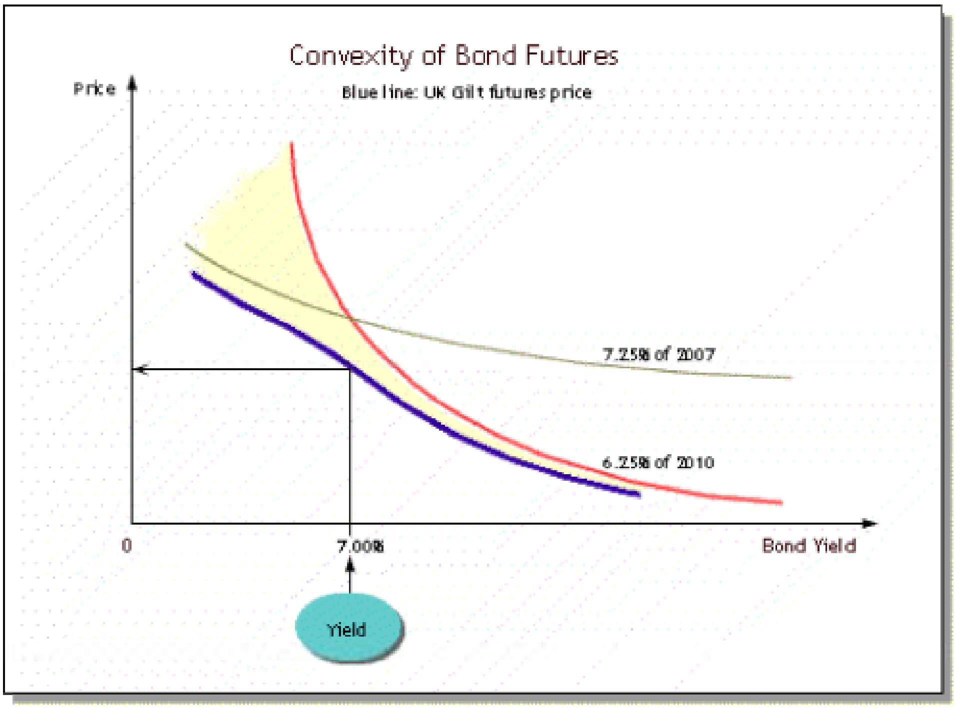
$$FP = \frac{SP + (SP + SA) \times RP \times N / YBasis - (FA - SA)}{CF}$$

Where:
FP = Theoretical bond futures price
CF = Conversion factor of the CTD bond
FA = Accrued interest at forward settlement date of CTD bond
SP = Spot clean price of CTD bond
SA = Accrued interest at spot settlement date of CTD bond
RP = Funding (i.e. repo) rate on the CTD bond
N = Carry period
YBasis = 360 or 365, depending on repo day-count convention

It follows that the market risk on the futures contract will reflect the BPV of the bond that is currently the CTD. Since the CTD may change, depending on the shape and level of the yield curve, the market risk on the futures is likely to vary as well.

8.2. Convexity

The figure below illustrates the relationship between the futures price and the prices of the two bonds that became the CTD in our example.



When market yields are low the futures price tracks the price of the low-BPV bonds, so its market risk is relatively low. At higher yields, the higher-BPV bonds tend to become the CTD and the futures price becomes more volatile.

Like callable bonds, bond futures may display negative convexity
(see Callable Bonds - Convexity).

As we shall see in later sections of this module, this pricing behaviour has important implications for:

- Traders who arbitrage the CTD against the bond futures
- Investors who use bond futures to manage the risks on their cash portfolios

9. Implied Repo Rate

Implied repo rate (IRP): the funding (or repo) rate that is implied in an actual futures price.

The IRP is used to:

- Measure the return potential of a cash-futures arbitrage trade
- Identify which bond is currently the CTD

The Formula

In section *Cheapest to Deliver* we modified the theoretical forward bond pricing formula to calculate the theoretical bond futures price, given the price of the CTD and its cost of funding.

In practice the bond futures seldom trades at fair (i.e. theoretical) value: it may trade rich or cheap to fair value. The IRP measures the extent to which the actual futures price is rich or cheap to fair value. It is calculated by turning the futures pricing formula around, so that we now solve for the repo rate instead of the futures price.

Implied Repo Rate

For the CTD bond:

$$FFP \times CF = (SP + SA) + (SP + SA) \times ARP \times N / Basis - FA$$

For any deliverable bond:

$$IRP = \left[\frac{(AFP \times CF + FA)}{(SP + SA)} - 1 \right] \times \frac{Basis}{N}$$

Where:
FFP = Fair bond futures price
AFP = Actual bond futures price
CF = Conversion factor
FA = Accrued interest at the forward settlement date
SP = Spot clean price
SA = Accrued interest at the spot settlement date
ARP = Actual funding (i.e. repo) rate
IRP = Implied repo rate
N = Carry period
Basis = 360 or 365, depending on repo day-count convention

The term inside the square brackets in the IRP formula is the percentage difference between the dirty price we would receive for this bond, if we sold it into the futures contract, and what the bond is worth today. The term outside the brackets annualises this percentage using the appropriate money market day-count convention.

The IRP is therefore the annualised gross return that you would make from buying a bond and selling it into the futures.

Example
Cash-futures Arbitrage

Settlement date:	3 August 1999
SEP Futures price:	103.67
Nr. days to delivery:	29
CTD bond:	7.25% maturing 7 December 2007
Coupon basis:	Semi-annual, Actual/Actual
Clean price:	105.17
Conversion factor:	1.0153346
Repo day-count basis:	Actl/365

? What is the IRP in these prices?

First we calculate the accrued interest on this bond, for cash and futures settlement:

Nr days from last coupon to cash settlement (7 June to 3 Aug)	=	57
Nr days from last coupon to futures settlement (7 June to 1 Sep)	=	86
Nr days in current coupon period (7 June to 7 Dec)	=	183

Accrued interest to cash settlement (SA) = 7.25 / 2 x 57/183
= 1.12910

Accrued interest to futures settlement (FA) = 7.25 / 2 x 86/183
= 1.70355

Next, we apply these figures to the IRP formula:

$$IRP = \left[\frac{(103.67 \times 1.0153346 + 1.70355)}{(105.17 + 1.12910)} - 1 \right] \times \frac{365}{29}$$

= 0.0786 or **7.86%**

? What would be the implication if the actual repo rate on this bond was 5.50%?

This implies an arbitrage profit:

- Buy the cash bond and simultaneously sell it into the futures; this locks in a gross return of 7.86%
- Carry the cash bond position until futures delivery, at a cost of 5.50%, so the net profit on the trade is 2.36% (= 7.86 - 5.50)

9.1. Using the Implied Repo Rate

Identifying Arbitrages

The example on the previous page illustrates how we can measure the arbitrage potential in a bond futures price by comparing its IRP with the actual repo rate (ARP):

- **If IRP > ARP then the futures trades rich to fair value (i.e. actual futures price > fair futures price): Buy the cash bond and short the futures!**
- **If IRP < ARP then the futures trades cheap to fair value (i.e. actual futures price < fair futures price): Short-sell the cash bond and buy the futures!**

You can see from the IRP formula that, other things being equal, the higher the actual futures price the higher is the IRP. Therefore, if the futures trades rich to fair value, its IRP must be greater than its actual repo rate, and vice-versa.

Identifying the CTD

If we think of the implied repo rate as the gross return that can be made from buying a bond and selling into the futures, then:

The CTD is the bond with the highest (IRP - ARP).

In other words, the CTD is the bond that yields the largest profit (or smallest loss) when it is arbitraged against the futures. Identifying which bond is currently the CTD is therefore a matter of calculating the IRP for each deliverable bond: the one with the highest IRP relative to its ARP is the CTD!

10. Gross Basis

Futures basis = Cash price of underlying – Futures price

When the term basis is used on its own in the context of bond futures, it usually means the **gross basis**.

Gross basis = Cash price of bond - Adjusted futures price
= Cash price of bond - (Futures price x Conversion factor)
= SP - FP x CF

10.1. Example - Basis Trading

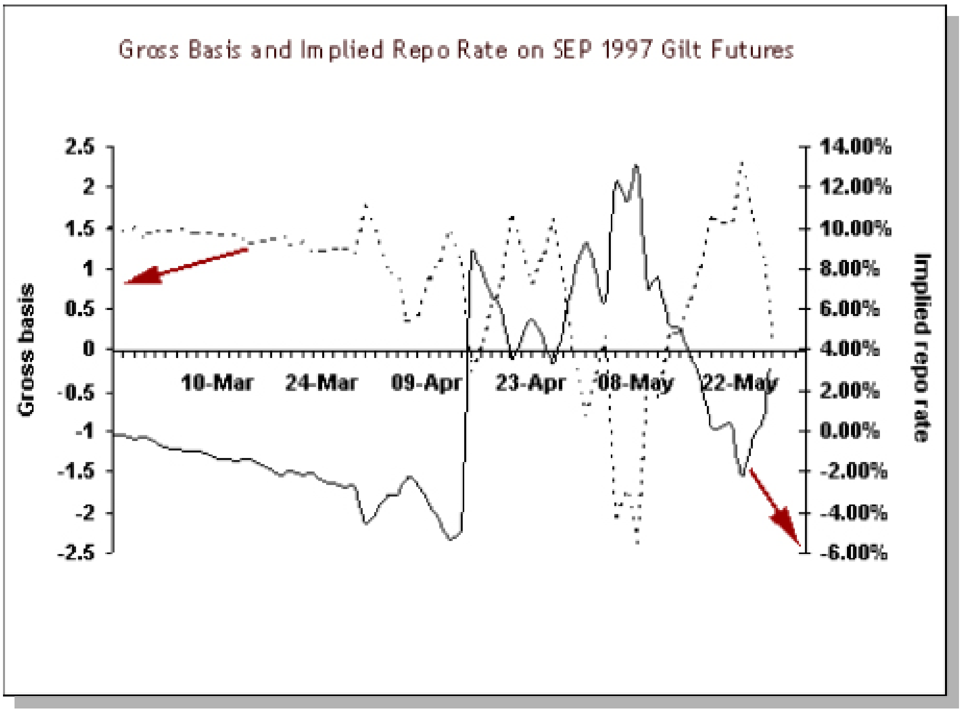
Basis traders take positions in anticipation of changes in the gross basis:

- Long the basis = Long the cash bond and short the futures
- Short the basis = Short the cash bond and long the futures

Situation

Settlement date:	23 May 1997
Futures contract:	LIFFE SEP long Gilt futures
Futures price:	90.49 (in decimal)
Contract specification:	GBP 50,000 of 9% notional bond
CTD:	6.25% of 2010
Coupon basis:	Semi-annual, Actual/365
Clean price:	71.54 (in decimal)
Conversion factor:	0.7896236

The figure on the next page shows the evolution of the gross basis and the implied repo rate on this bond over the preceding 3 months.



? Why is there a tendency for the gross basis to vary inversely with the implied repo rate?

The implied repo rate measures the extent to which the futures price trades rich or cheap to fair value. Other things being equal, the higher the implied repo rate, the richer the futures trades and (by definition) the lower is its gross basis.

? What is the gross basis on this bond now?

Gross basis = 71.54 - (0.7896236 x 90.49)
= **+0.087**

? What position(s) should the trader create if she believed that the gross basis should increase at this point in the delivery cycle?

The trader should **buy the basis**:
- Buy the cash bonds
- Sell the futures

? How much of the CTD bond should the trader buy, if she intends to short 100 futures contracts?

The idea is to create a position that generates profit or loss only if the gross basis changes. In our example, the gross basis would remain constant if every point rise in the futures price was associated with a 0.7896048 point rise in the price of the cash bond. Therefore, to create a position that is market-neutral, the trader selling 100 futures contracts should buy proportionately more than 50,000 x 100 bonds:

$$\text{Cash bond position} = \frac{50{,}000 \times 100}{\text{Conversion factor}}$$

$$= \frac{50{,}000 \times 100}{0.7896236}$$

= **GBP 6,332,130**, rounded.

Thus, the amount of bonds to be carried on a basis trade depends on their conversion factor: the higher the conversion factor, the smaller the amount of bonds that need to be carried.

Risk-weighted Basis Positions

$$\text{Bond portfolio size} = \frac{\text{Futures contract size} \times \text{Number of contracts}}{\text{Conversion factor}}$$

$$\text{Number of futures contracts} = \frac{\text{Bond portfolio size}}{\text{Futures contract size}} \times \text{Conversion factor}$$

The Outcome

Settlement date: 30 May 1997
Futures price: 91.23
CTD: 6.25% of 2010
Clean price: 72.31 (in decimal)

Gross basis = 72.31 - (0.7896236 x 91.23)
= **+0.273**

The basis rose by 0.186 (0.273 - 0.087), so the trader should have made a gross profit.

Date	Cash position	Futures position
23 May	6,332,130 x 71.54/100 = - 4,530,005.80	Sell 100 contracts @ 90.49
30 May	6,332,130 x 72.31/100 = +4,578,763.20	100 x 50,000 x (0.9049 - 0.9123) = - 44,500.00
Net	**+ 48,757.40**	**-37,000.00**

The combined profit of GBP 11,757.40 (= 48,757.40 - 37,000.00) does not take into account the net cost of carrying the trade. We shall do this in the next section.

10.2. What Drives the Gross Basis?

There are three main factors:

- **Bond yields**: One of the variables in the gross basis formula is the cash price of the deliverable bond, which depends on its yield. The formula suggests that the higher the price of the bond (the lower its yield), the higher will be the gross basis, but in fact this is not so.

  The difference between the price of the CTD and the futures price reflects the net cost of carrying that bond to futures delivery. At a higher price (lower yield) the net carry cost increases and this has the effect of reducing the basis. In other words, if yields fall the bond price may rise, but the futures price should rise proportionately more!

- **Funding rates**: the higher the repo rate, the higher will be the cost of carrying the underlying bond, so the futures price should rise relative to the cash price and this reduces the basis.

  For example, in the figure above the sterling yield curve inverted during April-May 1997 and the carry on the CTD became negative. As a result, the adjusted futures price exceeded the cash price and the gross basis was negative.

- **Time to futures delivery**: as we saw in section *Cheapest to Deliver*, over time:

  - The adjusted futures price converges to that of the CTD
  - So the gross basis converges to zero

The convergence is rarely smooth, as bond yields and funding rates change all along.

11. Net Basis

Net basis = Gross basis - Net carry
= (SP - FP x CF) - [(FA - SA) - (SP + SA) x RP x N / Year basis]

= SP + (SP + SA) x RP x N / Year basis - (FA - SA) - FP x CF
= Theoretical forward price - Adjusted futures price

Also known as: **Basis net of carry**, **Value basis**.

The net basis calculates the all-in profitability of a cash & carry arbitrage, taking into account the net carry on a basis position:

- Negative net basis means:
    - The net carry benefit exceeds the gross basis
    - Adjusted futures price is higher than theoretical forward price
    - There is an arbitrage profit in a long basis position

- Positive net basis means:
    - The net carry benefit is smaller than the gross basis
    - Adjusted futures price is lower than theoretical forward price
    - There is an arbitrage profit in a short basis position

11.1. Example – Cash-futures arbitrage

Settlement date:	30 May 1997
Futures contract:	LIFFE SEP 1997 long Gilt futures
Futures price:	91.23
Contract size:	GBP 50,000
First delivery date:	1 September 1997
CTD:	6.25% of 25 November 2010
Coupon basis:	Semi-annual, Actual/365
Clean price:	72.31 (in decimal)
Conversion factor:	0.7896236
Repo rate:	8.00%

? Is there an arbitrage in the current market?

One way to determine whether there is an arbitrage is to compare the actual futures price with the theoretical futures price, calculated using the formula we developed in section *Cheapest to Deliver*:

$$\mathbf{FP} = \frac{\mathbf{SP + (SP + SA) \times RP \times N / YBasis - (FA - SA)}}{\mathbf{CF}}$$

Where:
FP = Theoretical futures price
CF = Conversion factor of CTD bond
FA = Accrued interest at forward settlement date of CTD bond
SP = Spot clean price of CTD bond
SA = Accrued interest at spot settlement date of CTD bond
RP = Funding (i.e. repo) rate on the CTD bond
N = Carry period
YBasis = 360 or 365, depending on repo day-count convention

Nr days from last coupon to cash settlement (25 May to 30 May)	=	5
Nr days from last coupon to futures settlement (25 May to 1 Sep)	=	99
Nr days in current coupon period	=	182.5
Nr days from cash to futures settlement (30 May to 1 Sep)	=	94

Accrued interest to cash settlement (SA) = 6.25 / 2 x 5/183
= 0.08562

Accrued interest to futures settlement (FA) = 6.25 / 2 x 99/183
= 1.69521

$$FP = \frac{72.31 + (72.31 + 0.08562) \times 0.08 \times 94 / 365 - (1.69521 - 0.08562)}{0.7896236}$$

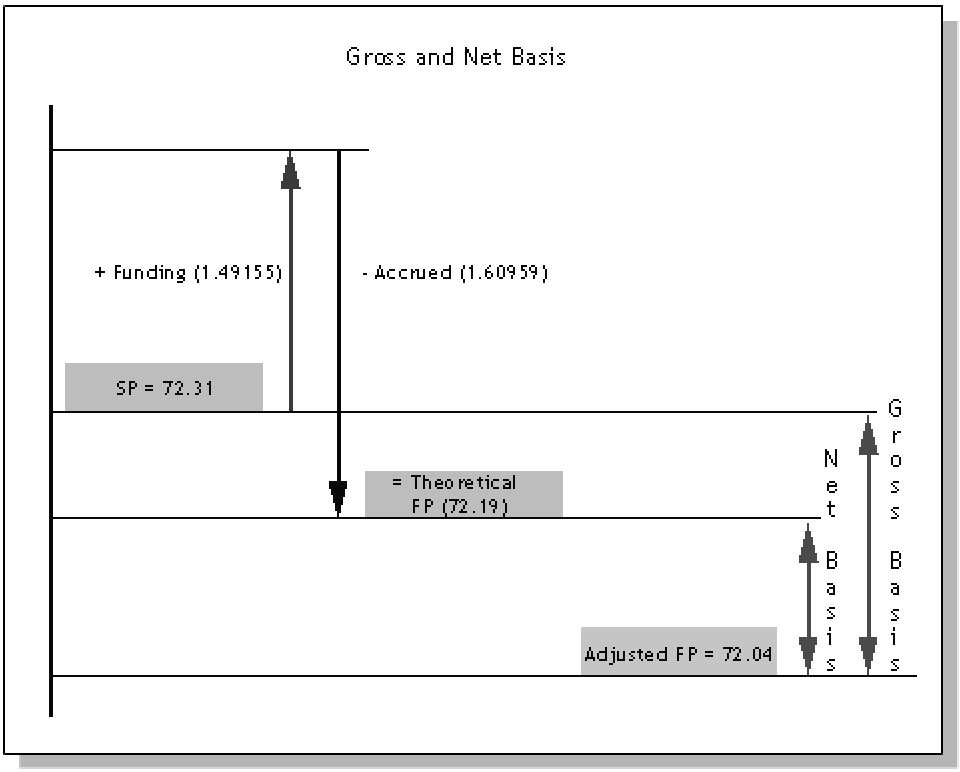
$$= \frac{72.31 + 1.49155 - 1.60959}{0.7896236}$$

= **91.43**

The futures is 0.20 points below fair value, so in principle there is an arbitrage profit. The net basis expresses the arbitrage profit potential on the trade in units of the underlying bond price:

Net basis = (SP - FP x CF) - [(FA - SA) - (SP + SA) x RP x N / Year basis]
= (72.31 - 91.23 x 0.7896236) - [1.60959 - 1.49155]
= 72.31 + 1.49155 - 1.60959 - 72.04
= 72.19 - 72.04
= **0.15**

Thus, the arbitrage profit is worth 0.15 points on the bond price. The figure below summarises the relationship between the gross basis and the net basis.



11.2. Net Basis and the Implied Repo Rate

In section *Implied Repo Rate* we saw that if the actual futures price is cheap to fair value, then the implied repo rate (IRP) must be less than the actual repo rate (ARP). We therefore have three alternative ways of expressing an arbitrage condition:

- **If actual futures price < theoretical futures price:**
  => IRP < ARP
  => Net basis > 0
  => Short cash bonds and buy the futures!

- **If actual futures price > theoretical futures price:**
  => IRP > ARP
  => Net basis < 0
  => Go long the cash bonds and short the futures!

Arbitrage Risks

In principle there is an arbitrage profit in the example above, which we could capture by shorting the cash bonds and buying the futures. But whenever you spot what looks like an arbitrage situation, you should always ask yourself the question: *Where's the catch?*

In this case one catch is that we would need to short the CTD bond until the futures delivery. It is not always possible to fix the cost of repoing in the bonds for the full term, so the trader carrying a short bond position is vulnerable to changes in the repo rate.

At the same time, as we explained in section *Cheapest to Deliver*, the bond futures has **negative convexity**. A short basis position (i.e. short the cash bonds and long the futures) therefore has negative convexity too. This means that movements in market yields can lead to a change in the CTD bond, and this would result in a trading loss.

We shall explore these important technical points in the next section.

Analytic systems
Examples of Bloomberg and Reuters bond futures CTD analysis functions

Below are sample screens from two widely-used providers of market information and analytics.

These examples are for illustration purposes only and do not form part of the IFID Certificate syllabus.

Bloomberg CTD summary

<HELP> for explanation, <MENU> for similar functions. N194 Comdty F3
Hit {NUMBER} <Go> to view detailed ranking/"F3 C" <Go> for Conventional Risk
16:53 CHEAPEST-TO-DELIVER SUMMARY
Mon 9/15

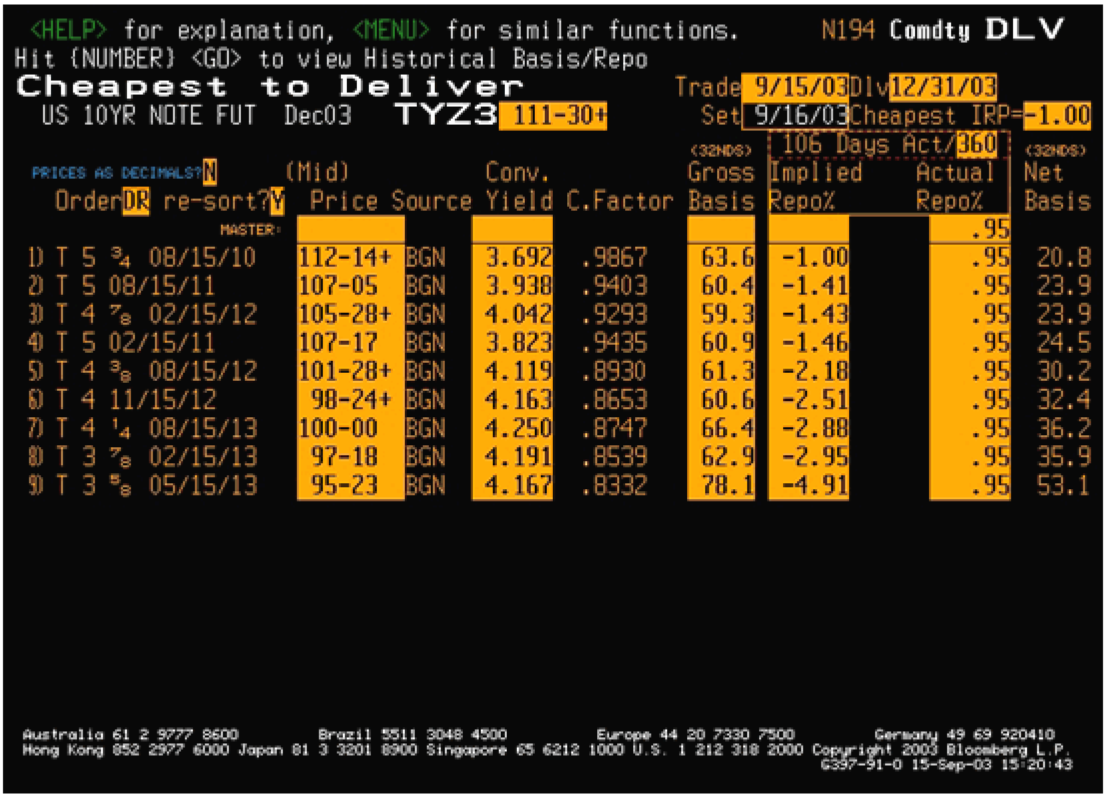
			DEC 2003				MAR 2004			
Contract	Cash Tkr	Futr	Price	Risk	CTD	Issue	Price	Risk	CTD	Issue
US 20yr 6%	T	1) USZ3	107-22	13.4	6 7/8	8/25	2) 106-11	13.4	6 7/8	8/02
US 10yr 6%	T	3) TYZ3	111-30+	6.57	5 3/4	8/10	4) 110-13+	7.07	5	2/11
US 5yr 6%	T	5) FVZ3	111-19	4.63	3	2/ 8	6) 110-16	4.88	2 5/8	5/ 8
US 2yr 6%	T	7) TUZ3	107-10	2.24	5 3/4	11/ 5	n/a			1/ 5
UK 10yr 7%	UKT	9) G Z3	117.86	8.67	8	9/13	10) 109.81	8.11	8	9/13
Euro-Bund 10yr 6%	DBR	11) RXZ3	113.46	8.26	5	7/12	12) 113.23	8.77	4 1/2	1/13
Euro-Bobl 5yr 6%	DBR	13) OEZ3	110.59	4.87	4 1/8	7/ 8	14) 110.35	5.38	3 3/4	1/ 9
Euro-Schatz 2yr 6%	DBR	15) DUZ3	105.81	2.12	6 1/2	10/ 5	16) 105.58	2.36	6	1/ 6
	CAN	17) CNZ3	108.33	7.61	5 1/4	6/12	18) 107.51	7.60	5 1/4	6/12
Jpn 10yr 6%	JGB	19) JBZ3	135.63	9.26#	225	12/10	20) 135.71	9.64	# 227	3/11
Jpn 10yr 6%	JGB	21) N Z3	136.05	9.26#	225	12/10	22) 135.76	9.64	# 227	3/11
Euro 10yr 3.5%	FRTR	23) MNZ3	94.07	7.58	4	4/13	24) 94.07	7.58	4	4/13
Euro 30yr 5.5% Bnd	FRTR		n/a			4/13	n/a			4/13
Swiss 10yr 6%	SWIS	27) FBZ3	127.65	10.3	4	2/13	28) 126.29	9.95	2 3/4	6/12
Spanish 10yr 4.0%	SPGB		n/a			7/13	n/a			7/13

Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 920410
Hong Kong 852 2977 6000 Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2003 Bloomberg L.P.
G337-91-0 15-Sep-03 16:53:17

Notes

- This screen summarises which bonds are currently CTD for the various bond futures contracts and therefore which bonds are driving the futures prices
- Notice that different delivery months of the same contract may have a different CTD and therefore the risk characteristics of each delivery month of the same futures contract may be different

Bloomberg CTD analysis


<HELP> for explanation, <MENU> for similar functions. N194 Comdty DLV
Hit {NUMBER} <GO> to view Historical Basis/Repo
Cheapest to Deliver Trade 9/15/03 Dlv 12/31/03
US 10YR NOTE FUT Dec03 TYZ3 111-30+ Set 9/16/03 Cheapest IRP=-1.00
106 Days Act/360
PRICES AS DECIMALS? N
Order DR re-sort? Y

	(Mid) Price	Source	Conv. Yield	C.Factor	(32NDS) Gross Basis	Implied Repo%	Actual Repo%	(32NDS) Net Basis
MASTER:							.95	
1) T 5 ¾ 08/15/10	112-14+	BGN	3.692	.9867	63.6	-1.00	.95	20.8
2) T 5 08/15/11	107-05	BGN	3.938	.9403	60.4	-1.41	.95	23.9
3) T 4 ⅞ 02/15/12	105-28+	BGN	4.042	.9293	59.3	-1.43	.95	23.9
4) T 5 02/15/11	107-17	BGN	3.823	.9435	60.9	-1.46	.95	24.5
5) T 4 ⅜ 08/15/12	101-28+	BGN	4.119	.8930	61.3	-2.18	.95	30.2
6) T 4 11/15/12	98-24+	BGN	4.163	.8653	60.6	-2.51	.95	32.4
7) T 4 ¼ 08/15/13	100-00	BGN	4.250	.8747	66.4	-2.88	.95	36.2
8) T 3 ⅞ 02/15/13	97-18	BGN	4.191	.8539	62.9	-2.95	.95	35.9
9) T 3 ⅝ 05/15/13	95-23	BGN	4.167	.8332	78.1	-4.91	.95	53.1

Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 920410
Hong Kong 852 2977 6000 Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2003 Bloomberg L.P.
G337-91-0 15-Sep-03 15:20:43

Notes

- This screen ranks the deliverable bonds in order of cheapness to deliver (i.e. implied repo rates)

- The negative implied repo rate for the CTD (as well as all the other deliverables) means that the futures is trading cheap to fair value, which means:

  - There is no arbitrage profit on a long cash & carry arbitrage against futures

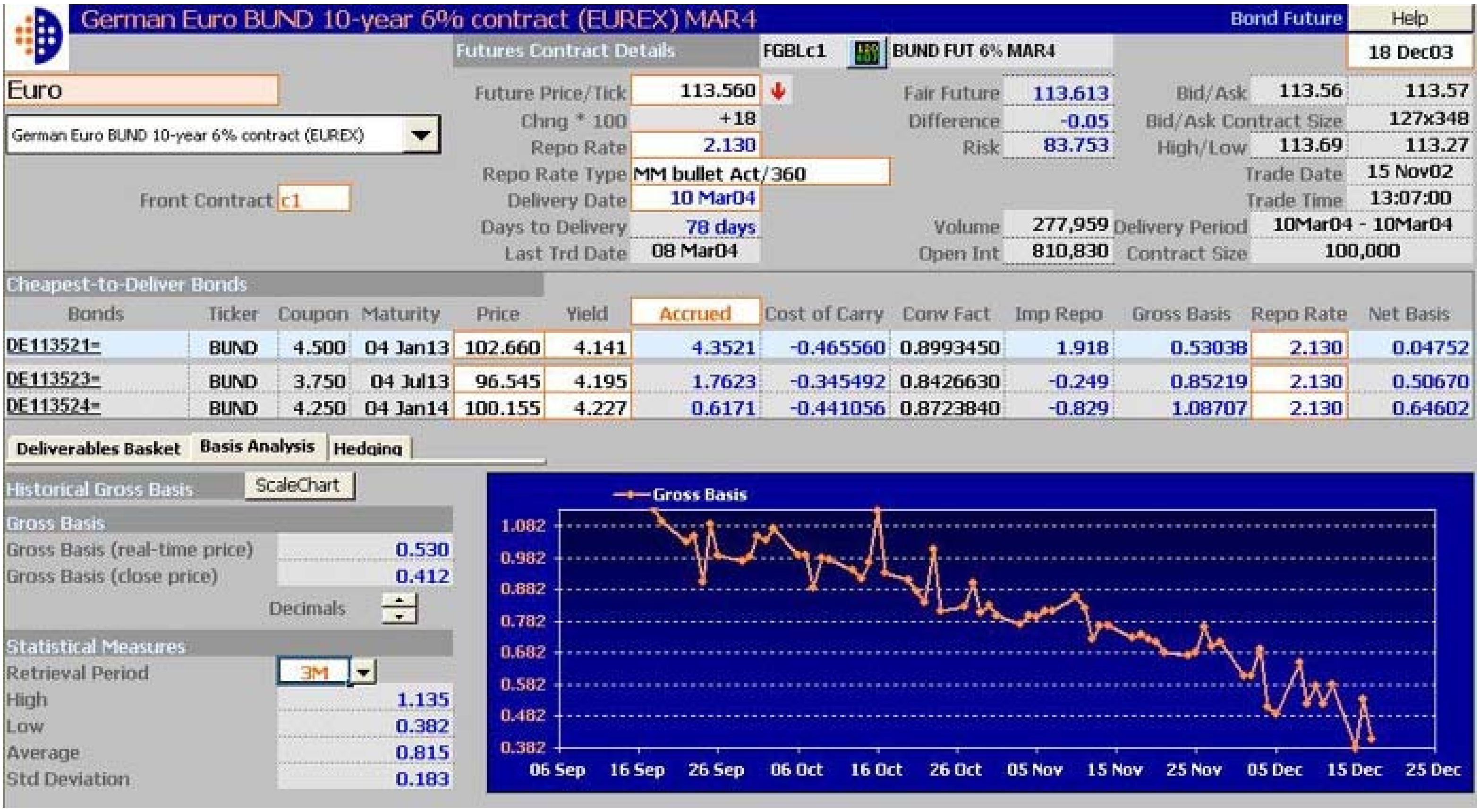
  - There may be an arbitrage profit on a short cash & carry arbitrage provided the CTD does not change!

- Notice the correlation between the implied repo rate and the net basis. The two measure the profitability of a cash-futures arbitrage but in different units:

  - The implied repo rate is the annualised gross percentage return on capital (before taking funding cost into account)

  - The net basis is the net cash return (including funding cost) per $100 nominal invested in the arbitrage

Reuters bond futures



Notes

- This screen ranks the deliverable bonds in order of cheapness to deliver (i.e. implied repo rates)
- The negative implied repo rate for the CTD (as well as all the other deliverables) means that the futures is trading cheap to fair value, as shown in the top-centre. This means:
    - There is no arbitrage profit on a long cash & carry arbitrage against futures
    - There may be an arbitrage profit on a short cash & carry arbitrage provided the CTD does not change!
- Notice the correlation between the implied repo rate and the net basis. The two measure the profitability of a cash-futures arbitrage but in different units:
    - The implied repo rate is the annualised gross percentage return on capital (before taking funding cost into account)
    - The net basis (=gross basis – cost of carry) is the net cash return (including funding cost) per $100 nominal invested in the arbitrage
- The lower panel displays the evolution of the gross basis on the current CTD: notice how it gradually converges to zero as the futures approaches delivery

12. Exercise 2

12.1. Question 1

Question 5

In this section we explore the price behaviour of bond futures in relation to the CTD and we illustrate the profits and risks associated with cash-futures arbitrage. Please launch the CTD Analysis spreadsheet and ensure the following data is specified correctly in the **Analysis** worksheet.

Cash market

Settlement date	1-Aug-99
Repo rate (RP)	5.2500%
Year basis	Actual/365

Futures market

Notional coupon	7.00%
1st delivery date	1-Sep-99
Difference	0.00

Yield vertices

	Raw Yld.	**Yld+Shift**	**Yld+Pivot**
Overnight	5.2500%	5.2500%	5.2500%
2 Yrs	5.7500%	5.7500%	5.7500%
4 Yrs	6.2500%	6.2500%	6.2500%
10 Yrs	6.3500%	6.3500%	6.3500%
20 Yrs	6.3700%	6.3700%	6.3700%
Shift		0.00%	0.00%

Deliverable bonds

Name	**Coupon Rate**	**Maturity Date**	**Coupon Frequency**	**Day-count Basis**	**Actual RP**
Treasury	7.2500	7-Dec-07	Semi-annual	Actl/Actl	5.25%
Treasury	9.0000	13-Oct-08	Semi-annual	Actl/Actl	5.25%
Treasury	8.0000	23-Sep-09	Semi-annual	Actl/Actl	5.25%
Treasury	5.7500	7-Dec-09	Semi-annual	Actl/Actl	5.25%
Treasury	6.2500	25-Nov-10	Semi-annual	Actl/Actl	5.25%
Treasury	9.0000	12-Jul-11	Semi-annual	Actl/Actl	5.25%

This specifies all the bonds that were deliverable into the LIFFE SEP 1999 long Gilt futures contract and prices the futures given the specified market conditions.

The CTD analysis model

The deliverable bonds are priced off a yield curve generated from a set of observed yield points, or **vertices**, labelled **Raw Yields**. A smooth yield curve is fitted through these vertices using cubic spline interpolation (for an explanation of this technique see Yield Curve Fitting - Cubic Splines).

The fields **Yld + Shift** and **Yld + Pivot** in the worksheet are just convenient means of allowing you to shift or pivot the curve without re-inputting all the yields again.

The worksheet calculates the theoretical futures from the CTD (i.e. the bond with the highest implied repo rate over the actual repo rate). It also allows you to enter an actual futures price by specifying a **Difference** between the actual and the theoretical futures prices. The gross and net basis for each bond, as well as the implied repo rates, is calculated from the actual futures price.

a) What is the theoretical futures price and what is the implied repo rate (IRP) on the current CTD? Type your answer in each box below and validate.

Theoretical futures price ☐

IRP (%) ☐

Please check the calculator settings, above, if your answers don't match!

b) Complete the table below showing the evolution of the gross basis on the CTD as we approach the futures delivery date.

Cash market Settlement date	Gross basis of CTD
1-Aug-99	☐
10-Aug-99	☐
20-Aug-99	☐
1-Sep-99	☐

c) What does a positive gross basis suggest?

- ○ The futures is trading cheap to fair value
- ○ There is positive carry
- ○ The futures price is higher than the price of the CTD
- ○ There is negative carry

d) Restore the cash settlement date to 1-Aug-99 and force the actual futures price below the theoretical price by entering 0.20 in the **Difference** field. Which of the following is/are true?

- ☐ The futures is trading rich to fair value
- ☐ The net basis on the CTD is 0.20
- ☐ There is an arbitrage opportunity in shorting the CTD and buying the futures
- ☐ There is an arbitrage opportunity in buying the CTD and buying the futures

e) The futures contract size is GBP 100,000. How many bond futures should you buy, if you were considering shorting GBP 20 million of the CTD bonds and carrying this position to delivery?

- ○ 203 contracts
- ○ 2,000 contracts exactly
- ○ 197 contracts
- ○ 200 contracts exactly

12.2. Question 2

Question 6

Use the same data in the **Analysis** worksheet as in *Question 1* and keep the actual futures price below the theoretical price by entering 0.20 in the Difference cell. We shall now calculate the profit potential of arbitraging the 7.25% of 2007 against the SEP futures:

- Short the 7.25% of 2007
- Buy the SEP futures

a) **Cash market settlement date:** **1-Aug-99**

(i)	Dirty price of 7.25% of 2007 (rounded to 5 decimal places)	
(ii)	Settlement amount on GBP 20 million of the 7.25% of 2007	
(iii)	Actual futures price	
(iv)	Nr. days to futures delivery	

b) Now move the **Cash settlement date** forward to 1-Sep-99 (the first delivery date) and change the value in the **Difference** field back to 0.00. In other words, we assume that by the first futures delivery date the gross and the net basis on the CTD will collapse to zero.

Cash settlement date: **1-Sep-99**

(v)	Actual futures price (EDSP)	
(vi)	Futures variation margin (1-Aug to 1-Sep) on 200 contracts	
(vii)	Interest earned on reverse repo: = (ii) @5.25% for (iv) days	
(viii)	Futures delivery invoice amount	
(ix)	Net arbitrage profit = (vi) + (vii) + (ii) - (viii)	
(x)	Profit as a percentage of capital: = (ix) / (ii) x 100 (to 2 decimal places)	

c) Restore the **Cash settlement date** back to 1-Aug-99 and the **Difference** field back to 0.20. We shall now explore the risk on this basis position if there is a change in the CTD. Force a parallel upward shift in the yield curve by entering +1.50% in the **Yld+Shift** field and calculate the mark-to-market profit/loss on your position:

Cash settlement date: **1-Aug-99**

(xi)	Dirty price of 7.25% of 2007 (rounded to 5 decimal places)	
(xii)	Settlement amount on GBP 20 million of the 7.25% of 2007	
(xiii)	Actual futures price	
(xiv)	Profit on cash bonds = (ii) - (xii)	
(xv)	Profit on bond futures	
(xvi)	Net Profit	

d) Which of the following is/are true?

- ☐ The futures price fell less steeply than the cash price
- ☐ The futures price fell more steeply than the cash price
- ☐ The CTD changed to a bond with a higher BPV
- ☐ The CTD changed to a bond with a lower BPV

e) Finally, watch what happens if the 5.75% of 2009 were to go on special. Change the *Actual RP* only on this bond to 4.00%. Which of the following is/are true?

- ☐ There is no change in CTD
- ☐ The future price also changes
- ☐ The futures price does not change
- ☐ The 5.75% 0f the 2009 becomes the CTD

13. Hedging

13.1. The Risk on a Bond Portfolio

The following example illustrates how bond futures may be used to manage the risk on a cash bond portfolio.

Settlement date:	10 July 1998
Position:	Long FRF 150 million of 7% French Government bond (OAT) maturing 25 October 2010 (annual, Actual/actual)
Price:	102.38
BPV:	0.08454
Scenario:	You expect the French bond market to react unfavourably to the forthcoming budget.

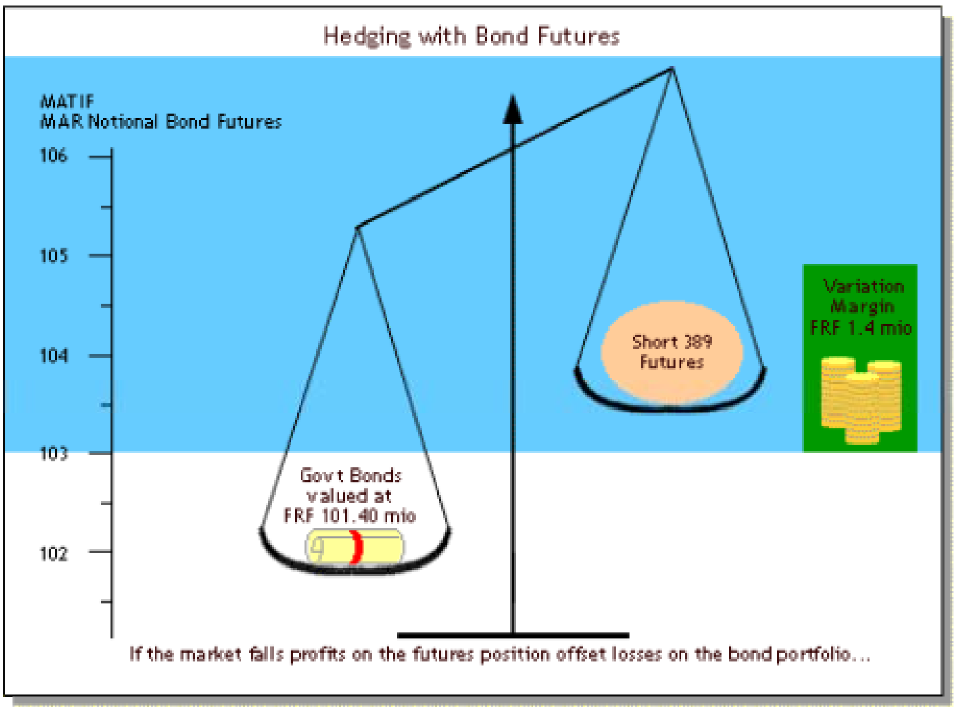
Strategy: **Hold the bond position but sell futures on the notional.**

Contract	MATIF Notional Bond Futures
Unit of trading	FRF 500,000 nominal notional French Government bond with 5½% coupon
Delivery months	March, June, September, December
Delivery day	Third Wednesday of delivery month
Last trading day	11:00 two business days prior to Delivery Day
Quotation	Percentage of face value (i.e. per FRF 100 nominal)
Minimum price movement (Tick Size)	0.02%
Tick value	$\frac{0.02}{100} \times 500{,}000$ = FRF 100.00

Deliverable bonds must have a maturity of 8½ - 10 years, so the bond you hold is not deliverable against the futures contract.

13.2. The Hedge Ratio

The idea is to create an offsetting position in the futures market that will generate profits equal to the losses that would be sustained on your bond, if the yield curve were to shift up.



Calculating the number of futures contracts you should sell to create a risk-weighted hedge - i.e. the **hedge ratio** - is not trivial because it depends on:

- The size of your portfolio
- Which bond is currently the cheapest to deliver (CTD) against the futures contract and therefore drives the futures price
- The market risk of your bond portfolio relative to that of the CTD

The diagram below outlines the links between the futures contract, the CTD and your cash bond portfolio. It also defines the industry-standard bond futures hedging formula.

Bond Futures Hedging Formula
(IFID exam formula)

Futures risk <---------------------> Risk of CTD <--------> Risk of cash portfolio

Futures risk-weighted hedge:

$$\text{Nr futures contracts} = \frac{\text{Portfolio size}}{\text{Contract size}} \times \text{Price factor of CTD} \times \frac{\text{BPV of cash portfolio}}{\text{BPV of CTD}}$$

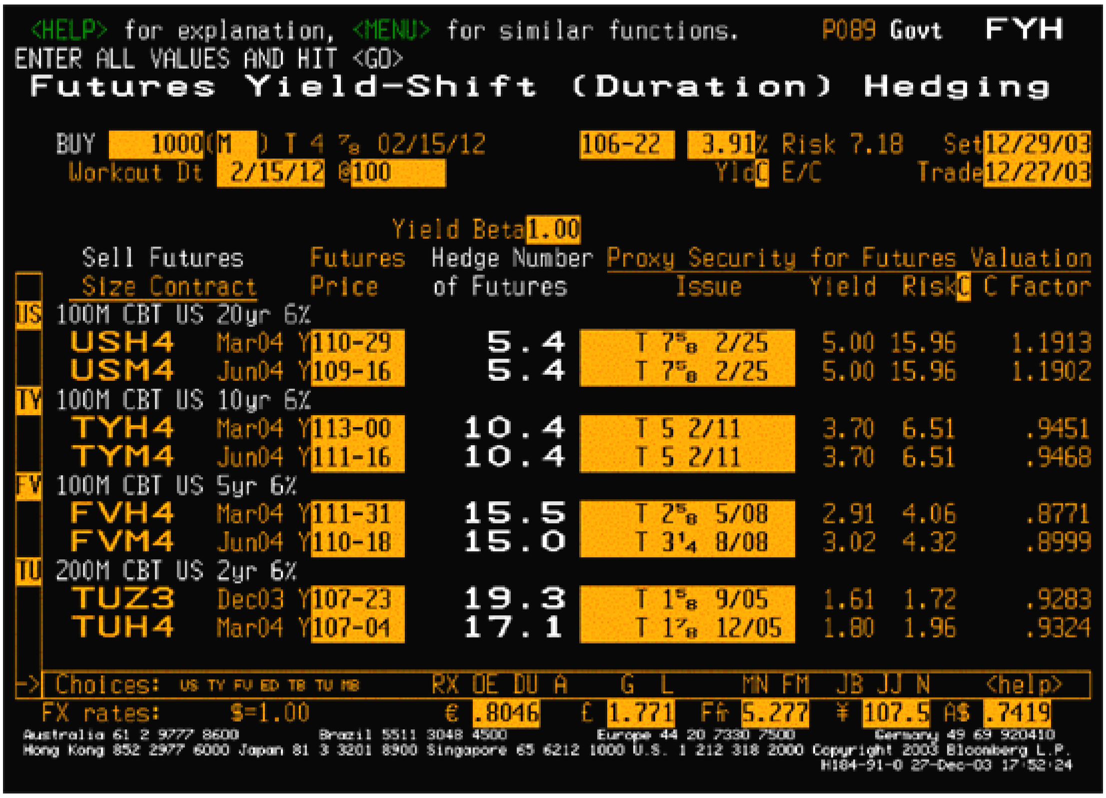
The first two terms in this formula establish a relationship between the price risk on the futures contract and the risk on the CTD. This was developed in section *Gross Basis*. The last term establishes a relationship between the risk on the CTD and the risk on your cash portfolio.

Analytic systems
Examples of Bloomberg and Reuters bond futures hedging analysis functions

Below are sample screens from two widely-used providers of market information and analytics.

These examples are for illustration purposes only and do not form part of the IFID Certificate syllabus.

Bloomberg futures hedging

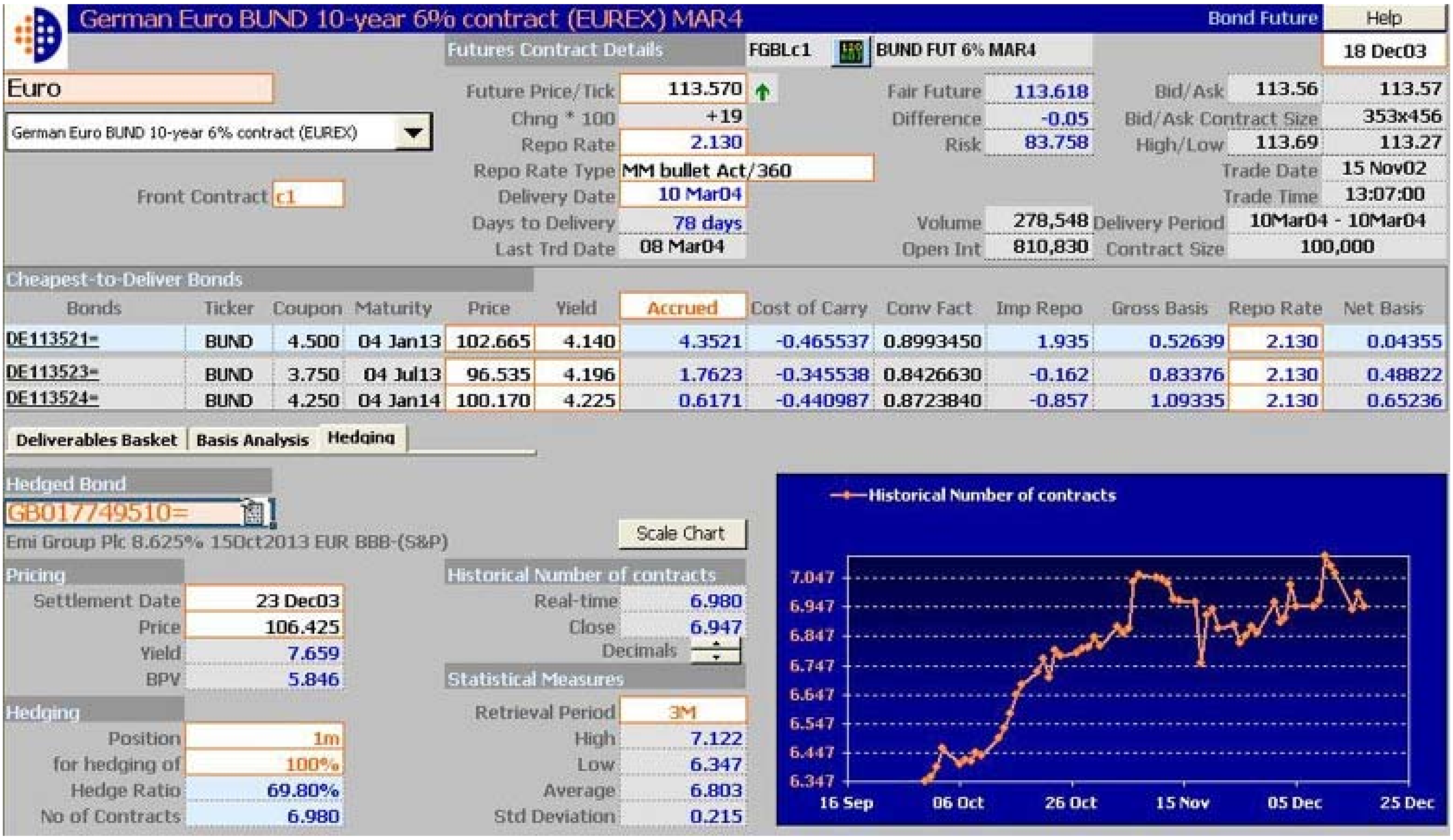


Notes

- This screen calculates how many of the selected futures contracts you should short in order to hedge just the market risk on the long position in the specified bond (i.e. a parallel shift in the yield curves)

- If you select a futures contract in a currency other than that of the bond in question, the calculation of the hedge ratio is of course also sensitive to the exchange rate, as you need to ensure that any losses on the bond will be made up by equal profits on the futures, when expressed in the bond's own currency

Reuters bond futures



Notes

- The lower panel in this screen calculates how many MAR 2004 Bund futures you should short in the current market in order to hedge the market risk on a EUR 1 million long position in the 8.625% EMI of 2013

- The number of contracts required is less than 10 (i.e. 1,000,000 / 100,000) because:

  - The BPV of the EMI bond is lower than that of the CTD Bund
  - The conversion factor on the CTD is less than 1

13.3. Hedge Effectiveness

In the example used in this section:

SEP futures: 102.70
Current CTD: 8.50% OAT maturing 25 October 2008
Price factor: 122.7773
BPV: 0.07996

$$\text{Nr futures contracts} = \frac{150{,}000{,}000}{500{,}000} \times 1.227773 \times \frac{0.08454}{0.07996}$$

= **389 contracts**, rounded.

The Outcome
French bonds fell after the budget and on 15 July, the day after the budget, market prices are as follows:

7% of 2010: 101.40
SEP futures: 102.33

Revaluation loss on the cash portfolio = Profit on futures trade

$$\frac{(101.40 - 102.38)}{100} \times 150 \text{ million} \qquad (10270 - 10233) \times 389 \times 100$$

= **- FRF 1,470,000** = **+FRF 1,439,300**

The hedge worked reasonably well though not perfectly, which is very often the case with bond futures hedging. This is because the relationship between the futures price and the value of your bond portfolio may be distorted by:

- A change in the CTD
- Changes in the repo rate (very likely, if yields rise across the board)
- A yield curve pivot - the current CTD that drives the futures is a 10-year bond, whereas your exposure is on a 12-year bond

14. Exercise 3

14.1. Question 1

Question 7

a) For settlement 4 August 2002, the 7¼% UK Gilt maturing on 7 December 2011 trades at 111.09375. If the security has a conversion factor of 1.0165266 and the DEC futures is trading at 109.65, what is the value of the gross basis?

Type your answer in the box below, rounded to 2 decimal places and validate.

14.2. Question 2

Question 8

Deliverable bond:	UK Gilt 9% maturing on 14 October 2012
Settlement date:	5 August 2002
Clean price:	125.125 (in decimal)
Accrued interest:	2.77869 (semi-annual, act/act)
Conversion factor:	1.1429955
Repo rate:	7.50%
SEP futures price:	109.490 (in decimal)
1st futures delivery date:	1 September 2002

a) What is the implied repo rate on this bond?
(Enter your answer in percent, rounded to 2 decimal places.)

b) Is there an arbitrage profit from buying this bond and selling it into the futures?

- Yes
- No

14.3. Question 3

Question 9

Consider the following statements:

(i) The implied repo rate is the funding rate implied in an actual futures price

(ii) The implied repo rate measures the gross return that can be made on a basis position held until futures delivery

(iii) The gross basis is the difference between the cash price of a deliverable bond and its adjusted futures price

(iv) The net basis is the net arbitrage profit or loss, per 100 nominal, that could be made on a basis position

(v) The net basis is the difference between the gross basis and the net carry

a) Which of these statements is/are true?

- Answers (i), (ii), (iii) and (iv) only
- All are true
- Answers (ii), (iii) and (v) only
- Answers (i), (iii) and (iv) only

14.4. Question 4

Question 10

Settlement date: 8 February 2000

Situation

You have a EUR 25 million long position in the following bond:

Security:	World Bank 5.75% EUR bond maturing 20 September 2005
Type:	Eurobond, annual, Act/Act.
Clean price:	104.75

You wish to hedge the market risk on this position using the March EUREX bond futures contract.

Name:	EUREX Bund Futures
Unit of trading	EUR 100,000 of a notional Bund with a 6% coupon and maturity of 8½ - 10½ years
Delivery Months	March, June, September, December
Delivery Day	Tenth calendar day of delivery month
Last Trading Day	Two business days prior to Delivery Day
Quotation	Percentage of face value, to 2 decimal places
Minimum Price Movement (Tick Size)	0.01%
Tick Value	0.01% x 100,000 = EUR 10.00

Below are the details of the deliverable bond that is currently the CTD on the March contract.

Security:	German Federal Government 5.50% maturing 11 June 2009
Type:	Annual, Act/Act.
Conversion factor:	0.964966
Clean price:	101.40

a) To hedge the market risk on this position, you should:

- ☐ Short the Futures
- ☐ Go long the Futures

b) How many contracts should you trade in order to create a risk-weighted hedge?

Instructions

Use the risk-weighted hedge formula developed in section *Hedging* and the bond pricing model supplied to calculate the BPVs of the two securities in question.

Number of contracts: ☐

c) Other things being equal, what would be the effect on the net position (long the World Bank bonds, short the bond futures) of an anti-clockwise pivot in the EUR yield curve?

- ☐ Indeterminate
- ☐ The net position should show a loss
- ☐ The net position should show a profit
- ☐ There should be neither a profit nor a loss on the net position

