ISMA CENTRE - THE BUSINESS SCHOOL
OF THE FINANCIAL MARKETS

UNIVERSITY OF READING
ENGLAND

IFID Certificate Programme

Rates Trading and Hedging

Yield Curve Dynamics

Table of contents

1. Overview 3

2. Official Rates 4
   2.1. Open Market Operations 4
   2.2. Discount Window Operations 6

3. Yield Curve Dynamics 7
   3.1. Long Yield Drivers 7
   3.2. Short and Long Yield Interaction 7

4. Yield Curve Shape 8
   4.1. Definition 8

5. Curve Fitting 11

6. Interpolation Methods 12
   6.1. Linear Interpolation 12
   6.2. Logarithmic Interpolation 13
   6.3. Cubic Splines 15

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1. Overview

We begin this module with a brief discussion of central bank **open market operations** and the role of the **discount window** and in this context we introduce some of the key money market rates that are associated with central bank monetary policy in some countries.

We then explain the key factors that affect yields at the longer end of the curve and discuss traditional theories about the shape of the curve, with some examples illustrating the complex interaction between different parts of the curve.

The module ends with a brief summary of some of the practical issues faced by the market analyst when constructing a yield curve:

- Which market yield data to select and which to discard?
- How to interpolate between known points on the curve?

The subject of yield curve interpolation is a specialist field. For the IFID Certificate exam you are only required to understand the different approaches to curve interpolation and their relative advantages and disadvantages.

Learning Objectives

By the end of this module, you will be able to:

1. Identify and interpret the key money market rates in the US, the Eurozone, the UK and Japan, in particular:
    - Official discount rates
    - Fed funds rate
    - ECB refinancing rate
2. Explain the impact of central bank open market operations on market liquidity and rates
3. Identify the main factors that cause the Treasuries curve to shift or to pivot
4. Outline the possible roles played by market expectations, liquidity preference and market segmentation in explaining the shape of the yield curve
5. Explain what is meant by:
    - A benchmark bond
    - An on-the-run bond
    - An off-the-run bond
6. Perform a linear interpolation between two adjacent yield points on the yield curve and explain the drawbacks of this approach.

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2. Official Rates

Central governments around the world are active in the money markets, not only to raise funds but also to control inflation and maintain stability in interest rates. They do so mainly through:

- **Open market operations**
  Buying and selling of eligible securities on a repo or (more rarely) outright basis (see Repurchase Agreements - Applications)

- **Discount window operations**
  Lending directly to financial institutions, where required

Obviously, such operations are of interest to the fixed income analyst and the trader, and in this section we take a brief look at how they work as well as the key money market rates that are associated with them.

2.1. Open Market Operations

Open market operations (OMO) involve the central bank buying or selling securities in the market in order to influence the amount of liquidity available in the commercial banking system, hence short-term interest rates.

One of the keys to inflation is controlling the amount of money that chases after goods. The bulk of the money supply is in the form of demand deposits and other interest bearing cash accounts, which are the liabilities of commercial banks and other deposit taking institutions. Controlling inflation is therefore largely a matter of controlling the ability of those financial institutions to grow their balance sheets.

This is what open market operations are designed to do, indirectly, and in order to understand how they work we need to understand first the central bank **cash reserve** system that operates in virtually all countries.

Central Bank Reserves

The bulk of the cash deposited in the commercial banking system has been lent out and the banking system works well as long as the public has faith that the banks have sufficient liquidity to meet their clients' daily needs for cash withdrawals, which are a small percentage of all the deposits held.

To support this faith in the system, banks and deposit taking institutions are normally required to maintain a cash reserve accounts with their central bank. The **reserve requirent** for each bank – the minimum balance that has to be kept in this account – is typically a percentage of the bank's deposit liabilities and varies depending on the nature of those liabilities, but it is generally a small percentage[1].

OMO at Work

To understand how OMOs work, remember the following:

- Every loan advanced by a commercial bank creates a new cash deposit. Initially, the money that is lent is credited into the borrower's account but later it lands on someone else's account, once the borrower has spent it.

- The minimum cash reserve requirement legally imposed on the banks means that any bank can only grow the size of its assets and liabilities in proportion to the amount of cash reserves that it has.

- The more cash the clients bring into a bank, the more cash that can be lent to the bank's clients or invested. But other things being equal, any such cash deposits must come out of the cash reserves of some other deposit taker, whose ability to lend or invest is therefore correspondingly curtailed.

[1] For example, in the US it ranges between 8% and 14% for demand deposits and interest bearing accounts with checking facilities, but it is lower for savings deposits with limited withdrawal rights.

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It follows that by varying the amount of central bank cash reserves that the entire commercial banking system holds, the central bank can (directly and indirectly) affect the size of those financial institutions' balance sheets and therefore the amount of money that they can lend – or at least its cost. This is how the central bank does it:

- **Liquidity injection**:
  The central bank buys securities from the private sector - government bonds, Treasury bills and other **eligible paper** - from the markets. The net amount of cash paid by the central bank (public sector) to the private sector is deposited with financial institutions, thereby increasing the amount of their cash reserves and allowing them to increase the size of their balance sheets. The result is to stimulate economic activity by encouraging corporate and consumer lending.

- **Liquidity drain**:
  The central bank sells some of the securities it holds to the markets. This results in a net outflow of cash reserves out of the banking system and a squeeze on the banks' reserve ratios, forcing each bank to raise its cost of lending in order to attract more depositors.

Nowadays, central banks typically deal with a small group of specialist securities dealers in their local market (know in the US as **primary dealers**) and trade securities with them on a repo basis, rather than outright.

Either way, the price or rate at which these transactions are done is typically arrived at through an auction process in which the primary dealers are invited to submit bids or offers and the central bank fixes the price of the auction at the level that clears the full amount of the intended transaction.

Central banks typically perform OMO daily as they implement the policy guidelines laid down at the weekly or fortnightly meetings of their monetary policy committees[2]. Below is a short description of how OMOs work in selected markets.

US markets

The Open Market Desk of the New York Federal District Reserve Bank intervenes daily in the repo market on the Fed's behalf, implementing the policy reviewed weekly by the Federal Open Market Committee (FOMC).

Most of these **system repo** interventions are on a term basis (typically 7 days) or an overnight basis. The size of the intervention and the rate at which the Fed does this business are immediately broadcast around the world, and in this context you should be aware of the following conventions:

- When the Fed is reported to have entered into **repurchase agreements**, it is the primary dealers that are doing the sale-and-repurchases (repos), so the fed is injecting liquidity into the banking system

- When the Fed is reported to have entered into **matched sale and repurchase** transactions, it is the Fed that is doing repo and therefore draining liquidity out of the banking system

Although the rates at which the Fed does repo daily is of interest, there is another US market rate that analysts follow very closely, that we need to explain.

The Fed Funds Rate

The money that the banks hold in reserve with the Fed earns no interest and it is therefore undesirable for a bank to keep more than the minimum requirement on deposit. On any day some banks will have surplus Fed funds (more than their reserve requirement) while others may have shortages and so an interbank **Fed funds market** exists for the lending of cash just for this purpose.

[2] We saw that relationship in module Time Value of Money – PV Sensitivities and we shall discuss it again in the context of bonds in module Interest Rate Risk – Macaulay Duration.

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Fed funds rate is the price of Fed reserves.

Most of the lending is done overnight and the interest rate prevailing in this market is the **Fed funds rate**. The Fed can influence the Fed funds rate immediately through open-market operations, which increases or decreases member banks' reserves and therefore their ability to increase the size of their balance sheets through customer lending. The Fed funds rate is therefore closely watched because it can reflect the authorities' intentions regarding monetary policy and interest rates.

Eurozone

The money market operations within the Eurozone are conducted on behalf of the European Central Bank (**ECB**) by the national central banks of the countries in the eurozone, implementing the policies reviewed fortnightly at the European ECB Board Meeting in Frankfurt. Eligible institutions are allowed to deal with the central banks' in such operations, which principally take two forms:

- **Main refinancing operation** (**MRO**)
  Every week, liquidity is injected to or withdrawn from the banking system via a two-week repo, which is used by the ECB to signal its rate intentions because it states the rate it requires within a tender. This rate, known as the **refinancing rate**, is the principal tool used by the ECB to signal its monetary policy intentions.

- **Long-term refinancing operation** (**LTRO**)
  Every month, liquidity is injected to or withdrawn from the banking system via a three-month repo. Unlike the MRO, which is used as a signalling tool, in the LTRO the ECB invites the institutions to put in tender bids in a conventional American-style auction.

2.2. Discount Window Operations

Commercial banks are the main originators of legal tender and are largely responsible for operating the domestic payment and money transmission system, therefore business problems in this sector can have very serious consequences for the rest of the economy and one of the roles of any central bank is to be the **lender of last resort** for the banking system.

In most OECD countries central banks operate a **discount window** facility, which allows eligible deposit takers to borrow reserve funds directly from the central bank at some official **discount rate**. This borrowing is typically collateralised with acceptable fixed income securities and in the old days the securities delivered were effectively bought by the central bank on a discount to par basis – hence the 'discount' rate.

In practice, the discount windows are nowadays rarely used by the money centre banks because normally those banks are able to manage their daily money market liquidity by themselves, either in the interbank market or in the repo market. Indeed, banks are reluctant to use the discount window for fear of being perceived by other market players as problem cases, unable to access normal funding routes. The discount window tends to be used mainly by smaller banks that require more major balance sheet restructuring or seasonal funding than is possible to obtain elsewhere.

In practice, the central banks nowadays use the discount rate more as a signalling device, to indicate the general direction of interest rate policy, rather than as an operational tool of money market intervention.

The basic discount rate is adjusted from time to time, in light of changing market conditions, to complement open market operations and to support the general thrust of monetary policy. US Federal Reserve.

In other words, the central bank uses the discount rate to point to the market where it would like rates to go, but the thing that gets the rates there are the OMOs.

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3. Yield Curve Dynamics

3.1. Long Yield Drivers

For long term government bond investor, inflation is Public Enemy Number 1.

While the short end of the yield curve is very under the control of the central bank, the long end is much more under the influence of long term fixed income investors, for whom, as we said in module Bond Pricing and Yield – Real Yields, the nominal yields that they will accept in order to hold government bonds depend critically on what future inflation they anticipate.

Investors locking into fixed nominal yields look for real yields - i.e. inflation-adjusted. So any economic information that has a potential bearing on future inflation also has an impact on nominal yields (and therefore on prices of long term securities):

- **Economic activity indicators** - e.g. unemployment, employment and payroll data; surveys of manufacturing orders, etc.
- **Financial indicators** - e.g. consumer credit granted, money supply growth, the exchange rate, commodity prices, etc.[3]

3.2. Short and Long Yield Interaction

The relationship between the short and the long ends of the curve are very complex and the example scenarios below are meant to illustrate how central bank money market intervention can have dramatically different effects on the curve, depending on how the market interprets that intervention.

Scenario 1

The economy is on full employment and overheating, with growing inflationary pressures. The central bank drains liquidity out of the system through OMOs and raises its official discount rate.

- **Market verdict: positive**
  The yield curve pivots clockwise as the bond market celebrates it as the right economic medicine, while conditions in the money market remain tight
- **Market verdict: negative**
  The yield curve shifts up in parallel (and may even pivot anti-clockwise, perhaps associated with a weakening currency) as the market considers the rise to be insuficient

Scenario 2

Economic activity is a little on the weak side, with relatively high unemployment and slow GDP growth. The central bank inject liquidity into the system to bring rates down.

- **Market verdict: positive**
  The yield curve shifts down in parallel as the bond market believes that a stronger economy will improve the government's future tax revenues and therefore reduce its future borrowing requirements and therefore new treasury issues.
- **Market verdict: negative**
  The yield curve pivots clockwise, as there is plenty of liquidity in the money markets but the bond markets either:
    - Dislike what they see as its potentially inflationary long term effects; or
    - Believe the rate fall will be insufficient to improve economic activity, which will worsen the government's future borrowing requirements

[3] Market analysts carefully pour over the minutes of such meetings for any clues on possible policy changes.

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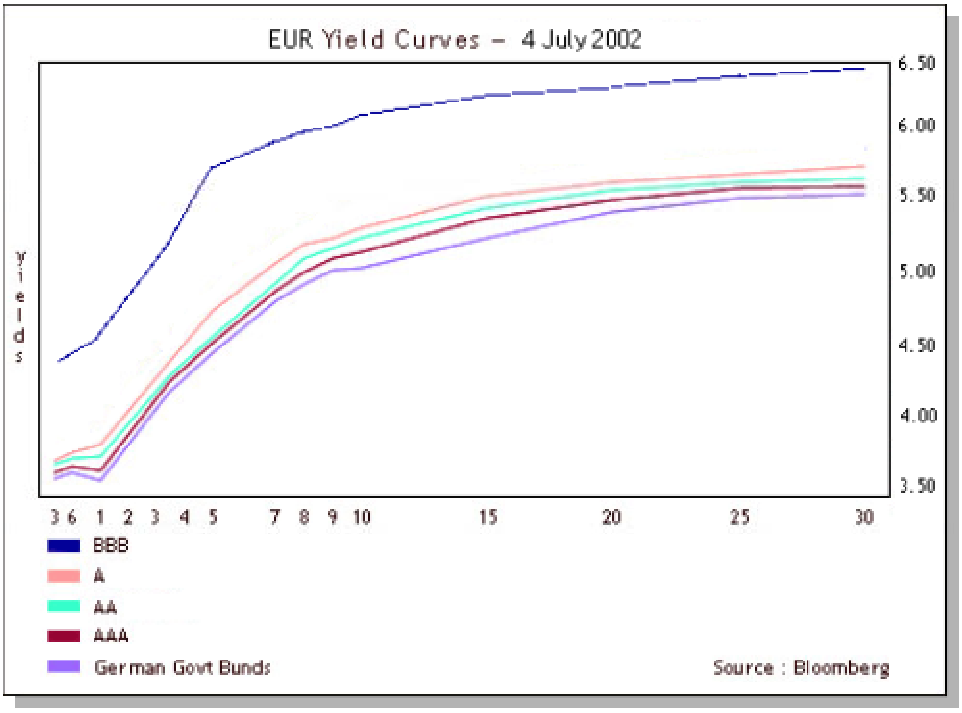
Conclusion

The different ways in which the market may respond to a given central bank intervention, depending on the economic and political context in which it occurs, opens the possibility of financial betting not only on the next central bank intervention but also on the way the market will react to it – i.e. **spread trading** the short against the long end of the curve. We shall discuss this type of strategy in detail in module Outright and Spread Trading.

Before that we need to understand the factors that drive the shape of the yield curve (the exact relationship between short and long term yields) and how to interpolate between different points on it.

4. Yield Curve Shape

4.1. Definition



? What does the shape of the curve reflect?

This question has been debated for over a hundred years, and is central to trading strategies that rely on future changes in its slope and shape (see for example Cash Market Strategies – Yield Curve Spreads).

In module Bond Pricing and Yield – The Yield Curve we suggested that part of the answer to this question must be market expectations about future rates and in module Spot and Forward Yields – Forward Yield Derivation we saw how to derive the path of forward rates implied in the shape of the curve. Here we consider whether market expectations really is the only factor driving forward rates and hence the shape of the curve.

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Below is a summary of the main theories that seek to explain the shape of the yield curve.

Market expectations

According to this theory, the forward rates that are currently implied in the shape of the yield curve are nothing more than an exact representation of the market's average **expectation** about future short-term rates:

- A positive curve (upward sloping) indicates the market expects short term rates to rise

  If an investor expects rates to rise within the next 12 months, he will demand a higher rate to lock into 2 or 3 year investments because by committing his funds for a long period he foregoes the opportunity of benefiting from higher rates in the near future.

- An inverted curve (downward sloping) would imply that the market anticipates rates to fall

  If the investor expects future rates to fall, he can afford to lock into long-term lending at a lower rate.

In later modules in this programme, we shall see how investors can enter into derivative positions that lock them today into the forward rates implied in the current yield curve. At the settlement date of those derivatives, the trader realises a profit or a loss if the actual market rate turns out to be different from the forward rate at which they had locked in[4].

According to the market expectations theory, in the long run investors who lock today into forward rates should make no systematic profits against the actual rates that eventually materialise. In other words, according to this theory forward rates are priced on a **risk-neutral** basis.

Liquidity preference

This theory highlights the fact that investing in bonds of different maturities carry different market risks. In particular, risk increases with maturity because of the close relationship between maturity and market risk.

Market risk: the sensitivity of the fixed income instrument to changes in market rates[5].

In other words, whereas the market expectations theory is risk-neutral (forward rates exactly represent average expected future rates), the liquidity preference theory claims that long term yields (and therefore the forward rates implied from them) carry a **risk premium**:

- Investors hold longer-term securities only if they are offered a yield premium which is higher than the average of the expected future rates

- In the long run, investors who lock into today's market forward rates should on average make systematic profits against the actual rates that eventually materialise

Current forward rates reflect both interest rate expectations and a liquidity premium.

[4] A discussion of how these economic and financial factors relate to inflation is outside the scope of the IFID Certificate syllabus.

[5] We saw that relationship in module Time Value of Money – PV Sensitivities and we shall discuss it again in the context of bonds in module Interest Rate Risk – Macaulay Duration.

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Market segmentation and preferred habitat

Both the **market segmentation** and the **preferred habitat** theories recognise that the yield curve reflects both expectations about future rates and a liquidity risk premium, but claim that this premium may increase or decrease with maturity, depending on the supply and demand balance for instruments in different maturity sectors (or **bands**).

One investor's medicine is another's poison!

According to the market segmentation theory, different categories of investors and borrowers are constrained by the nature of their assets and liabilities to operate in parts of the yield curve. For example, pension funds and life assurance companies tend to have long-term liabilities and for them investing in short-dated paper is riskier (from a market risk perspective) than investing in longer maturities (see module Interest Rate Risk – Using Duration). These institutions need the predictable cash flows of long-term bonds in order to service their long-term liabilities

According to the preferred habitat theory, some investors and borrowers change their positioning along the yield curve depending on how they see the markets evolving. For example, in a bear market when rates are rising investors will switch to short maturities and so the liquidity premium for longer maturities will rise. Conversely, in a bull market when rates are falling investors will move to the longer maturities in an attempt to **gear up** their potential profits from future rate falls[6], so in this scenario the liquidity premium for longer maturities may be smaller than that for shorter maturities.

Therefore, the risk premium can be seen as the appropriate compensation to lenders and borrowers to induce them to shift from one maturity sector to another whenever the demand and supply associated to each maturity band do not match.

Conclusion

Market expectations may not be the only factor driving the shape of the yield curve but in the liquid OECD markets it is widely regarded to be by far the most significant one.

In the less liquid emerging markets, where the different bands of the yield curve may be less evenly populated, the other factors mentioned above may play a more important role in shaping the curve. In those markets, forward rates may show a distorted picture of how the market really expects interest rates to move.

[6] The longer the maturity of the investment the larger its PV sensitivity to changes in yield, as we saw in module Time Value of Money – PV Sensitivities.

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5. Curve Fitting

One of the first problems that a fixed income security analyst typically faces when constructing a yield curve is to decide which bonds to include in its construction. Of course, only bonds of the same rating should be included, but even then it is unlikely that a smooth curve can be drawn joining the yields of all the issued securities:

- Some of the bonds will be on-the-run and therefore **benchmark issues** while other will be off the run, whose yields may be distorted by their low market liquidity[7]

- Not all the bonds will have the same coupon rates, so some may trade at a premium while others at a discount to face value. As we saw in module Yield Curve Analysis, the coupon on a bond does have an impact on its yield, so strictly speaking only bonds with the same rating and trading close to par should be included in the yield curve construction

- In many countries capital gains are taxed at different rates than coupon income, giving some investors a strong incentive to buy low coupon bonds, if capital gains tax is lower than income tax, or to buy high coupon bonds if vice versa. This will distort the gross yields on bonds with different coupon sizes.

- In some countries institutional fund managers don't like investing in high coupon bonds simply because their pull down towards par, as they approach maturity, make their valuation at the end of each reporting date look worse and worse. Of course, this negative price effect is offset by their higher current yields, but for some fund managers this type of window dressing is important in those markets high coupon bonds tend to trade at higher yields than low coupon bonds with the same rating and maturity.

Faced with such practical difficulties, analysts have taken two different approaches to yield curve fitting:

1. Include all the bonds with the same rating and fit a non-linear statistical regression curve that best represents all the yield observations

2. Include only on-the-run or benchmark bonds and remove the ones that are believed to have yield distortions because of taxation, window dressing or other factors. Then fit a curve that passes through the **yield vertices** represented by that small selection of 'reliable' bonds.

While the first approach may produce quite reasonable 'average' views of the term structure of interest rates, it suffers from the weakness that a statistically fitted curve may not in fact pass through any one of the observed yield vertices. In other words, none of the yield points on this fitted curve are actually traded in the market. Therefore method 2 is typically used, making yield curve construction in practice much more of an art than a science!

? Having selected the bonds that will be used to represent the entire curve, how do we fit a curve passing through the known vertices: Straight lines? Non linear?

The choice of method is important because the **interpolated yield** for a maturity that lies in between two known vertices may be a few basis points different, depending on the interpolation method used – and in the high-BPV bond market a few basis points easily translates into a lot of cash!

In the next section we outline 4 different methodologies that could be used in yield curve fitting.

[7] See for example Bond Futures - Pricing, Interest Rate Futures – Pricing and Interest Rate Swaps – Forward Swaps.

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6. Interpolation Methods

Normally the whole yield curve is built from just a few discrete set of observed yield points (or **yield vertices**). Yields for maturities in between the observed vertices must be estimated by interpolating the fitted curve.

Ideally, the fitted curve must:

- Be a **continuous function** of term to maturity - i.e. there must be no gaps in the curve
- Pass through all the observed yield vertices
- Be smooth, because any 'sharp angles' on the curve will result in sudden jumps in the derived forward yields

In the following sections we examine 3 different techniques of yield curve interpolation and we assess the extent to which each method satisfies these criteria.

6.1. Linear Interpolation

The simplest form of interpolation is to draw a straight line joining each pair of yield vertices.

Example

Consider the following observed yield vertices:

Maturity (Years)	**YTM**
$T_0 = 0$[8]	$Y_0 = 4.00\%$
$T_1 = 2$	$Y_1 = 5.00\%$
$T_2 = 4$	$Y_2 = 6.50\%$
$T_3 = 10$	$Y_3 = 6.75\%$

? What is the estimated yield for a 6-year maturity?

A linearly interpolated yield curve is represented by three linear equations, $Y_i(t)$, each one joining two adjacent vertices T_i and T_{i+1}, as follows:

$$\mathbf{Y_i(t) = Y_i + \frac{(t - T_i)}{(T_{i+1} - T_i)} \times (Y_{i+1} - Y_i)}$$

Thus, to estimate the yield on an instrument maturing in t = 6 years
(between T_2 and T_3):

$$Y_2(6) = 6.50 + \frac{(6 - 4)}{(10 - 4)} \times (6.75 - 6.50)$$

$$= \mathbf{6.58\%}$$

[8] The bond market often pays a small premium (i.e. earns a lower yield) for benchmark issues, because investors find them a useful reference for pricing other new issues. This gives governments, international agencies and large corporations with reliable credit ratings an incentive to make **benchmark issues** in large amounts and for standard maturities such as 5, 7, 10, 15 and 30 years.

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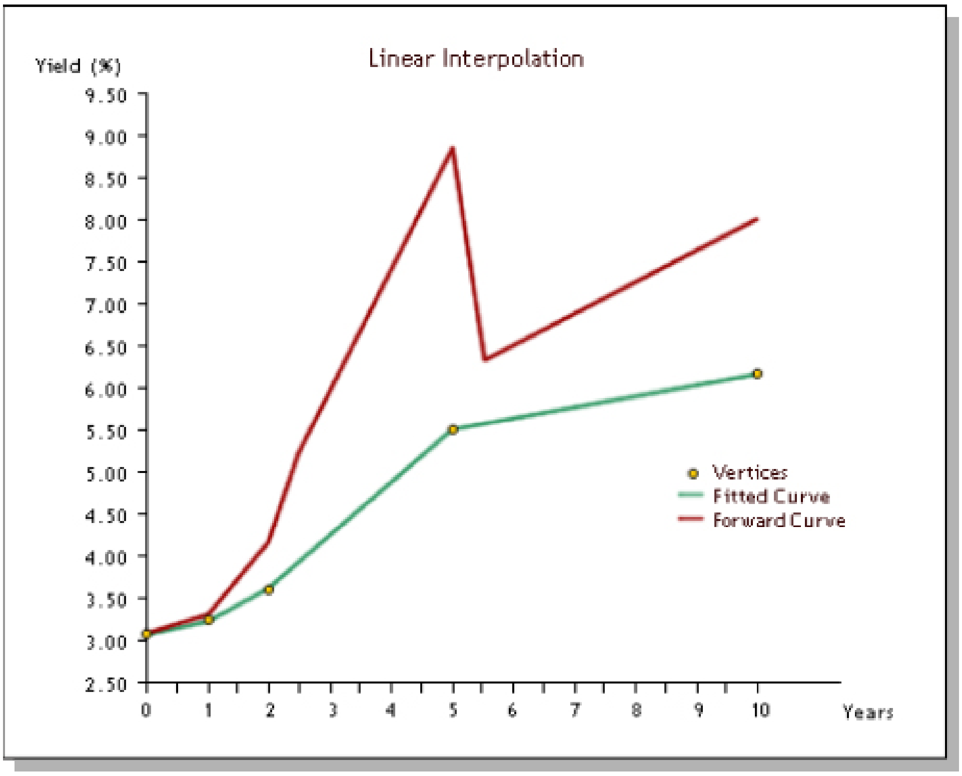
Limitations

The problem with linear interpolation is that the curve can have sharp angles at the vertices where two straight lines meet, resulting in sharp and economically unreasonable jumps in the derived forward rates each side of the angle. The figure below shows the structure of the linearly interpolated coupon curve and its derived strip of 1-year forward yields.



6.2. Logarithmic Interpolation

Another approach often used is to apply linear interpolation to the natural logarithms[9] of the set of discount factors, rather than to the yields themselves.

Example

Maturity (Years)	**YTM**
$T_0 = 0$	$Y_0 = 4.00\%$
$T_1 = 2$	$Y_1 = 5.00\%$
$T_2 = 4$	$Y_2 = 6.50\%$
$T_3 = 10$	$Y_3 = 6.75\%$

? What is the estimated yield for a 6-year maturity, using logarithmic interpolation?

[9] For a definition of the natural logarithm, see module Time Value of Money – Periodic & Continuous Compounding.

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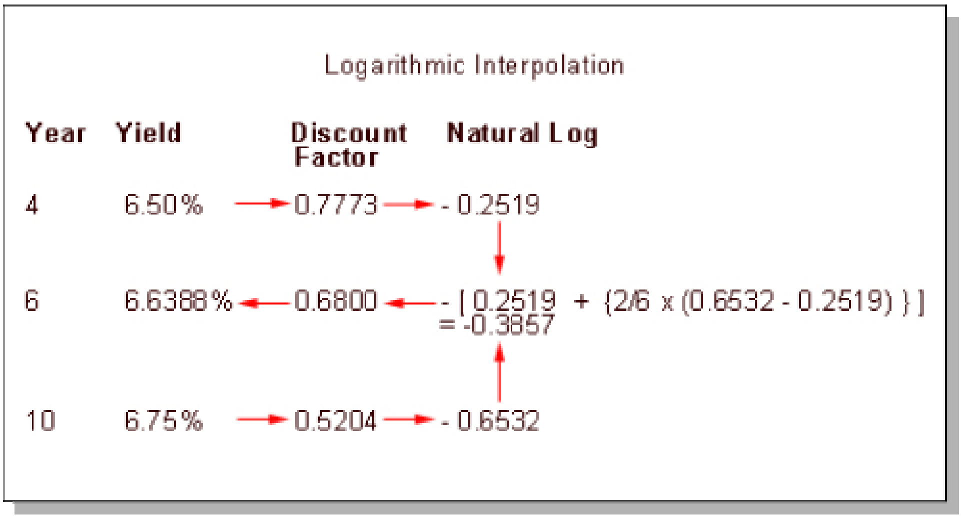
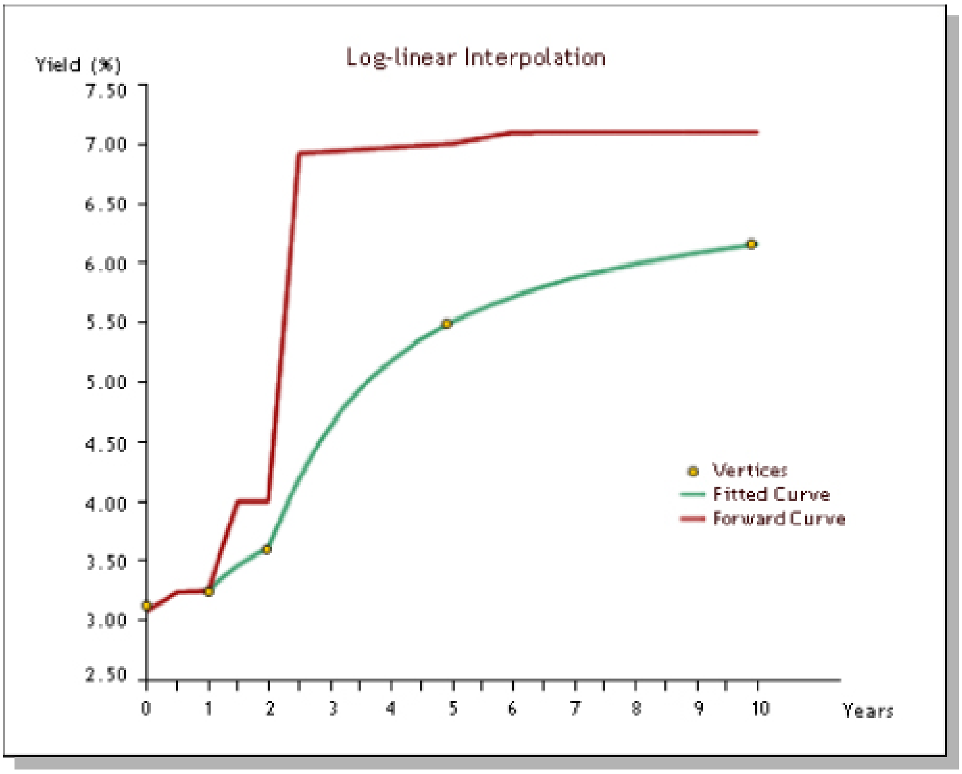
The figure below shows the sequence of calculations required:

1. Calculate the discount factors for years 4 and 10
2. Take the natural logarithms of these discount factors
3. Perform a linear interpolation on these logarithms
4. Take the anti-log of the result, to get the implied interpolated discount factor
5. Calculate the implied yield in this discount factor



Limitations

Notice how in a positive yield curve the logarithmically interpolated yield comes out somewhat higher than the one calculated earlier by simple straight-line interpolation. In an inverted curve the interpolated points would be correspondingly lower. This is a desirable feature, as it reduces the sharpness of any angles on the curve. However, it still leaves undesirable jumps in the derived forward yields, as the figure below shows.



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6.3. Cubic Splines

The splining[10] technique is commonly used in the dealing room because it is a technique that makes for a smooth yet firm fitted curve. The idea is to connect each pair of yield vertices by fitting a unique cubic equation between them. The whole yield curve is then represented by a daisy-chain of cubic equations.

Example

Consider the same observed yield vertices as before:

Maturity (Years)	YTM
$T_0 = 0$	$Y_0 = 4.00\%$
$T_1 = 2$	$Y_1 = 5.00\%$
$T_2 = 4$	$Y_2 = 6.50\%$
$T_3 = 10$	$Y_3 = 6.75\%$

There are four observed vertices and we fit three cubic equations, $Y_{(i,t)}$, each one connecting two adjacent vertices T_i and T_{1+1}, as follows:

$Y(0,t) = 0.022 \times t^3 + 0.413 \times t + 4.000$ - vertex T_0 - T_1
$Y(1,t) = -0.047 \times t^3 + 0.411 \times t^2 - 0.410 \times t + 4.548$ - vertex T_1 - T_2
$Y(2,t) = 0.008 \times t^3 - 0.249 \times t^2 + 2.230 \times t + 1.029$ - vertex T_2 - T_3

To find the coefficients for each cubic equation[11] we impose certain conditions that make the system of equations solvable. In particular, we require that at the point where any two curves join together their slopes and curvatures are identical. This makes for smooth transitions across the yield vertices and prevents the forward rates from jumping wildly.

? What is the estimated yield for a 6-year maturity?

Applying the third cubic equation, which spans the 4 - 10 year vertices:

$$Y_{(2,t)} = 0.008 \times 6^3 - 0.249 \times 6^2 + 2.230 \times 6 + 1.029$$
$$= \mathbf{7.17\%}$$

Notice that the result obtained here is significantly different from that obtained through linear interpolation: in cash terms what interpolation method is used will translate into a large price difference on a 6-year bond!

The figure on the next page shows the shape of the yield curve generated by the three equations shown above. Notice that now the shape of the forward curve is not only smooth but also more stable than the forward curve derived for the same data by polynomial interpolation.

[10] The word spline comes from the carpenter's tool used for drawing smooth curves on a piece of board.

[11] The exact technique for finding the correct set of cubic equations lies outside the scope of the IFID Certificate syllabus.

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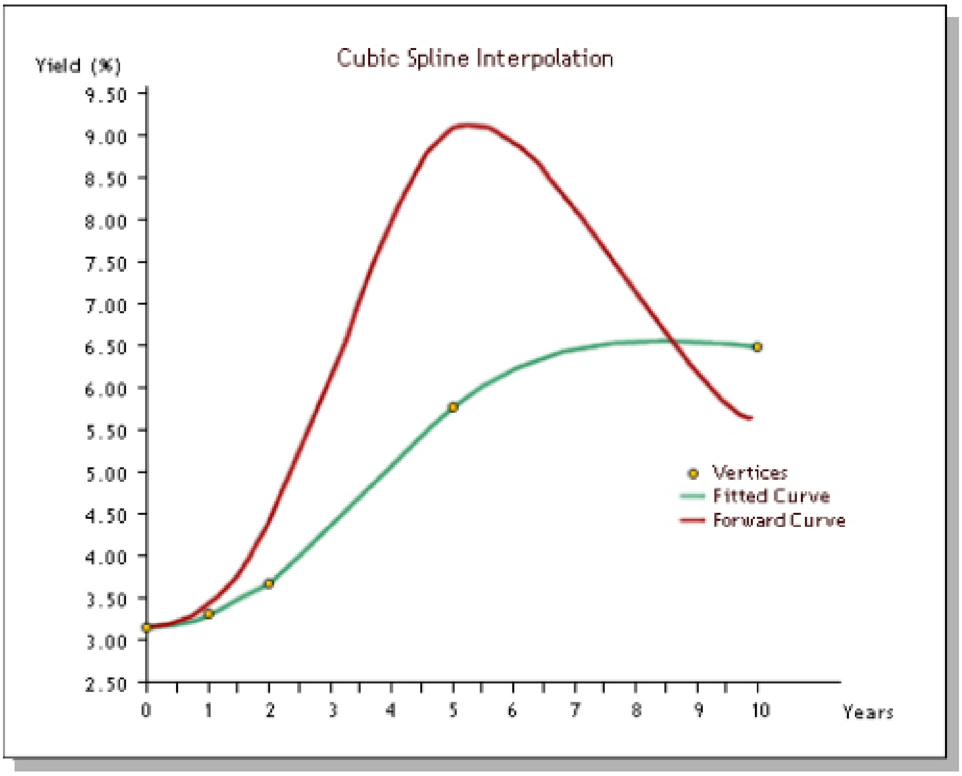

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