ISMA CENTRE - THE BUSINESS SCHOOL OF THE FINANCIAL MARKETS

UNIVERSITY OF READING
ENGLAND

IFID Certificate Programme

Rates Trading and Hedging

Answers to Exercises

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1. Securities Financing

Question 1

Settlement date: 11 October 2000

Situation:
A bond dealer purchases GBP 50 million nominal of the 5¾% UK Treasury maturing 7 December 2009, at a clean price of 103.98. On the settlement date, there will be 126 days of accrued interest.

The dealer finances his purchase by repoing out the gilts to another investment bank on the following terms:

Repo type:	Classic repo
Repo buyer:	Investment bank
Repo seller:	Bond dealer
Collateral:	5¾% Treasury 7 December 2009
Type:	Semi-annual, Actual/Actual
Nominal amount:	GBP 50,000,000
Clean price:	103.98
Dirty price:	105.960 (rounded to 3 decimal places, by convention)
Value date of repo:	11 October 2000
Maturity of repo:	18 October 2000
Term of repo:	7 days
Repo rate:	6.22% (Actual/365)
Haircut:	2%

The haircut in this transaction is applied using the PSA/ISMA Global Master Repurchase Agreement formula (see section *Managing Collateral*):

$$\textbf{Start proceeds} = \textbf{Nominal value} \times \frac{\textbf{Dirty price of collateral}}{(\textbf{100} + \textbf{Percentage haircut})}$$

What are the cash flows through this repo (all figures to the nearest pound)?

a) Start proceeds GBP: 51941176

Explanation
Start by calculating the market value of the collateral, using the dirty price (i.e. price with accrued interest):

$$\frac{105.960}{100} \times 50{,}000{,}000$$

= **GBP 52,980,000**

Next, we apply the haircut formula to the market value of the collateral to arrive at the start proceeds:

$$\textbf{Start proceeds} = \textbf{Nominal value} \times \frac{\textbf{Dirty price of collateral}}{(100 + \textbf{Percentage haircut})}$$

$$= \frac{\text{Market value of collateral}}{(100 + \text{Percentage haircut})}$$

$$= \frac{52{,}980{,}000}{(1 + 0.02)}$$

= GBP **51,941,176** rounded to the nearest pound.

b) Payable by:

- ○ Repo Seller
- ◉ Repo Buyer

Explanation
The start proceeds are **payable by the repo buyer** (i.e. Investment bank).

c) End proceeds GBP: 51941176
Repo interest GBP: 61959
Total GBP: 52003135

Explanation
Given that this is a classic repo, the end proceeds are equal to the start proceeds.

The repo interest is:

51,941,176 x 0.0622 x 7 / 365
= GBP 61,959

Therefore:

End proceeds:	GBP	**51,941,176**
Repo interest:	GBP	**61,959**
Total:	GBP	**52,003,135**

d) Payable by:

- [ ] Repo Buyer
- [x] Repo Seller

Explanation
Repo seller.

e) Which of the following statements is/are true about this transaction?

- [x] The Repo buyer suffers credit risk if the bond price falls
- [x] The Repo seller suffers credit risk if the bond price rises
- [ ] The Repo buyer suffers credit risk if the bond price rises
- [ ] The Repo seller suffers credit risk if the bond price falls

Explanation

- The repo buyer suffers credit risk if the bond price falls
- The repo seller suffers credit risk if the bond price rises

Question 2

Settlement date: 7 April 2000

Situation
You are a market maker in Italian government bonds (BTP). For settlement 7 April 2000 you sell a customer EUR 25 million nominal of the 5% BTP maturing 15 February 2003 at a clean price of 100.59.

As you do not have the BTP in your inventory, and you think it is a little overpriced at the moment, you decide to cover the short position by reversing in the bond for 10 days from a mutual fund. The terms of the transaction are:

Repo type:	Buy/sell-back
Buyer:	Market maker (you)
Seller:	Mutual fund
Collateral:	5% BTP 15 February 2003
Type:	Semi-annual, Actual/Actual
Nominal amount:	EUR 25,000,000
Value date:	7 April 2000
Maturity:	17 April 2000
Term:	10 days
Clean price at value date:	100.59
Accrued interest at value date:	0.714285714%
Accrued interest at maturity:	0.851648352%
Repo rate:	3.834% (Actual/360)

What are the cash flows through this buy/sell-back (all figures rounded to the nearest EUR)?

a) Start proceeds EUR: 25326071

Explanation
Start by calculating the market value of the collateral:

Dirty price at value date	= Clean price	+ Accrued interest
	= 100.59	+ 0.714285714
	= 101.304285714	

$$\frac{101.304285714}{100} \times 25{,}000{,}000$$

= **EUR 25,326,071** rounded to the nearest EUR.

b) Payable by:

- ○ Repo Seller
- ● Repo Buyer

Explanation
Since there is no haircut on the buy/sell-back, this is also the start proceeds, and it is **payable by the repo buyer** (i.e. Market maker).

c) End proceeds EUR: 25353043

Explanation
The repo interest is:

25,326,071 x 0.03834 x 10 / 360
= EUR 26,972

Since this is a buy/sell-back, the end proceeds include the repo interest:

End proceeds	= Start proceeds	+ Repo interest
	= 25,326,071	+ 26,972
	= **EUR 25,353,043**	

d) Payable by:

- ○ Repo Buyer
- ● Repo Seller

Explanation
Payable by the **repo seller**.

e) If the buy/sell-back was quoted as a forward price, what would this be (rounded to 2 decimal places)?

orward price: 100.56

Explanation
Using the formula presented in section Sell/Buy-backs:

$$\textbf{Forward price} = \frac{\textbf{End proceeds - Accrued interest at maturity}}{\textbf{Nominal value}} \times 100$$

$$= \frac{25{,}353{,}043 - (0.851648352 / 100 \times 25{,}000{,}000)}{25{,}000{,}000} \times 100$$

= **100.56**, rounded to 2 decimal places.

f) What is the breakeven on your short position - i.e. the price at which you should buy back the bond after 10 days in order to cover the net cost of carrying the position? (Your answer to 2 decimal places.)

Breakeven price: 100.56

Explanation
As explained in section *Sell/Buy-backs*, the market maker's breakeven is the same as the forward price on the buy/sell-back - i.e. **100.56.**

3. Outright and Spread Trading

Question 1

Settlement date: 15 March 2004

As the business climate in the UK continues to improve, you feel uncertain about the general direction of UK rates in the coming weeks, as it is uncertain how soon the Bank of England will start to reverse the easy-rates policy that it has followed in the past couple of years.

However, you believe that the UK bond market will start to discount slightly higher inflation in the near term, so the UK Gilts (government bonds) curve is likely to steepen.

You want to profit from your market view by entering into a market-neutral yield curve steepener spread position using the following bonds:

- 7½% of 7 December 2006 @ 107.34
- 5¾% of 7 December 2009 @ 104.21

a) Which one of these bonds should you buy and which one should you short-sell in order to set up your strategy?

- ( ) Go long both bonds
- (•) Long the 2006 and short the 2009
- ( ) Short both bonds
- ( ) Short the 2006 and long the 2009

Explanation
You expect the yield on the 2006 to fall relative to the yield on the 2009, therefore:

- Long the 7½% of 7 December 2006 @ 107.34
- Short the 5¾% of 7 December 2009 @ 104.21

b) **Coupon interest on UK Gilts is calculated on a semi-annual, Act/Act basis.**

Using a financial calculator or the bond pricing model on the left, calculate the yield and the risk factor on each of these bonds (risk factor = 100 x BPV per 100 nominal). Enter your prices and yields in percentages, both rounded to 3 decimal places, and your risk factors rounded to 4 decimal places.

	Clean Price	Dirty Price	Yield	Risk factor
7½% of 7 December 2006	107.340	109.369	4.603	2.6540
5¾% of 7 December 2009	104.210	105.765	4.897	5.0683
Yield spread (2009 – 2006)			0.294	

Explanation

	Clean price	Dirty Price	Yield	Risk factor
7½% of 7 December 2006	107.340	**109.369**	**4.603%**	**2.6540**
5¾% of 7 December 2009	104.210	**105.765**	**4.897%**	**5.0683**
Yield spread (2009 – 2006)			**0.294%**	

c) If you plan to trade GBP 50 million of the 2006s, how many of the 2009s should you trade against that in order to make your net position market neutral? Enter your answer in absolute size, to the nearest pound.

	Nominal amount traded
7½% of 7 December 2006	50,000,000
5¾% of 7 December 2009	2618234

Explanation

	Nominal amount traded
7½% of 7 December 2006	50,000,000
5¾% of 7 December 2009	= 50,000,000 x 2.6540 / 5.0683 = **26,182,349**

d) How much net cash will this trade require you to invest? Use the dirty prices calculated in (b), rounded to 3 decimal places, and enter your results rounded to the nearest £100 and including the sign of the cash flows.

	Cash proceeds
7½% of 7 December 2006	-5468450
5¾% of 7 December 2009	2769180
Net	-2699270

Explanation

	Cash proceeds
7½% of 7 December 2006	-50,000,000 x 109.369 / 100 = GBP **-54,684,500**
5¾% of 7 December 2009	26,182,349 x 105.765 / 100 = GBP **27,691,800**
Net	GBP **-26,992,700**

e) How much money do you stand to make or lose on this trade for each basis point change in the yield spread between the two bonds?

- ○ Nil, the position is market neutral
- ○ GBP 1,327,000
- ○ GBP 26,540
- ● GBP 13,270

Explanation
By construction, the total risk weight on each bond position is the same:

	Risk weight (per 1 bp)
7½% of 7 December 2006	= 2.6540/100 x 50,000,000 / 100 GBP **13,270**
5¾% of 7 December 2009	= 5.0683/100 x 26,182,349 / 100 GBP **13,270**

Therefore, each basis point change in the yield on either bond (or their spread) creates a profit or loss on this position of GBP 13,270.

Question 2

Settlement date: 15 March 2004

Similar scenario as in *Question 1* except that you now feel uncertain both about the general direction of UK rates and whether the Gilts curve is likely to steepen or to flatten.

However, you take the view that the 'elbow' or 'belly' that is currently noticeable around the 4-5 year part of the curve is likely to become even more pronounced, as the market discounts higher inflation rates in the medium term.

In other words, you expect yields in the 2008 - 2009 part of the yield curve to rise relative to yields on other parts of the curve and you want to profit from this view by entering into the following **risk-neutral butterfly** (or **barbell**) position:

- Long the 7½% of 7 December 2006 @ 107.34 (the 'S bond')
- Short the 5¾% of 7 December 2009 @ 104.21 (the 'M bond')
- Long the 5% of 7 March 2012 @ 100.42 (the 'L bond')

The position should be built such that it makes a profit only if the yield on the M bond (the 'body' of the butterfly) rises relative to weighted average yield on the S and the L bonds (the two 'wings') but should be risk-neutral in every other respect. The exercise below will show you, step by step, how to construct such a strategy[1].

[1] Exactly the same strategy would be used by a traditional fund manager, who would like to switch out of the 5¾% of 2009 and into a combination of the 7½% of 2006 and the 5% of 2012, which she feels will perform better, while maintaining the same overall market risk on her portfolio.

a) Using a financial calculator or the Excel model on the left, calculate the yield and the risk factor on each of the 3 bonds involved. Enter your yields in percentages, rounded to 3 decimal places, and your risk factors rounded to 4 decimal places.

	Clean price	**Dirty Price**	**Yield**	**Risk factor**
7½% of 7 December 2006	107.34	109.369	4.603	2.6540
5¾% of 7 December 2009	104.21	105.765	4.897	5.0683
5% of 7 March 2012	100.42	100.529	4.935	6.5460

Explanation

	Price	**Dirty Price**	**Yield**	**Risk factor**
7½% of 7 December 2006	107.340	**109.369**	**4.603%**	**2.6540**
5¾% of 7 December 2009	104.210	**105.765**	**4.897%**	**5.0683**
5% of 7 March 2012	100.420	**100.529**	**4.935%**	**6.5460**

b) If you plan to short GBP 50 million of the 2009s (bond M), what would be the total risk weight of the body of this butterfly – i.e. the profit/loss on the GBP 50 million position for a 100 basis point change in yield?

Total risk weight = Q_m x R_m / 100

Where:
Q_m = Nominal amount of bond M
R_m = Risk factor on bond M, per 100 nominal

Enter your answer as an absolute amount, rounded to the nearest pound.

2534150

c) If you plan to short £50 million of the 2009s, what nominal amount of the 2006s and the 2012s should you buy in order to make this trade:

- **Market risk-neutral**:
  With parallel shifts (up or down) of the yield curve, losses on the wings should be exactly offset by profits on the body and vice versa

- **Pivot risk-neutral**:
  With simple yield curve pivots, any net losses on one of the wings (and possibly also on the body) should be exactly offset by profits on the other wing – i.e. the total risk weight on each wing should be the same

Enter your answer in absolute amounts, rounded to the nearest £100.

Hint

In symbols:

- Market risk-neutral:

  $$\mathbf{Q_m \times R_m = Q_s \times R_s + Q_l \times R_l}$$

- Pivot risk-neutral:

  $$\mathbf{Q_s \times R_s = Q_l \times R_l}$$

  Where:
  Q_i = Nominal amount of bond i (S, M or L)
  R_i = Risk factor on 100 nominal of bond i

For pivot neutrality, each wing of the butterfly has to carry ½ of the total risk weight of the body. In symbols:

$Q_s \times R_s = Q_l \times R_l = ½ \times Q_m \times R_m$

and therefore:

$$\mathbf{Q_s = ½ \times Q_m \times R_m / R_s}$$

$$\mathbf{Q_l = ½ \times Q_m \times R_m / R_l}$$

	Nominal Amount
7½% of 7 December 2006	4774210
5¾% of 7 December 2009	-50,000,000
5% of 7 March 2012	1935650

Explanation

	Nominal Amount
7½% of 7 December 2006	= ½ x 50,000,000 x 5.0683 / 2.6540 = **47,742,100**, rounded.
5¾% of 7 December 2009	-50,000,000
5% of 7 March 2012	= ½ x 50,000,000 x 5.0683 / 6.5460 = **19,356,500**

d) How much net cash will this trade require you to invest? Use the results calculated above and enter the amounts rounded to the nearest pound and including the sign of the cash flows.

	Cash proceeds
7½% of 7 December 2006	-5215057
5¾% of 7 December 2009	5288250
5% of 7 March 2012	-1945889
Net	-1879145

Explanation

	Cash Proceeds
7½% of 7 December 2006	= 47,742,100 x 109.369 /100 = GBP **-52,215,057**
5¾% of 7 December 2009	= 50,000,000 x 105.765 /100 GBP **52,882,500**
5% of 7 March 2012	= 19,356,500 x 100.529 /100 GBP **-19,458,896**
Net	GBP **-18,791,453**

The position is market and pivot-neutral but as you can see, not cash neutral: the trader has to put up GBP 18,791,453 of capital to fund it (net). A different type of butterfly can be constructed, where the amount of the wing bonds is set so that the total cost of buying the wings exactly equals the total proceeds from the sale of the body – i.e. the butterfly is cash-neutral, as well as market-neutral.

Details about how to construct this type of butterfly are given below for your future reference only and you are not required to remember any of the formulas on how to do it for the IFID Certificate exam.

However, you are expected to be able to explain that:

- A market-neutral and pivot-neutral butterfly is very unlikely to be cash-neutral as well
- A cash-neutral and market-neutral butterfly is very unlikely to be pivot-neutral as well

The cash-neutral butterfly

In symbols:

- Market risk-neutrality:

  $$\mathbf{Q_m \times R_m = Q_s \times R_s + Q_l \times R_l}$$

- Cash-neutrality:

  $$\mathbf{Q_m \times DP_m = Q_s \times DP_s + Q_l \times DP_l}$$

  Where:
  Q_i = Nominal amount of bond i (S, M or L)
  DP_i = Dirty price of bond i

By a process of substitution in the above equations, we arrive at the solutions for Q_s and Q_l:

$$Q_l = Q_m \times \frac{(R_m \times DP_s - DP_m \times R_s)}{(R_l \times DP_s - R_s \times DP_l)}$$

$$Q_s = Q_m \times \frac{(R_m \times DP_l - DP_m \times R_l)}{(R_s \times DP_l - R_l \times DP_s)}$$

Using the same bonds as we used in the pivot-neutral butterfly that we constructed earlier, we obtain:

$$Q_l = 50{,}000{,}000 \times \frac{(5.0683 \times 109.369 - 105.765 \times 2.6540)}{(6.5460 \times 109.369 - 2.6540 \times 100.529)}$$

$= 30{,}460{,}816$

$$Q_s = 50{,}000{,}000 \times \frac{(5.0683 \times 100.529 - 105.765 \times 6.5460)}{(2.6540 \times 100.529 - 6.5460 \times 109.369)}$$

$= 20{,}353{,}616$

Proof

Net proceeds from sale of the butterfly's body (bond M):
= $Q_m \times DP_m$
= 50,000,000 x 105.765/100
= GBP 52,882,500

Net cost of wings:
= $(Q_s \times DP_s) + (Q_l \times DP_l)$
= (20,353,616 x 109.369/100) + (30,460,816 x 100.529/100)
= GBP 52,882,500

Question 3

Settlement date: 15 March 2004

You work on the origination desk of a bank and today your bank has done a bought deal for GBP 500 million of Abbey National bonds with a coupon of 5.375% and maturity of 30 December 2009 at a clean price of 99.98 (Annual, E30/360).

a) You are planning to hedge the risk on this position using the 6¼% UK Gilt maturing on 25 November 2010, which is currently being offered at 107.48 (semi-annual, Act/Act). What is the nominal amount of this bond that you should short?

Enter your answer as an absolute nominal amount, rounded to the nearest £100.

413002200

b) Having shorted the amount of 6¼% of 2010 calculated in the previous question, what residual risks does your position still have?

- [ ] Market risk
- [x] Credit risk
- [x] Yield curve pivot risk
- [x] Funding risk

Explanation

- **Credit risk**
  Shorting a government bond still leaves us exposed to a widening of the yield spread of the Abbey National bond over UK Gilts

- **Yield curve pivot risk**
  We have a little bit of pivot risk, having shorted a 2010 bond to hedge the market risk on a 2009 bond. In particular, the spread position will lose money if the yield curve were to pivot clock-wise: the yield on the Gilt fell relative to that on the Abbey National bond

- **Funding risk**
  The Abbey National bond has to be funded while the short Gilt has to be repoed in until the issue has been placed with investors. The problem is that it cannot be known in advance when the issue will be placed in full, so the bank cannot enter into term funding and repo agreements to carry the full position.

4. Futures Trading

Question 1

a) Complete the trading blotter below showing your daily profit/loss and cash flows, and the totals for the whole campaign. By the end of each day the balance on your margin account should have the minimum required to run the position. Enter your answer in each box below and validate.

Date	Position	Profit/Loss	Cashflow Out - /In +	Balance Margin A/c
1 June	BUY 10 SEPs @ 102.38	0.00	Initial margin: -25000.00	25000.00
	Settlement price: 102.10	-2800.00		22200.00
			Variation margin: -2800.00	25000.00
2 June	Settlement price: 102.23	1300.00		26300.00
			Variation margin: 1300.00	25000.00
3 June	SELL to close 10 SEPs @ 102.45	2200.00		27200.00
			Margin back: 27200.00	0.00
Net (1 to 3 June):		700.00	700.00	

Explanation

Date	Position	Profit/Loss	Cashflow Out - /In +	Balance Margin A/c
1 June	BUY 10 SEPs @ 102.38	**0.00**	Initial margin: 10 x 2,500 = **(25,000)**	**25,000**
	Settlement price: 102.10	(10210 – 10238) x 10 x 10 = **(2,800)**		**22,200**
			Variation margin: **(2,800)**	**25,000**
2 June	Settlement price: 102.23	(10223 – 10210) x 10 x 10 = **1,300**		**26,300**
			Variation margin: **1,300**	**25,000**
3 June	SELL to close 10 SEPs @ 102.45	(10245 – 10223) x 10 x 10 = **2,200**		**27,200**
			Margin back: **27,200**	**0.00**
Net: 1 to 3 June		**700**	**700**	

5. Bond Futures

Question 1

a) By allowing a range of government bonds to qualify for delivery against the same futures contract, the futures exchange tries to:

(i) Make the derivative represent a whole section of the yield curve, rather than just a single point on it

(ii) Increase the liquidity of the futures contract

- Only (ii) is correct
- Neither is correct
- Only (i) is correct
- Both are correct

Explanation
Both are correct.

The only qualification is that, as we explain in a later section, in practice the futures is driven by only one bond (the cheapest to deliver) so most of the liquidity tends to be focused on that bond and the derivative simply reflects conditions on the part of the yield curve that it represents.

Question 2

The conversion factor attempts to establish some value equivalence between the futures price and the price of each deliverable bond.

a) If the coupon rate on a deliverable bond is higher than that on the notional bond, its conversion factor is:

- Less than 1
- Greater than 1

Explanation
Greater than 1.

The conversion factor is the price of the deliverable bond to yield the coupon rate on the notional by the first futures delivery date.

Question 3

Settlement date: 6 June 2003

Situation
Today is the last trading date of the June Bund futures and you are short 75 contracts, against which you intend to deliver the 5.50% maturing on 11 June 2012 (annual, actual/actual), which is currently the cheapest to deliver with a conversion factor of 0.965967.

Name:	EUREX Bund Futures
Unit of trading	EUR 100,000 of a notional Bund with a 6% coupon and maturity of 8½ - 10½ years
Delivery Months	March, June, September, December
Delivery Day	Tenth calendar day of delivery month
Last Trading Day	Two business days prior to Delivery Day
Quotation	Percentage of face value, to 2 decimal places
Minimum Price Movement (Tick Size)	0.01%
Tick Value	0.01% x 100,000 = EUR 10.00

a) What is the face value of the bonds that you should deliver?

EUR 7500000

Explanation

Nominal amount of bonds delivered = 75 x 100,000
= EUR **7,500,000**

b) If the EDSP is 107.08 and the CTD has 5.46986301% of accrued interest, what will be your invoice amount, rounded to the nearest Euro?

EUR 8167921

Explanation
Futures delivery settlement:

$$\text{Invoice amount} = \left(\frac{\text{EDSP}}{100} \times \text{Conversion factor} + \text{Accrued interest}\right)$$

$$\times \text{Contract size} \times \text{Number of contracts}$$

$$= \left(\frac{107.08}{100} \times 0.965967 + \frac{5.46986301}{100}\right) \times 7{,}500{,}000$$

= EUR 8,167,921

Question 4

In this exercise you will use a forward bond pricing model based on the formula developed in section *Pricing*.

Launch the forward price and yield calculator (by clicking on the link shown on the left of the screen) and please ensure that the model contains the following data:

Bond structure		**Funding**	
Coupon rate	7.5000%	Repo rate	6.00%
Maturity	6-Apr-09	Day count: Actual/	360
Coupon period	Semi		
Year basis	Actl/Actl		

Spot date		**Forward date**	
Settlement	4-Nov-02	Settlement	4-May-03
Accrued	0.59753	Accrued	0.57377
Clean price	88.50000	Clean price	87.44453
Yield	9.965%	Yield	10.388%
Basis point value	4.3189	Risk factor	3.9957

a) Why is the forward clean price lower than the cash price of this bond?

- ○ Because the market is bearish
- ◉ Because the yield on the bond is higher than the funding rate
- ○ Because the market has negative carry
- ○ Because the coupon rate on the bond is lower than the repo rate

Explanation
Because the yield on the bond is higher than the funding rate.

In other words, this market has positive carry

b) What is the repo rate that would make the forward clean price of this bond equal to 89.00?

Instructions

You may try entering different repo rates until you find one that makes the forward price equal to 89.00. Alternatively, you may let Excel find it for you. Select **Tools | Goal Seek** from the Excel menu and enter the data shown in bold into the dialog box

Set cell: **Forward_price**
To value: **89.00**
By changing cell: **Reinvest**

Repo rate: 9.49

Explanation
9.49%.

In other words, a forward price of 89.00 (when the cash price is 88.50) implies a repo rate of 9.49%. The higher the forward price relative to the cash price, the higher is the **implied repo rate**. The implied repo rate is a key concept in bond futures, as we shall explain in a later section.

c) Restore the repo rate back to 6.00% and complete the table below, *entering your figures rounded to 2 decimal places*:

Spot Settlement	Forward Clean price	Basis = Cash price - Forward price
4-Nov-2002	87.44	1.06
4-Feb-2003	88.00	0.50

Explanation

Spot Settlement	Forward Clean price	Basis = Cash price - Forward price
4-Nov-2002	**87.44**	**1.06**
4-Feb-2003	**88.00**	**0.50**

d) Which of the following is (are) true, *other things being equal*?

- [x] A rise in the funding rate is beneficial to a long forward position
- [ ] A rise in funding rate makes a long forward position less profitable
- [x] If carry is positive, a long forward position becomes more profitable over time
- [x] If carry is negative, a long forward position becomes less profitable over time

Explanation

- **A rise in the funding rate is beneficial to a long forward position**
- **If carry is positive a long forward position becomes more profitable over time**
- **If carry is negative a long forward position becomes less profitable over time**

In a positive carry market the forward price converges up to the cash price, making a long forward position increasingly profitable. At the same time, as we saw in b), a rise in the funding rate raises the forward price relative to the cash price.

Question 5

In this section we explore the price behaviour of bond futures in relation to the CTD and we illustrate the profits and risks associated with cash-futures arbitrage. Please launch the model and ensure the following data is specified correctly in the **Analysis** worksheet.

Cash market

Settlement date	1-Aug-99
Repo rate (RP)	5.2500%
Year basis	Actual/365

Futures market

Notional coupon	7.00%
1st delivery date	1-Sep-99
Difference	0.00

Yield vertices

	Raw Yld.	Yld+Shift	Yld+Pivot
Overnight	5.2500%	5.2500%	5.2500%
2 Yrs	5.7500%	5.7500%	5.7500%
4 Yrs	6.2500%	6.2500%	6.2500%
10 Yrs	6.3500%	6.3500%	6.3500%
20 Yrs	6.3700%	6.3700%	6.3700%
Shift		0.00%	0.00%

Deliverable bonds

Name	Coupon Rate	Maturity Date	Coupon Frequency	Day-count Basis	Actual RP
Treasury	7.2500	7-Dec-07	Semi-annual	Actl/Actl	5.25%
Treasury	9.0000	13-Oct-08	Semi-annual	Actl/Actl	5.25%
Treasury	8.0000	23-Sep-09	Semi-annual	Actl/Actl	5.25%
Treasury	5.7500	7-Dec-09	Semi-annual	Actl/Actl	5.25%
Treasury	6.2500	25-Nov-10	Semi-annual	Actl/Actl	5.25%
Treasury	9.0000	12-Jul-11	Semi-annual	Actl/Actl	5.25%

This specifies all the bonds that were deliverable into the LIFFE SEP 1999 long Gilt futures contract and prices the futures given the specified market conditions.

The CTD analysis model

The deliverable bonds are priced off a yield curve generated from a set of observed yield points, or **vertices**, labelled **Raw Yields**. A smooth yield curve is fitted through these vertices using cubic spline interpolation (for an explanation of this technique see Yield Curve Fitting - Cubic Splines).

The fields **Yld + Shift** and **Yld + Pivot** in the worksheet are just convenient means of allowing you to shift or pivot the curve without re-inputting all the yields again.

The worksheet calculates the theoretical futures from the CTD (i.e. the bond with the highest implied repo rate over the actual repo rate). It also allows you to enter an actual futures price by specifying a **Difference** between the actual and the theoretical futures prices. The gross and net basis for each bond, as well as the implied repo rates, is calculated from the actual futures price.

a) What is the theoretical futures price and what is the implied repo rate (IRP) on the current CTD? Type your answer in each box below and validate.

Theoretical futures price 103.45

IRP (%) 5.25

Please check the calculator settings, above, if your answers don't match!

Explanation

Theoretical futures price = **103.45**
IRP (%) = **5.25**

b) Complete the table below showing the evolution of the gross basis on the CTD as we approach the futures delivery date.

Cash market Settlement date	Gross basis of CTD
1-Aug-99	0.14
10-Aug-99	0.10
20-Aug-99	0.05
1-Sep-99	0.00

Explanation

Cash market Settlement date	Gross basis of CTD
1-Aug-99	**0.14**
10-Aug-99	**0.10**
20-Aug-99	**0.05**
1-Sep-99	**0.00**

c) What does a positive gross basis suggest?

- ○ The futures is trading cheap to fair value
- ◉ There is positive carry
- ○ The futures price is higher than the price of the CTD
- ○ There is negative carry

Explanation
There is positive carry.

d) Restore the cash settlement date to 1-Aug-99 and force the actual futures price below the theoretical price by entering 0.20 in the **Difference** field. Which of the following is/are true?

- ☐ The futures is trading rich to fair value
- ☑ The net basis on the CTD is 0.20
- ☑ There is an arbitrage opportunity in shorting the CTD and buying the futures
- ☐ There is an arbitrage opportunity in buying the CTD and buying the futures

Explanation

- The net basis on the CTD is 0.20
- There is an arbitrage opportunity in shorting the CTD and buying the futures

e) The futures contract size is GBP 100,000. How many bond futures should you buy, if you were considering shorting GBP 20 million of the CTD bonds and carrying this position to delivery?

- ○ 203 contracts
- ○ 2,000 contracts exactly
- ○ 197 contracts
- ◉ 200 contracts exactly

Explanation
200 contracts exactly.

Unlike basis trading, where we seek to profit from changes in the gross basis, here we do not need to risk-weight the position because the intention is to carry it into the futures delivery. We just need to buy enough futures contracts to cover the short position on the bonds.

Question 6

Use the same data in the **Analysis** worksheet as in *Question 1* and keep the actual futures price below the theoretical price by entering 0.20 in the Difference cell. We shall now calculate the profit potential of arbitraging the 7.25% of 2007 against the SEP futures:

- Short the 7.25% of 2007
- Buy the SEP futures

a) **Cash market settlement date:** **1-Aug-99**

(i) Dirty price of 7.25% of 2007 (rounded to 5 decimal places)	106.26133
(ii) Settlement amount on GBP 20 million of the 7.25% of 2007	21252266
(iii) Actual futures price	103.25
(iv) Nr. days to futures delivery	31

Explanation

Cash market settlement date:	**1-Aug-99**
(i) Dirty price of 7.25% of 2007 (rounded to 5 decimal places)	105.17185 + 1.08948 **= 106.26133**
(ii) Settlement amount on GBP 20 million of the 7.25% of 2007	106.26133/100 x 20,000,000 = GBP **21,252,266.00**
(iii) Actual futures price	**103.25**
(iv) Nr. days to futures delivery	**31**

b) Now move the **Cash settlement date** forward to 1-Sep-99 (the first delivery date) and change the value in the **Difference** field back to 0.00. In other words, we assume that by the first futures delivery date the gross and the net basis on the CTD will collapse to zero.

Cash settlement date:		**1-Sep-99**
(v)	Actual futures price (EDSP)	103.52
(vi)	Futures variation margin (1-Aug to 1-Sep) on 200 contracts	+54000.00
(vii)	Interest earned on reverse repo: = (ii) @5.25% for (iv) days	+94761.82
(viii)	Futures delivery invoice amount	21362197.5
(ix)	Net arbitrage profit = (vi) + (vii) + (ii) - (viii)	+38830.26
(x)	Profit as a percentage of capital: = (ix) / (ii) x 100 (to 2 decimal places)	0.18

Explanation

Cash settlement date:		**1-Sep-99**
(v)	Actual futures price (EDSP)	**103.52**
(vi)	Futures variation margin (1-Aug to 1-Sep) on 200 contracts	(10352 - 10325) x 10 x 200 **= +54,000.00**
(vii)	Interest earned on reverse repo: = (ii) @5.25% for (iv) days	21,252,266 x 0.0525 x 31/365 **= +94,761.82**
(viii)	Futures delivery invoice amount	(103.52 x 1.0153346 + 1.70355)/100 x 100,000 x 200 **= 21,362,197.56**
(ix)	Net arbitrage profit = (vi) + (vii) + (ii) - (viii)	**= +38,830.26**
(x)	Profit as a percentage of capital: = (ix) / (ii) x 100 (to 2 decimal places)	**0.18**

c) Restore the **Cash settlement date** back to 1-Aug-99 and the **Difference** field back to 0.20. We shall now explore the risk on this basis position if there is a change in the CTD. Force a parallel upward shift in the yield curve by entering +1.50% in the **Yld+Shift** field and calculate the mark-to-market profit/loss on your position:

Cash settlement date:		**1-Aug-99**
(xi)	Dirty price of 7.25% of 2007 (rounded to 5 decimal places)	96.93396
(xii)	Settlement amount on GBP 20 million of the 7.25% of 2007	19386792.0
(xiii)	Actual futures price	93.58
(xiv)	Profit on cash bonds = (ii) - (xii)	+1865474.0
(xv)	Profit on bond futures	-1934000.00
(xvi)	Net Profit	-68526.00

Explanation

Cash settlement date:		**1-Aug-99**
(xi)	Dirty price of 7.25% of 2007 (rounded to 5 decimal places)	95.84448 + 1.08948 **= 96.93396**
(xii)	Settlement amount on GBP 20 million of the 7.25% of 2007	96.93396/100 x 20,000,000 **= 19,386,792.00**
(xiii)	Actual futures price	**93.58**
(xiv)	Profit on cash bonds = (ii) - (xii)	21,252,266 – 19,386,792 **= +1,865,474.00**
(xv)	Profit on bond futures	(9358 – 10325) x 10 x 200 **= -1,934,000.00**
(xvi)	Net Profit	**-68,526.00**

d) Which of the following is/are true?

- [ ] The futures price fell less steeply than the cash price
- [x] The futures price fell more steeply than the cash price
- [x] The CTD changed to a bond with a higher BPV
- [ ] The CTD changed to a bond with a lower BPV

Explanation

- **The futures price fell more steeply than the cash price**
- **The CTD changed to a bond with a higher BPV**

The futures contract tends to display negative convexity:

- In a bear market bonds with higher BPVs become the CTD, causing the futures price to fall proportionately more than the fall in the price of the previous CTD
- In a bull market bonds with lower BPV become the CTD, causing the futures price to rise proportionately less than the price of the previous CTD

Either way, this creates mark-to-market losses for someone holding a short basis position. In contrast, a long basis position has positive net convexity, so a change in the CTD would result in a trading profit.

There is considerably more risk in shorting the basis than in going long the basis. Traders must balance the profit potential of the short basis arbitrage against the risk that the CTD might change.

Since the risk of a CTD change is related to the volatility of the yield curve, the positive net basis that typically prevails in the bond futures market is effectively a premium for selling yield volatility.

e) Finally, watch what happens if the 5.75% of 2009 were to go on special. Change the *Actual RP* only on this bond to 4.00%. Which of the following is/are true?

- [ ] There is no change in CTD
- [x] The future price also changes
- [ ] The futures price does not change
- [x] The 5.75% 0f the 2009 becomes the CTD

Explanation

- **The futures price also changes**
- **The 5.75% of 2009 becomes the CTD**

The change in the CTD causes the futures price to fall, which worsens the losses on our basis trade. Those who were long the 5.75% of 2009 and short the futures would profit from this.

This illustrates how it is possible for traders to influence which bond is the CTD by manipulating the repo rate on specific issues. A bond futures arbitrageur must understand the liquidity of the underlying repo market!

Question 7

a) For settlement 4 August 2002, the 7¼% UK Gilt maturing on 7 December 2011 trades at 111.09375. If the security has a conversion factor of 1.0165266 and the DEC futures is trading at 109.65, what is the value of the gross basis?

Type your answer in the box below, rounded to 2 decimal places and validate.

-0.37

Question 8

Deliverable bond:	UK Gilt 9% maturing on 14 October 2012
Settlement date:	5 August 2002
Clean price:	125.125 (in decimal)
Accrued interest:	2.77869 (semi-annual, act/act)
Conversion factor:	1.1429955
Repo rate:	7.50%
SEP futures price:	109.490 (in decimal)
1st futures delivery date:	1 September 2002

a) What is the implied repo rate on this bond?
(Enter your answer in percent, rounded to 2 decimal places.)

7.25

b) Is there an arbitrage profit from buying this bond and selling it into the futures?

- [ ] Yes
- [x] No

Explanation
No: the implied repo rate is lower than the actual repo rate, so the return on the position does not cover the cost of funding it.

Question 9

Consider the following statements:

(i) The implied repo rate is the funding rate implied in an actual futures price
(ii) The implied repo rate measures the gross return that can be made on a basis position held until futures delivery
(iii) The gross basis is the difference between the cash price of a deliverable bond and its adjusted futures price
(iv) The net basis is the net arbitrage profit or loss, per 100 nominal, that could be made on a basis position
(v) The net basis is the difference between the gross basis and the net carry

a) Which of these statements is/are true?

- [ ] Answers (i), (ii), (iii) and (iv) only
- [x] All are true
- [ ] Answers (ii), (iii) and (v) only
- [ ] Answers (i), (iii) and (iv) only

Explanation
They are all are true!

Question 10

Settlement date: 8 February 2000

Situation
You have a EUR 25 million long position in the following bond:

Security:	World Bank 5.75% EUR bond maturing 20 September 2005
Type:	Eurobond, annual, Act/Act.
Clean price:	104.75

You wish to hedge the market risk on this position using the March EUREX bond futures contract.

Name:	EUREX Bund Futures
Unit of trading	EUR 100,000 of a notional Bund with a 6% coupon and maturity of 8½ - 10½ years
Delivery Months	March, June, September, December
Delivery Day	Tenth calendar day of delivery month
Last Trading Day	Two business days prior to Delivery Day
Quotation	Percentage of face value, to 2 decimal places
Minimum Price Movement (Tick Size)	0.01%
Tick Value	0.01% x 100,000 = EUR 10.00

Below are the details of the deliverable bond that is currently the CTD on the March contract.

Security:	German Federal Government 5.50% maturing 11 June 2009
Type:	Annual, Act/Act.
Conversion factor:	0.964966
Clean price:	101.40

a) To hedge the market risk on this position, you should:

- (●) Short the Futures
- ( ) Go long the Futures

Explanation
Short the futures.

b) How many contracts should you trade in order to create a risk-weighted hedge?

Instructions

Use the risk-weighted hedge formula developed in section *Hedging* and the bond pricing model supplied to calculate the BPVs of the two securities in question.

Number of contracts: 165

Explanation
Number of futures contracts:

$$= \frac{\textbf{Portfolio size}}{\textbf{Contract size}} \ \textbf{X} \ \textbf{Price factor of CTD} \ \textbf{x} \ \frac{\textbf{BPV of cash portfolio}}{\textbf{BPV of CTD}}$$

Using the bond pricing model or other financial calculator, we calculate the BPV of the two securities in question:

BPV of bond in portfolio = 0.04980
BPV of CTD = 0.07292

Therefore, number of futures contracts:

$$= \frac{25{,}000{,}000}{100{,}000} \times 0.964966 \times \frac{0.04980}{0.07292}$$

= 164.75 or **165** contracts, rounded.

c) Other things being equal, what would be the effect on the net position (long the World Bank bonds, short the bond futures) of an anti-clockwise pivot in the EUR yield curve?

- Indeterminate
- The net position should show a loss
- The net position should show a profit
- There should be neither a profit nor a loss on the net position

Explanation
The net position should show a profit.

This is because, in effect, we have a spread position:

- Long a 2005 bond
- Short a 2009 bond (the CTD)

Therefore, a rise in the yield on the 2009, *relative to the yield on the 2005*, should result in a net profit.

6. Short Interest Futures

Question 1

Date: Monday 10 June

Situation
A corporate treasurer is informed by her sales department that her company will receive a payment of EUR 30 million for value 15 September, by way of advance on a supply order. The cash will be available for a period of six months, after which it will be required to purchase the necessary equipment.

Amid market talk of a possible cut in European interest rates, the treasurer is keen to lock into a forward deposit rate now. At this point she checks the latest futures prices.

Three Month Euribor Futures - EUR 25 per 0.01%

Month	Price	Delivery	Nr. days from today
JUN	96.35	16 June	6
SEP	96.25	15 September	97
DEC	96.10	15 December	188
MAR	95.96	15 March	279

a) Which trade would you recommend to the treasurer?

- ( ) Buy 30 SEPs
- ( ) Sell 30 SEPs
- (•) Buy a strip of 30 SEPs and 30 DECs
- ( ) Sell 60 SEPs

Explanation
The treasurer should buy the strip of 30 SEP and 30 DEC contracts. This matches exactly the period during which the cash will be available for deposit.

On 13 September, the last trading day of the SEP contract, the two futures positions should be closed. On that day the treasurer can fix the six month rate for her deposit. The six month LIBOR at the time should be fairly close to the six month rate implied in the strip of 2 three-month futures.

Notice that the futures position hedges against a fall in LIBOR but does not hedge against:

- Changes in the bid/offer spread in the cash Eurocurrency market - LIBID may change by more than LIBOR
- Liquidity risk - the treasurer still has to place the funds in the cash market and may not even get LIBID for it

b) If the futures order in a) was filled at the market prices shown, what would be the implied six month LIBOR achieved? Type in your answer in the box below, to 2 decimal places, and validate.

3.84

c) On 13 September, the last trading day of the SEP contract, EUR rates are as follows:

Cash Deposits
3 months 3 1/8 - 1/4
6 months 3 1/4 - 3/8

DEC Futures 96.59

What is the effective deposit rate achieved by the treasurer? Enter your answer to 2 decimal places.

3.74

7. Swaps

Question 1

Market Conditions
USD 4 year swap: 7.50 - 7.60%
Basis: Semi-annual, Act/360 against 6 month LIBOR

Situation
Hybex Electrics is a highly rated company with a considerable amount of fixed rate liabilities. The Treasurer would like to increase the percentage of floating rate debt in the company's liabilities. There is currently a 7 5/8% USD 100 million Eurobond (annual, 30/360) outstanding with four years to maturity.

Gartside Trust is a medium-sized fixed income fund. The manager of this fund feels strongly that interest rates will rise and would like to take advantage of this perceived trend. The fund currently has USD 100 million invested in fixed rate corporate US bonds yielding 8.00% (semi-annual, 30/360). The manager would like to convert this onto a LIBOR basis for a period of four years.

a) How could Hybex achieve its objectives using swaps, and what swap rate would apply to them?

Hybex should become a:

- ( ) Payer
- (•) Receiver

Explanation
Receiver

b) of fixed @ (rate) :

7.50

c) on a swap against _____ of LIBOR

- (•) payment
- ( ) receipt

Explanation
Payment

d) How could Gartside achieve its objectives using swaps, and what swap rate would apply to them?

Gartside should become a:

- (•) Payer
- ( ) Receiver

Explanation
Payer

e) of fixed @ (rate)

7.60

f) on a swap against _____ of LIBOR

- ○ Payment
- ● Receipt

Explanation
Receipt

Question 2

USD 4 year swap: 7.50 - 7.60%
Basis: Semi-annual, Act/360 against 6 month LIBOR

In *Question 1*:

- Hybex swaps a 7 5/8% USD Eurobond issue (annual, 30/360)
- Gartside swaps an investment in a bond yielding 8.00% (semi-annual, 30/360)

Calculate the all-in LIBOR spread achieved by these parties after the swap. Express the spreads as percentages on a semi-annual Act/360 basis, rounded to 2 decimal places.

a) Hybex's LIBOR spread:
-0.12

b) Gartside's LIBOR spread:
0.29

Question 3

In this exercise, you will explore the pricing of some of the interest rate swap structures covered in this module.
Please launch the *Swaps Pricing Model* on the left and ensure the following data is specified in the model:

Curve analysis

Years	Swap Rate
1	6.00%
2	6.70%
3	6.85%
4	6.90%
5	6.92%
Shift	0.00%
Pivot	0.00%

Swap details

Maturity	5 years	
Notional	10,000,000	
Pay/receive	Receive	
	Fixed	**LIBOR**
Coupon	6.9200%	6.0000%
Settlement	Annual	Annual
Day count	30/360	ACT/365

The values in the cells labelled **Notional Multiple** should initially be set to 1 throughout.

a) What is the net present value (NPV) of this swap position (to the nearest pound)?

0

b) Is the fixed rate receiver a net payer or receiver of cash flow at the first settlement date (in 365 days)?

- [ ] Payable
- [x] Receiver

Explanation
Receiver.

c) What will be the net settlement amount payable or receivable at the first settlement date? Enter your answer rounded to the nearest pound.

92000

d) Now shift the entire swap curve up by 50 basis points by entering 0.50% in the **Shift** cell. What is the swap NPV, to the nearest pound?

-156331

e) What is the BPV, or delta, of this swap position (in pounds to the nearest £5 and ignoring the sign)?

Instructions

Enter 0.01% in the **Shift** cell and check the **Swap NPV**.

3180

f) If you were hedging the market risk on this position with a government bond that has a BPV of 0.04550 per £100 nominal, how much of this bond should you buy or sell? Enter your answer in £millions to 1 decimal place.

- [ ] Buy £6.9 million
- [x] Short £6.9 million
- [ ] Short £14.8 million
- [ ] Short £0.1 million

Explanation
Short £6.9 million.

The idea is to create a risk-weighted hedge: we therefore want to short an amount of bonds, Q, such that the BPV of the swap position exactly matches the BPV of the bond position:

$$Q \times \frac{0.04550}{100} = 3{,}180$$

$$Q = \frac{3{,}180}{0.04550} \times 100$$

= £6,989,011
= £**6.9** million, rounded to the nearest pound.

Please note: this bond position only hedges the market risk on the swap but, as we shall see in the next section, leaves the net position exposed to the risk of changes in the spread between swap rates and treasury yields.

g) Make sure that the **Shift cell** has a value of zero again. What is the rate for a 1 year into 3 years forward swap?

Instructions

1. Enter zero into the **Notional Multiple** cells for years 1 and 5 (this eliminates any cash flows for those two years)
2. Enter 1 into the **Notional Multiple** cells for years 2 to 4
3. Find the rate for the fixed leg of the swap that makes its NPV = 0. Select **Tools | Goal Seek** from the Excel menu and enter the data shown in bold into the dialog box:

Set cell:	**NPV**
To value:	**0**
By changing cell:	**Coupon**

7.24

Question 4

In this exercise, you will explore the pricing and risk characteristics of asset swaps using a simple Excel-based model. Please launch the model now and check that the data below has been correctly specified.

Swap Curve

Years	Swap rate
O/N	3.00%
1	3.10%
2	3.50%
3	4.80%
4	5.65%
5	6.12%
6	6.50%
7	6.78%
8	6.97%
9	7.00%
10	7.00%
Shift	0.00%

Asset Swap

		Fixed		Floating
Coupon	7.50%	7.00%		3.09%
		+Spread		0.00%
Maturity	10-Feb-08	10 –Feb-08		
Amount	1,000,000	1,000,000		
Effective	15-Mar-99	15-Mar-99		
Payment freq.	Annual	Annual		Annual
Day count	30/360	30/360		act/365
Yield	8.05%			

Please ensure that Type of Swap in the analysis table is set to Par in / par out.

a) On the face of it, if the bond yields 8.05% and the swap rate is 7.00%, what spread over LIBOR should the investor receive on the asset swap? Enter your answers in percent.

Bond yield:	8.05
Less swap rate	7.00
= LIBOR +	1.05

Explanation

Bond yield = **8.05%**
Less swap rate = **7.00%**
= LIBOR + = **1.05%**

b) What is the price of the bond to be asset-swapped? Enter your answers to 4 decimal places.

Clean price	96.5718
Dirty price	97.3010

Explanation

Clean price **= 96.5718**
Dirty price **= 97.3010**

Please check your input data if your answers don't match!

c) What is the total cost of buying £1 million of this bond?

973010

d) How much additional capital must the investor put up, over-and-above the cost of the bond, for a par in / par out swap for £1 million?

26990

e) The current market rate for a vanilla swap effective on 15 March 1999 and maturing on 10 February 2008 (i.e. matching the dates on the bond) is 7.00%. But if you look in the **Cash Flows** worksheet you will see that the net present value (NPV) of this swap is not zero. Why is that?

- [x] There is accrued interest on the fixed leg
- [ ] There is accrued interest on the floating leg
- [ ] The first LIBOR has not been fixed at the current market rate
- [x] There is an up-front cash flow = Par – Bond dirty price

Explanation

- **There is accrued interest on the fixed leg**
- **There is an up-front cash flow = Par – Bond dirty price**

The 7.00% rate is for a vanilla swap. Here the investor pays the full rate on the fixed leg of the swap, but the first settlement date is only some 11 months away, so the fixed leg already has 1 month of accrued interest. On the other hand the rate for the floating leg is fixed against an 11 month LIBOR, so there is no accrued interest on this side. This is the figure shown against the heading **NPV of 7% vanilla swap against LIBOR flat** in the **Analysis** table.

At the same time, the investor makes an up-front payment to the swap counterparty equal to the difference between the bond's dirty price and par. This is the figure shown against the heading **PV of 1 or 2** in the **Analysis** table.

f) To create a clean par in / par out structure, the fixed rate on the payer swap has to be adjusted so that it matches the coupon rate on the underlying bond. Please change the fixed rate on the swap to 7.50%.

After this adjustment, what is the NPV of the interest payer swap, from the point of view of the investor?

Swap NPV (£)

-67295

g) Who should be compensated for paying the higher fixed rate on the swap?

- ( ) The swap counterparty
- (•) The investor

Explanation
The investor

h) So far the investor:

- Pays par for the asset swap (when in fact the underlying bond trades below par)

- Exchanges all the bond's coupons against just LIBOR flat (when in fact the current swap rate is lower than the bond's coupon rate)

- Will have to pay a full coupon on the fixed leg of the swap, at the first settlement date, but will only receive LIBOR for the 332 days in the first interest period

What spread over LIBOR should the swap counterparty pay the investor to compensate her for all these factors? Enter your answer in % to 2 decimal places.

Instructions

Add a spread over LIBOR to the floating leg of the swap such that the NPV of the interest payer swap is zero.

You can do this by trial-and-error or you can let Excel find it for you. Select **Tools | Goal Seek** from the Excel menu and enter the data shown in bold into the dialog box:

Set cell: **NPV_Swap**
To value: **0**
By changing cell: **Swap_floating_spread**

1.01

i) What is the **risk factor** on this par / par asset swap package (bond plus structured payer swap) - i.e. the gain or loss, in pounds per £100 nominal, for a 100 basis point parallel shift in market rates?

Net market risk

0.4171

j) How much would a client investing £1 million in this asset swap stand to lose if market rates were to rise by 10 basis points? Enter you answer to the nearest pound.

417

8. Option Concepts

Question 1

Below are premium prices for options on IBM stock, which currently trades at USD 65.00.

CME - IBM Options (USD per share)

	Calls		Puts	
Strike	MAR	JUN	MAR	JUN
60.00	7.80	8.30	2.25	2.50
70.00	3.30	4.20	6.75	7.10

a) Complete the table below: type your answer in each box and validate. Enter 'U' where the potential loss is unlimited or very large.)

Strategy	Max. Loss/ (U)limited	Max. Profit/ (U)nlimited	Breakeven	Intrinsic Value	Time Value
Long MAR 60 call	7.80	U	67.80	5.0	2.80
Long MAR 70 call	3.30	U	73.30	0.0	3.30
Long JUN 70 call	4.20	U	74.20	0.0	4.20
Short JUN 70 call	U	4.20	74.20	0.0	4.20
Long MAR 60 put	2.25	U	57.75	0.0	2.25
Short MAR 60 put	U	2.25	57.75	0.0	2.25
Short MAR 70 put	U	6.75	63.25	5.0	1.75

Explanation

Strategy	Max. Loss	Max. Profit	Breakeven	Intrinsic Value	Time Value
Long MAR 60 call	**7.80**	**U**	60 + 7.80 = **67.80**	65 - 60 = **5.0**	7.80 - 5.0 = **2.80**
Long MAR 70 call	**3.30**	**U**	70 + 3.30 = **73.30**	**0.0**	**3.30**
Long JUN 70 call	**4.20**	**U**	70 + 4.20 = **74.20**	**0.0**	**4.20**
Short JUN 70 call	**U**	**4.20**	70 + 4.20 = **74.20**	**0.0**	**4.20**
Long MAR 60 put	**2.25**	**U**[2]	60 - 2.25 = **57.75**	**0.0**	**2.25**
Short MAR 60 put	**U**[3]	**2.25**	60 - 2.25 = **57.75**	**0.0**	**2.25**
Short MAR 70 put	**U**[4]	**6.75**	70 - 6.75 = **63.25**	70 - 65 = **5.0**	6.75 - 5.0 = **1.75**

[2] Strictly speaking, maximum profit = $57.75 (60.00 - 2.25)
[3] Strictly speaking, maximum loss = $57.75 (60.00 - 2.25)
[4] Strictly speaking, maximum loss = $63.25 (= 70.00 - 6.75)

Question 2

Below are premium prices for options on IBM stock, which currently trades at USD 65.00.

CME - IBM Options (USD per share)

	Calls		Puts	
Strike	MAR	JUN	MAR	JUN
60.00	7.80	8.30	2.25	2.50
70.00	3.30	4.20	6.75	7.10

Which one or more factors should someone who is bullish on IBM take into account when comparing the performance of:

a) The MAR 60 call with the MAR 70 call?

- [ ] The MAR 70 call has more downside risk
- [x] The MAR 70 call has less downside risk
- [ ] The MAR 70 call has a lower breakeven
- [x] The MAR 70 call has a higher breakeven

Explanation
The MAR 70 call is cheaper because it is OTM, whereas the MAR 60 is already ITM, so **the downside on the MAR 70 is smaller**.

On the other hand **the MAR 70 has a higher breakeven** level, so IBM has to rise more strongly before this option makes a profit.

b) The MAR 70 call with the JUN 70 call?

- [ ] The JUN 70 call has less downside risk
- [ ] The JUN 70 call has a lower chance of expiring ITM
- [x] The JUN 70 call has a better chance of expiring ITM
- [x] The JUN 70 call has more downside risk

Explanation
The **JUN 70 call** is more expensive than the MAR 70, so its **downside is greater**.

On the other hand it has a **longer expiry**, hence a **higher probability** that IBM will go through $70.00.

c) Which one or more of the strategies below would be appropriate to someone who is bearish on a market?

- [x] Sell calls
- [ ] Sell puts
- [ ] Buy calls
- [x] Buy puts

Explanation
Alternative strategies:

Buy puts
Sell calls

Both strategies make a profit if the underlying market falls. However, the risk/return profiles are very different:

Long puts: limited downside; (virtually) unlimited upside
Short calls: limited upside; unlimited downside

9. Options Pricing and Risks

Question 1

In these exercises we explore the pricing behaviour of options using the standard European option pricing model described in section *Analytic Models*. Please launch the model and begin by setting the **Market data** as per the table below.

Market data

	100.00
	100.00
	1.00
	6.00%
	6.00%
	10.00%

a) Complete the table below, showing the premium price for a call with different strikes. Type your answer into each box and then validate.

Strike	Call Price
$102.00	2.93
$100.00	3.76
$98.00	4.74

Explanation

Strike	Call Price
$102.00	**$2.93**
$100.00	**$3.76**
$98.00	**$4.74**

b) What is the relationship between the strike and the premium price

For a call if the strike is higher the premium is:

- ○ Higher
- ◉ Lower

Explanation
For a call: the higher the strike - the more OTM the option - the lower is its premium.

c) For a put if the strike is higher the premium is:

- ◉ Higher
- ○ Lower

Explanation
For a put: the higher the strike - the more ITM the option - the higher is its premium

Question 2

Premium and Time Value

Set the **Market data** in the pricing model as per *Question 1*.

a) Complete the table below for the $100 strike call.

Underlying Price	Option Price	Change	Intrinsic Value	Time Value
85.00	0.19	---	0.0	0.19
90.00	0.67	0.48	0.0	0.67
95.00	1.78	1.11	0.0	1.78
100.00	3.76	1.98	0.0	3.76
105.00	6.65	2.89	4.71	1.94
110.00	10.32	3.67	9.42	0.90

Instructions

- Since the option holder cannot be forced to exercise, intrinsic value can only be either positive or zero; it cannot be negative

- For a European option:
  Intrinsic value = Present value of { Forward – Strike }

  You can read this value from the **Mark-to-market** cell in the spreadsheet

Explanation

Underlying Price	Option Price	Change	Intrinsic Value	Time Value
85.00	**$0.19**	--	**$0.0**	**$0.19**
90.00	**$0.67**	**$0.48**	**$0.0**	**$0.67**
95.00	**$1.78**	**$1.11**	**$0.0**	**$1.78**
100.00	**$3.76**	**$1.98**	**$0.0**	**$3.76**
105.00	**$6.65**	**$2.89**	**$4.71**	**$1.94**
110.00	**$10.32**	**$3.67**	**$9.42**	**$0.90**

Notice how the option's price becomes progressively more sensitive to changes in the underlying as the option goes further into the money:

- Between $90 and $95 the option reflects just 22.2% of the action in the underlying (= 1.11 / 5.00 x 100)

- Between $105 and $110 it reflects 73.4% of the action (= 3.67 / 5.00 x 100)

This sensitivity is the option's delta, which we shall explore in Option Risks - Delta.

b) Where is time value highest?

- [x] When the option is ATM
- [ ] When the option is ITM
- [ ] Indeterminate
- [ ] When the option is OTM

Explanation
When the option is ATM. Time value decreases as the call moves ITM or OTM.

Question 3

Premium and Time

Set the **Market data** in the pricing model as per *Question 1.*

a) Complete the table below for the $100 strike call.

Expiry (years)	Option Price	Change
1.0	3.76	---
0.8	3.40	-0.36
0.6	2.98	-0.42
0.4	2.46	-0.52
0.2	1.76	-0.70
0.0	0.0	-1.76

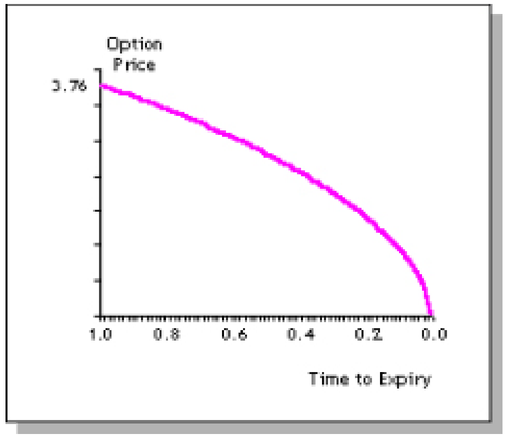
Explanation

Expiry (years)	Option Price	Change
1.0	**$3.76**	**---**
0.8	**$3.40**	**- $0.36**
0.6	**$2.98**	**- $0.42**
0.4	**$2.46**	**- $0.52**
0.2	**$1.76**	**- $0.70**
0.0	**$0.00**	**- $1.76**

b) Describe the relationship between time value and time to expiry.

- [ ] Time value decays evenly
- [ ] Time value decays unpredictably
- [x] Time value decays faster over time
- [ ] Time value decays more slowly over time

Explanation
The relationship is not linear: time value (hence the option price) decays at ever-faster rates as the option approaches expiry.



The speed of time value decay (the slope of the price - time curve) is the option's **theta**, which we shall explore in Option Risks - Theta. In this case, the theta of the 1 year option, for a 2.4 month (0.2 of a year) change in time, is \$0.36, becoming larger as the expiry approaches.

! Because of their time value decay, long positions in options with less than two weeks to expiry are very expensive to carry!

Time value reflects the perceived cost of managing the market risk on an option position and the figure above indicates that this risk is not proportional with time to expiry.

? Why is the risk on, say, a 1 year option less than twice the risk on a 6 month option?

If you examine closely the pricing formula presented in section *Analytic Models*, you will see a term $\sigma \times \sqrt{T}$. This is the option's **effective volatility** - the annual volatility pro-rated for the option's time to expiry.

Effective volatility = $\sigma \times \sqrt{T}$

Where:
σ = Volatility of underlying price (in percentage per annum)
T = Time to expiry, in years or fraction

The pattern of time value decay line shows that the risk on an option proportional to the **square root of time**.

Square Root of Time Rule

Total price risk = Daily price risk x $\sqrt{\text{Time}}$

This rule is derived from the formula for the standard deviation of the return on a portfolio of assets, when the returns on the various assets held are uncorrelated. The same approach can be applied in the context of holding a single asset over a period of time, n, if we imagine that each day the asset is held represents one element of a 'portfolio' of n assets.

Example

Suppose we hold an asset over two days (n = 2). The return on the 'portfolio' over this period is:

$$\mathbf{R_2 = R_t + R_{t-1}}$$

Where:
R_2 = Return on a portfolio over two days
R_i = Return on asset on day i

The variance of the return over the same period is:

$$\begin{aligned}
\sigma_2^2 &= E\,(R_2 - \mu)^2 \\
&= E\,[\,(R_t + R_{t-1}) - (\mu_t + \mu_{t-1})\,]^2 \\
&= E\,[\,(R_t - \mu_t) + (R_{t-1} - \mu_{t-1})\,]^2 \\
&= E\,(R_t - \mu_t)^2 + 2 \times E(R_t - \mu_t) \times (R_{t-1} - \mu_{t-1}) + E\,(R_{t-1} - \mu_{t-1})^2 \\
&= \sigma_t^2 + 2 \times \sigma_{t,t-1} + \sigma_{t-1}^2
\end{aligned}$$

Where:
σ_2^2 = Variance of return over 2 days
μ = The mean return over 2 days
E(x) = Shorthand for expected (mean) value of x. In other words, $E(x) = \Sigma(x) / n$
$\sigma_{t,t-1}$ = Covariance of R_t with R_{t-1}

If the daily returns on a financial instrument are normally (or log-normally) distributed, then they are also serially uncorrelated. Therefore $\sigma_{t,t-1} = 0$. Moreover, if we also assume the asset has constant volatility, then $\sigma_t = \sigma_{t-1} = \sigma$ and the variance formula reduces to:

$$= 2 \times \sigma^2$$
$$= \sqrt{2} \times \sigma$$

The standard deviation - hence the price risk - of the asset over the two days is given by its daily volatility multiplied by the square root of time.

The square root of time rule is an important result in finance: it is also used in risk management to model the risk on a trading portfolio (see Market Value at Risk - VAR and Time).

Question 4

Premium and Volatility

Set the **Market data** in the pricing model as per *Question 1*.

a) For the $100 strike option with 1 year to expiry, what is the premium price for the following levels of volatility:

Volatility	Option Price	Change
0%	0.0	---
5%	1.88	+1.88
10%	3.76	+1.88
15%	5.63	+1.87
20%	7.50	+1.87
25%	9.37	+1.87

Explanation Volatility	Option Price	Change
0%	**$0.0**	**---**
5%	**$1.88**	**+$1.88**
10%	**$3.76**	**+$1.88**
15%	**$5.63**	**+$1.87**
20%	**$7.50**	**+$1.87**
25%	**$9.37**	**+$1.87**

b) Describe the relationship between option price and volatility.

- [ ] Volatility has an unpredictable effect on premium
- [x] For ITM options premium accelerates as volatility increases
- [x] For ATM options premium is proportional to volatility
- [x] For OTM options premium accelerates as volatility increases

Explanation

- **For an ATM option**: the relationship is linear - i.e. double the volatility and you also double the option price
- **For ITM or OTM options**: option premium increase progressively faster as volatility is increased

The sensitivity of the option price to changes in volatility is the option's **vega**, which we shall explore in Option Risks - Vega.

c) The $100 strike call with 1 year to expiry trades at $5.25. What is its implied volatility?

Instructions

In Excel select **Tools | Goal Seek**. In the dialog box, enter the following (shown in **bold**):

Set cell: **Call_price**
To value: **5.25**
By changing cell: **Volatility**

13.98

Question 5

Premium and Cost of Funding

Set the **Market data** in the pricing model as per *Question 1.*

a) Complete the table below.

LIBOR	Call Price	Put Price
4.00%	2.92	4.82
6.00%	3.76	3.76
8.00%	4.72	2.86

Explanation

LIBOR	Call Price	Put Price
4.00%	**$2.92**	**$4.82**
6.00%	**$3.76**	**$3.76**
8.00%	**$4.72**	**$2.86**

b) Other things being equal, when interest rates increase calls become more expensive and puts become cheaper because:

- [x] Buying the call instead of the underlying saves on capital
- [ ] The market believes the underlying price will rise
- [x] The put seller hedges by shorting the underlying, so earns the interest on the proceeds
- [x] The call seller has to carry the underlying

Explanation

- **Buying the call instead of the underlying saves on capital**
- **The put seller hedges by shorting the underlying, so earns the interest on the proceeds**
- **The call seller has to carry the underlying**

The option price is derived from the cost of hedging its market risk:

- For a call seller the hedge is a long position in the underlying instrument; therefore the higher the cost of funding the underlying, the higher is the option price

- For a put seller the hedge is a short position in the underlying; therefore the higher the cost of funding, the higher is the positive carry on the short position

The sensitivity of the option's price to changes in the cost of funding is captured by the option's **rho** (see Option Risks - Rho & Phi). n this case the call's rho for a 1% rise in the funding rate, from 4.00% to 5.00%, is approximately:

= (3.76 - 2.92) / 2
= **$0.42**

Question 6

Premium and Profit/loss

Set the **Market data** in the pricing model as per *Question 1*.

a) If you bought the $100 strike 1 year call at the price calculated in *Question 1(a)* and after 3 months (0.25 years) the underlying price rose to $102.00 but volatility fell to 6%, what is the profit/loss on the trade?

-0.66

b) You bought the $100 strike 1 year call at the price calculated in *Question 1(a).* Calculate your profit/loss at the option's expiry in the following scenarios:

Underlying Price	Initial Premium Paid	Expiry Value	Profit/loss
98.00	$3.76	0.00	-3.76
100.00	$3.76	0.00	-3.76
102.00	$3.76	2.00	-1.76
104.00	$3.76	4.00	+0.24

Explanation

Underlying Price	Initial Premium Paid	Expiry Value	Profit/loss
98.00	$3.76	**$0.00**	**-$3.76**
100.00	$3.76	**$0.00**	**-$3.76**
102.00	$3.76	**$2.00**	**-$1.76**
104.00	$3.76	**$4.00**	**+$0.24**

c) What does the option's price at expiry represent?

- ○ The strike
- ○ Time value
- ● Intrinsic value
- ○ The price of the underlying

Explanation
At expiry there is no more time value, so all of the option's price represents just **intrinsic value**.

d) What is the breakeven on your trade?

103.76

Question 7

Suppose we have the following position in treasury bond options:

Position	Delta
Short $1,000,000 in 102.30 DEC calls	0.60
Long $2,000,000 in 100.27 DEC puts	0.35
Short $500,000 in 100.50 MAR puts	0.70
Long $300,000 in MAR futures	1.00

a) What would you do to make this position delta-neutral?

- ○ Sell cash bonds
- ● Buy cash bonds

Explanation
Buy USD, because the position is short delta.

The key is to convert each position into a spot equivalent: multiply the contract value of each option trade by its delta.

- Short call and long put positions have negative deltas
- Short puts and long calls have positive deltas

Position	**Delta**	**Spot Equivalent**
Short $1,000,000 in 102.30 DEC calls	0.60	- $600,000
Long $2,000,000 in 100.27 DEC puts	0.35	- $700,000
Short $500,000 in 100.50 MAR puts	0.70	+ $350,000
Long $300,000 in MAR futures	1.00	+ $300,000
Net		**- $650,000**

b) Enter the nominal amount of bonds you would trade: type your answer in the box below, to the nearest dollar, and then validate.

650000

Question 8

In this and the following questions we shall explore some of the risk management implications of trading options, using a specially developed spreadsheet. Launch the *Options Strategist* and set the contract **Specification** as per the table below.

Underlying instrument	UK Gilts
Currency of payment	GBP
Underlying year basis	Act/365
Funding year basis	Act/365
Contract Size	100,000
Option style	American
Underlying	Futures
Minimum price change	1/100[5]
Value of one tick	10.00 (per tick)

Case Study - May

Following the recent fiscal and political uncertainties in the UK, you observe that the LIFFE JUN UK Gilt futures options are trading with implied volatilities of around 14%, and you feel these should settle back to their normal 7.00 - 8.00% levels very soon.

Enter the following initial and current market data in the **Trades** worksheet of the spreadsheet:

Initial market data

Spot price	95.80
GBP rate[6]	3.50%
UK Gilts yield[7]	4.80%
Volatility	14.00%

Current market data

Days elapsed	0
Spot price	95.80
Volatility	14.00%

a) What trade would you put in place if you wanted to **sell volatility** (i.e. **short vega**) but did not have a strong view on the direction of UK Gilts?

○ Buy calls only

○ Buy puts only

○ Sell puts only

◉ Sell both calls and puts

Explanation
Sell both calls and puts

[5] I.e.: minimum price change = 0.01%.
[6] The funding rate from cash settlement up to the expiry date of the JUN futures contract.
[7] The yield on the JUN CTD bond, which for simplicity in this exercise is assumed to have a conversion factor of 1.0000, so it's easier to calculate the theoretical bond futures price (see Bond Futures – Cheapest to Deliver).

This is the classic strategy for selling volatility and is known as a straddle or a strangle (see Options Strategies - Volatility Trading). The strikes of the two options are typically set so that the net position is delta-neutral to begin with. This way the position is short vega but has no net exposure to changes in the futures price: any losses on the short call position should be offset by profits on the short put and vice-versa.

b) In this case study, we shall implement a short volatility position by selling calls and delta-neutralising them with Gilt futures.

With the June futures trading at 95.68 you decide to sell 20 JUN 97.00 calls expiring in 34 days. In column 1 of the **Trades** worksheet enter:

Positions	***1***
Number of NKI contracts	-20
Contract type	Call
Strike	97.00
Initial days to expiry	34

What is the premium price of these options, in ticks?

102

c) What is the total premium earned on the trade, in USD?

20400

d) What is the delta of these options? Enter your answer in percent, to 1 decimal place, *including the sign of the delta*.

-38.1

e) How many Gilt futures would you need to buy or sell in order to make this position delta neutral? Enter the number of contracts, rounded to the nearest unit.

8

f) In column 2 of the **Trades** worksheet enter the number of contracts calculated in (e) for your delta hedge:

Positions	***1***	***2***
Number of futures contracts	-20	**???**
Contract type	Call	Underlying
Strike	97.00	95.68
Initial days to expiry	34	34

Complete the table below to test the effectiveness of your futures hedge by moving the **Current spot price** up and down by 0.10%.

Spot price	**95.70**	**95.80**	**95.90**
Current futures price	95.58	95.68	95.78
97.00 calls (in ticks)	98	102	106
Profit/loss:			
Futures position	-800.0	0.0	800.0
Options position	800.0	0.0	-800.0
Net	0.0	0.0	-0.0

Explanation

Spot price	**95.70**	**95.80**	**95.90**
Current futures price	**95.58**	**95.68**	**95.78**
97.00 calls (in ticks)	**98**	**102**	**106**
Profit/loss:			
Futures position	(9558 – 9568) x 10 x 8 = £**-800**	**0**	(9578 – 9568) x 10 x 8 = £+**800**
Options position	(102 – 98) x 10 x 20 = 20,400 – 19,600 = £+**800**	**0**	(102 – 106) x 10 x 20 = 20,400 – 21,200 = £-**800**
Net	**0**	**0**	**0**

g) Does the delta hedge work?

- ○ Not at all
- ◉ Pretty much

Explanation
Pretty much! Losses on the options position are exactly offset by profits on the futures hedge and vice-versa. The net position is delta-neutral.

Question 9

Same case as in *Question 2*. Make sure you restore **Current spot price** back to 95.80 and keep the long position in the 8 futures contracts as well. Now change the **Days elapsed** in the spreadsheet to 15.

a) What would be the net profit/loss on the position (options and futures) at this point if the options traded at the following current volatility levels:

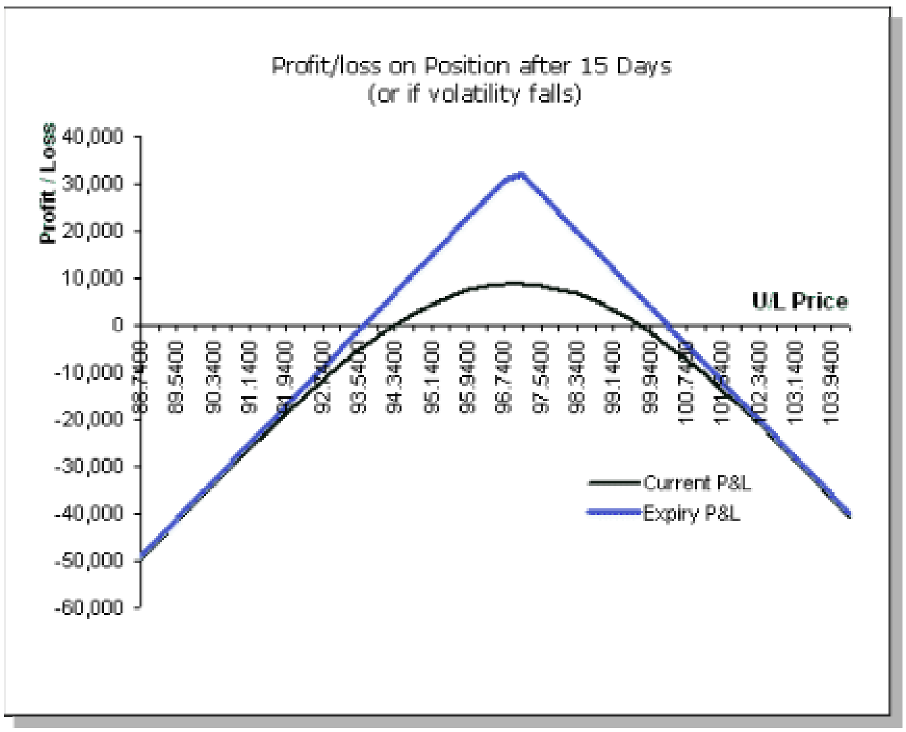
Volatility	**Profit/loss ($)**
7%	17480
14%	7280
18%	880
19%	-920

Explanation

Volatility	Profit/loss ($)
7%	**+17,480**
14%	**+7,280**
18%	**+880**
19%	**-920**

Notice how after 19 days volatility can rise against us all the way to 18% before the position loses money. The profits earned on the theta of this position alone is worth some 4 volatility points (18% – 14%). A position that is long theta can generate a very significant positive carry effect for the option trader.

If you look at the **Current P&L** worksheet in the model, you will notice that the passage of time or any fall in the volatility at which the options currently trade have the effect of shifting the current P&L curve upwards, toward its expiry P&L line.



b) Explain the results.

○ The position is short theta and long vega

○ The position is long theta and long vega

● The position is long theta and short vega

○ The position is short theta and short vega

Explanation
The position is long theta and short vega; see section *Summary of Greeks*.

Question 10

Same case as in *Question 2*, and make sure you restore the **Days elapsed** back to zero and the **Current volatility** back to 14%.

a) A few hours after the trades were put on the Gilts futures jumps up one full point (100 ticks) to 96.68, on account of a very strong pound. Complete the table below.

Spot price	**95.80**	**96.80**
Current futures price	95.68	96.68
97.00 calls (in ticks)	102	144
Profit/loss:-		
Futures position ()		8000
Options position (USD)		-8400
Net (USD)		-400

Explanation

Spot price	**95.80**	**96.80**
Current futures price	**95.68**	**96.68**
97.00 calls	**102**	**144**
Profit/loss:-		
Futures position		+100 x 10 x 8 = £**+8,000**
Options position		(102 - 144) x 10 x 20 = (20,400 – 28,800) = **- £8,400**
Net		£**-400**

b) Explain the results:

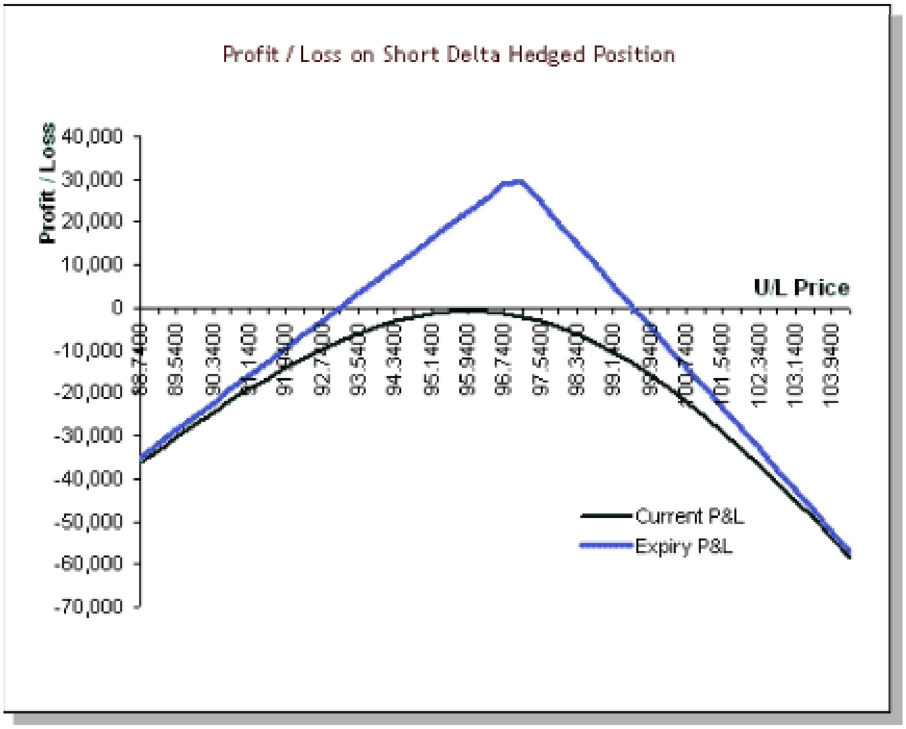
- [x] Loss on the option position is greater than profit on the futures hedge
- [ ] Profit on the option position is less than profit on the futures hedge
- [ ] The option's price fell by more than its original delta had predicted
- [x] The option's price rose by more than its original delta had predicted

Explanation

- **The option's price rose by more than its original delta had predicted**
- **The loss on the options was greater than the profit made on the futures hedge**.

The net loss arises because the option's price rose by more than the 39 ticks originally predicted by its delta (= 102 x 0.381). With such a large jump in the Gilts market, delta itself changed as the options moved closer to the money. What was an adequate hedge when the options traded with a delta of 38.1% became insufficient when delta increased.

If the market had fallen, instead of rising, the net result would have been a loss as well. Delta on the calls would have decreased, so the profit on the option position would have been less than the losses made on the futures hedge. The figure below shows the mark-to-market profit/loss profile on the net position for different levels of the market. (You can see this in the**Current payoff** worksheet).



c) Which of the following is now true about the position?

- ☐ The position is gamma-positive
- ☐ The position is still delta-neutral
- ☒ The position is short delta
- ☐ The position is long delta

Explanation
The position is short delta

As we show in the figure above, the position is now delta-negative (or **short delta**): further rises in the underlying price will result in additional losses on the net position. Indeed, the losses would become proportionately larger if the index continues to rise.

d) What additional risks are you now taking?

- ☒ The UK Gilts market could rally further
- ☐ The UK Gilts market could fall back again

Explanation
The UK Gilts market could rally further.

A further full-point rise in the futures price would cost this position a lot more than the £400 already lost on it.

e) What would you now do if you were concerned about the risk of a further rally in the UK Gilts market?

- ( ) Nothing
- ( ) Hope for a fall in the Gilts market, then close the position
- (•) Buy at least 2 more futures contracts
- ( ) Sell at least 2 futures contracts

Explanation
Buy at least 2 more futures contracts.

At the very least the position should be made delta-neutral again by adjusting the size of the futures hedge. At a futures level of 96.68 the options have a delta of 47.6%.

Therefore, the number of futures contracts required to make the position delta-neutral now is:
= 0.476 x 20
= 9.52 or **10 contracts**, rounded.

Since we only have 8 futures contracts, we should now buy at least 2 more.

Question 11

Same case as in *Question 4*. Having taken your losses, you now need to reset the *Options Strategist* to the current market level:

- Set both the **Initial spot price** and the **Current spot price** to 96.80
- Change the **Strike** of the futures position to the current futures price of 96.68
- In column 2 of your **Position** adjust your futures hedge to make the option position delta-neutral again

a) After having rebalanced your hedge, the JUN Gilts futures then gives up all the gains achieved earlier in the day, to close back at 95.68. Complete the table below and comment on the results.

Spot price	**96.80**	**95.80**
Current futures price	96.68	95.68
97.00 calls (in ticks)	144	102
Profit/loss:-		
Futures position ()		-10000
Options position (USD)		8400
Net (USD)		-1600

Explanation

Spot price	**96.80**	**95.80**
Current futures price	**96.68**	**95.68**
97.00 calls	**144**	**102**
Profit/loss:-		
Futures position		-100 x 10 x 10 = £-**10,000**
Options position		(144 - 102) x 10 x 20 = (28,800 – 20,400) = +**£8,400**
Net		**- £1,600**

The new loss arises because the actual fall in the option's price is less than the 69 ticks predicted by its delta (= 0.476 x 144). Delta moved back to 38.1% so the position is now over-hedged and the losses on the futures hedge outweighed the gains on the options!

Worse than that, the position is now delta-positive, so a further fall in UK Gilts will result in proportionately larger net losses. To re-hedge the book we now need to sell off the 2 futures contract that we had bought earlier - at a loss!

Adding the loss made here, when the market fell back, to the loss calculated in *Question 4* (when the market rose):

Cumulative loss so far = 400 + 1,600 = £**2,000**

These losses are effectively crystallised when we re-sell, at 95.68, the futures contract that we bought at 96.68:

Loss on futures trading = (9668 – 9568) x 10 x 2 contracts = £**2,000**

At this rate it won't be very long before the premium income of £20,400, which we earned at the start of this campaign, vanishes altogether!

The Moral of this story…

Managing delta when you are short options (or **short premium**) may be a very costly business in practice: although time is on your side (the position is theta-positive), every time the underlying moves significantly the position loses money and also becomes unhedged. And every time you rebalance your hedge you lock in some losses.

As in any budgeting exercise, the trader selling volatility must ensure that the premium charged at the outset is sufficient to cover the trading losses that he expects to make in managing the hedge.

A trader selling options has to ensure that he charges enough implied volatility to cover the cost of delta-hedging the actual volatility that he expects to see in the market.

Moreover, a short premium position is potentially explosive:

- It becomes increasingly delta-negative in a rising market, forcing you to buy ever-increasing amounts of the underlying and thus risking that you could yourself be pushing the market even higher

- It becomes increasingly delta-positive in a falling market, forcing you to short ever-increasing amounts of the underlying and risking that you could yourself be driving the market further down!

A short premium position that may be manageable within normal ranges of market volatility can quickly become unmanageable in extreme volatility conditions. Underlying market liquidity may be insufficient to permit adequate re-hedging and so the assumptions underpinning all option pricing models - that it should be possible at all times to maintain the correct hedge ratio - are no longer valid (see Option Pricing - Binomial Model).

In such conditions, the concept of a fair option price becomes meaningless. Like plutonium, options can be useful provided they are handled with care!

b) What is the value of your position gamma, for a 100 tick move in the Gilts futures price? Enter your answer in percent to one decimal place.

9.5

c) Compare your position with that of a trader who was long vega. To remain delta-neutral, and lock in the profits, the trader who is long vega would have to:

- [ ] Sell the underlying when the market falls
- [x] Buy the underlying when the market falls
- [ ] Buy the underlying when the market rises
- [x] Sell the underlying when the market rises

Explanation

- **Sell the underlying when the market rises**
- **Buy the underlying when the market falls**

This makes it much easier to manage delta. On the other hand the position is theta-negative, so without price volatility the profits gradually erode away.

10. Interest Rate Options

Question 1

a) Which of these positions would you enter into if you believed swap rates were likely to fall?

- [x] Buy a receiver swaption
- [ ] Sell a receiver swaption
- [x] Sell a payer swaption
- [ ] Buy a payer swaption

Explanation

- **Buy a receiver swaption**
- **Sell a payer swaption**

Question 2

Date: 10 March 1998
Scenario: You issued a 5 year GBP 100 million Floating Rate Note on which you pay 3 month LIBOR + 0.20% and you are worried that UK interest rates may rise on a five year view. Current 3 month LIBOR: 7.45%

Strategies:
1. Buy a 5 year 7.65% cap on 3 month LIBOR (19 caplets)
2. Buy a 3 months into 4¾ years 7.65% payer swaption

a) Which one or more of the following best describe these two strategies?

- [x] The swaption potentially locks you into a fixed rate
- [x] The cap is a strip of options on forward LIBORs
- [x] The cap protects you against rate rises but allows you benefit if they fall
- [x] The Swaption is an option on a swap (i.e. a strip of LIBORs)

Explanation
They are all correct

b) Other things being equal, which one of the following is preferable?

- ( ) The swaption
- ( ) Cannot determine from the information given
- ( ) Both are equally beneficial
- (•) The cap

Explanation
The cap

c) Which strategy do you think will be more expensive?

- ◯ Cannot determine from the information given
- ◯ The swaption
- ◉ The cap
- ◯ Both should cost the same

Explanation
The cap

Question 3

In this exercise we explore the price behaviour of European caps and floors using an Excel-based pricing model. Please ensure the following data is specified in the model:

Option type	Cap/floor
Notional Amount	$100,000,000
Valuation date	01-Jan-00
Reset period	Semi-annual
Year basis	Actual/360
Effective (yrs)	0.0
Term (max. 10 yrs)	10.0
Cap rate	8.00%
Floor rate	0.00%
Volatility	21.0%

	Raw Yld.	Yld+Shift	Yld+Pivot
Overnight	3.00%	3.00%	3.00%
1 Yr	3.20%	3.20%	3.20%
2 Yrs	3.60%	3.60%	3.60%
5 Yrs	5.00%	5.00%	5.00%
10 Yrs	6.00%	6.00%	6.00%
		0.00%	0.00%

The yield curve is generated from rates quoted in the money market and the swap market for various fixed maturities. The model uses cubic splines to fit a smooth swap curve passing through the specified yield vertices (see Yield Curve Fitting).

a) Complete the table below, showing the price of a cap, in percentage per annum and the capped interest rate to a borrower funding at LIBOR flat. Type your answer in each box and validate.

Cap rate	Cap price	Capped cost
8.00%	0.71	8.71
7.00%	0.95	7.95
6.00%	1.27	7.27

Explanation ap rate	Cap price	Capped cost
8.00%	**0.71%**	**8.71%**
7.00%	**0.95%**	**7.95%**
6.00%	**1.27%**	**7.27%**

b) What would be the price of the 7% cap, in percentage per annum, if other things being equal the yield curve pivoted clockwise by 50 basis points? (Enter -0.50 in the **Yld+Pivot** cell in the model.)

0.61

c) Interpret the result:

- [x] A flatter yield curve makes all the caplets OTM
- [x] A flatter yield curve reduces the implied forward rates
- [ ] A flatter curve implies a less volatile market
- [ ] If the curve is flatter there is less demands for caps, so it becomes cheaper

Explanation

- **A flatter yield curve reduces the implied forward rates**
- **A flatter yield curve makes all the caplets OTM**

You can see from the yield curve chart (and also in the **Caplets & Floorlets** worksheet) in the pricing model that many of the more distant forward rates were previously higher than the 7% cap rate. After the curve is flattened they are all below the cap rate, so all the caplets are now OTM.

d) Restore the yield curve to its original setting (i.e. reset the **Yld+Pivot** cell in the model to zero). What is the price of the 7% cap, in percentage per annum, for the following levels of volatility?

Volatility	Cap price
31.00%	1.40
26.00%	1.17
21.00%	0.95

Explanation

Volatility	Cap price
31.00%	**1.40%**
26.00%	**1.17%**
21.00%	**0.95%**

e) Restore the volatility back to 21%. A borrower considering a 7% cap would like to reduce its net cost to 75 basis points per annum, net, by selling a floor as well. Which one (or more) of the following is true?

- [x] The borrower will not benefit if LIBOR fell below the floor rate
- [ ] The floor would have to be set at 5.00%
- [ ] The borrower has sold a collar
- [x] The floor could be set at 3.68%

Explanation

- **The floor could be set at 3.68%**
  Try this floor rate and you should see the cost of the collar falling to zero.

- **The borrower will not benefit if LIBOR fell below the floor rate**
  The buyer of the collar is still compensated by the counterparty if LIBOR rises above 7%. However, having sold a 3.68% floor as well, they now have to compensate the counterparty for the difference between LIBOR and the floor rate, if LIBOR falls below the floor rate.

f) The borrower now wishes to buy a zero-cost collar. What rate should be set on the floor to achieve zero-cost structures for the following cap rates? Enter the floor rates rounded to the nearest 1 decimal place.

Cap rate	Floor rate
8.0%	4.90
7.0%	5.40
5.9%	5.90

Instructions

You may find the answers by trial-and-error, or you can let the **Goal Seek** function in the Excel **Tools** menu do most of the work for you! Enter the following pre-set labels (in **bold**) in the Goal Seek dialog box:

Set cell: **C22** (the net cost of the collar)
To value: **0.0**
By changing cell: **Floor_rate**

Explanation

Cap rate	Floor rate
8.0%	**4.9%**
7.0%	**5.4%**
5.9%	**5.9%**

g) Which of the following statements are true?

- [ ] Selling the floor always neutralises the benefits of buying the cap
- [x] For a zero-cost collar, the lower the cap rate, the higher must be the floor rate
- [x] A zero-cost collar with a floor rate equal to the cap rate is a synthetic swap!
- [ ] A zero-cost collar with the floor rate equal to the cap rate is a swaption

Explanation
For a zero-cost collar, the lower the cap rate, the higher must be the floor rate.
The more the buyer wants to benefit from a lower cap rate, the more they have to forego the benefit of falling interest rates by selling higher floors.

A zero-cost collar with a floor rate equal to the cap rate is a synthetic swap!

- Whenever LIBOR is higher than 5.9% the collar buyer receives the difference
- Whenever LIBOR is lower than 5.9% the buyer pays the difference

This is equivalent to paying a fixed rate of 5.9% and receiving LIBOR - i.e. a swap. In fact, when we convert the synthetic swap rate generated by this structure from a money market basis (Act/360) to a bond basis (Act/Act), we find that the synthetic swap is just two basis point different from the 10 year swap rate specified in the underlying yield curve:

Swap rate (bond basis) = 5.9 x 365 / 360
= 5.98 or **6.0**%, rounded.

What we see here is the put-call parity principle: for European options, ATM forward calls and puts must have the same price (see Option Pricing - Put Call Parity).In this case the cap price equals the floor price if the strikes are both set at the underlying swap rate.

11. Options Strategies

Question 1

Trading Options - Summary

Place the following strategies in their appropriate box on the matrix below:

Long the underlying	(LU)
Short the underlying	(SU)
Long a call or call spread	(LC)
Short a call or call spread	(SC)
Long a put or put spread	(LP)
Short a put or put spread	(SP)
Long a straddle or strangle	(LS)
Short a straddle or strangle	(SS)
No Strategy	(NS)

a)

		View on underlying market		
		Rising	**Don't know**	**Falling**
View on volatility	**Rising**	LC	LS	LP
	Don't know	LU	NS	SU
	Falling	SP	SS	SC

Explanation

		View on underlying market		
		Rising	**Don't know**	**Falling**
View on volatility	**Rising**	**LC**	**LS**	**LP**
	Don't know	**LU**	**NS**	**SU**
	Falling	**SP**	**SS**	**SC**

12. Introduction to Exotic Options

Question 1

a) Select one or more from this list.

- [x] The larger the rebate, the more expensive is the contract
- [ ] The larger the rebate the cheaper is the contract
- [ ] The lower the outstrike the cheaper is the option
- [x] The lower the outstrike the more expensive is the option

Explanation

- The larger the rebate, the more expensive is the contract
- The lower the outstrike the more expensive is the option

Question 2

In this exercise, we compare the expiry payoff of a knock-out put option with that of a conventional European put with the same strike and expiry. The details of the knock-out put are as follows:

Type: Knock-out put
Strike: $100
Outstrike: $120
Rebate: None

a) Complete the table below, showing the payoff of the two options in different market scenarios.

Underlying Price at expiry	Highest Underlying price	European Put	Knock-out Put
110	125	0	0
90	110	10	10
90	125	10	0

Explanation

Underlying Price at expiry	Highest Underlying price	European Put	Knock-out Put
110	125	**0**	**0**
90	110	**10**	**10**
90	125	**10**	**0**

b) Which of the following statements is/are true about a long position in a knock-out put option that has no rebate clause?

- [x] The knock-out put is always cheaper than the European put
- [ ] The knock-out put is always more expensive than the European put
- [x] The knock-out put is riskier than the European put
- [ ] The knock-out put is less risky than the European put

Explanation

- **The knock-out put is always cheaper than the European put**

  because

- **The knock-out put is riskier than the European put**

Question 3

You are NOT required for the IFID Certificate exam to perform valuations on exotic options.

However, this exercise involving the use of some exotic valuation models may help you gain a closer understanding of the ways in which some of the structures introduced in this module respond to changes in underlying market conditions.

In this first question we explore the price behaviour of European cash-or-nothing digital options. When you have launched the model, please select the **Digital** worksheet and ensure the following data is correctly specified:

Type	Call
Spot price	100.00
Strike	100.00
Expiry	1.00
Cash payout	10.00
Funding rate	3.00%
Yield	3.00%
Volatility	10.00%

a) What is the premium price of this option? Enter the result in the box and validate.

Please check your calculator settings, above, if your answer does not match!

4.66

b) Complete the table below, showing the evolution of two digital calls over time: one with a strike of $100 and the other with a strike of $90.

Expiry	Strike = 110	Strike = 90
1.00	1.53	8.17
0.50	0.82	9.13
0.25	0.27	9.74
0.00	0.00	10.00

Explanation

Expiry	Strike = 110	Strike = 90
1.00	**1.53**	**8.17**
0.50	**0.82**	**9.13**
0.25	**0.27**	**9.74**
0.00	**0.00**	**10.00**

c) Which of the following statements is/are true?

- [x] A position long an OTM call is short theta
- [ ] A position short an ITM call is long theta
- [ ] A position short an OTM call is short theta
- [x] A position long an ITM call is long theta

Explanation

- **A position long an OTM call is short theta**
- **A position long an ITM call is long theta**

Note how the sign of theta depends on whether the option is OTM or ITM. This is one of the characteristic features of exotic options.

d) Restore the expiry to 1 year and complete the table below showing the price of the two digital calls (the $110 and the $90 strikes) for different volatility levels.

Volatility	Strike = 110	Strike = 90
10.00	1.53	8.17
5.00	0.26	9.52
0.00	0.00	9.70

Explanation

Volatility	Strike = 110	Strike = 90
10.00	**1.53**	**8.17**
5.00	**0.26**	**9.52**
0.00	**0.00**	**9.70**[8]

[8] With 0% volatility the probability of the $90 call being exercised is 100%, so its price is just the present value of $10:

e) Which of the following statements is/are true?

- [x] A position long an ITM call is short vega
- [x] A position short an OTM call is short vega
- [ ] A position short an ITM call is short vega
- [x] A position long an OTM call is long vega

Explanation

- A position long an OTM call is long vega
- A position long an ITM call is short vega
- A position short an OTM call is short vega

f) Restore the volatility to 10%, the strike to $100 and change the expiry to 0.08 years (i.e. approximately 1 month). Complete the table below, showing the sensitivity of the price of this call to changes in the underlying price.

Spot price	Call price	Change in price (i.e. Delta)	Change in delta (i.e. Gamma)
85.00	0.00	-----	-----
90.00	0.00	0.00	-----
95.00	0.34	+0.34	+0.34
100.00	4.93	+4.59	+4.25
105.00	9.54	+4.61	+0.02
110.00	9.97	+0.43	-4.18
115.00	9.98	+0.01	-0.42

Explanation

Spot price	Call price	Change in price (i.e. Delta)	Change in delta (i.e. Gamma)
85.00	**0.00**	**-----**	**-----**
90.00	**0.00**	**0.00**	**-----**
95.00	**0.34**	**+0.34**	**+0.34**
100.00	**4.93**	**+4.59**	**+4.25**
105.00	**9.54**	**+4.61**	**+0.02**
110.00	**9.97**	**+0.43**	**-4.18**
115.00	**9.98**	**+0.01**	**-0.42**

Price = $10.00 x $e^{-0.03 \times 1.0}$
= **$9.70**
#/fn1#

g) Which of the following statements is/are true?

- [x] A position long an OTM call is long gamma
- [x] A position long an ITM call is short gamma
- [ ] A position long an OTM call is short gamma
- [ ] A position long an ITM call is long gamma

Explanation

- **A position long an OTM call is long gamma**
  The price of the OTM call accelerates as the option approaches the strike.

- **A position long an ITM call is short gamma**
  The price of the ITM call decelerates as the option moves more deeply ITM

Question 4

In this question, we explore the price behaviour of European barrier options using an Excel-based model. When you have launched the models, please select the **Barrier** worksheet and ensure the following data is correctly specified:

Flavour	Up-and-out-call
Spot price	95.00
Strike	100.00
Barrier	120.00
Rebate	0.00
Expiry	1.00
Funding rate	3.00%
Yield	3.00%
Volatility	10.00%

a) What is the premium price of this option, and what is the price of an equivalent conventional European call? Enter the result in the box and validate.

Premium up-and-out call [1.50]

Premium European call [1.83]

Please check your calculator settings, above, if your answer does not match!

Explanation

Premium up-and-out call **1.50**
Premium European call **1.83**

b) Complete the table below, showing the evolution of an OTM and an ITM up-and-out call over time.

Expiry	OTM: Spot price = $95	ITM: Spot price = $105
1.00	1.50	3.56
0.50	0.91	4.88
0.25	0.38	5.27
0.00	0.00	5.00

Explanation

Expiry	OTM: Spot price = $95	ITM: Spot price = $105
1.00	**1.50**	**3.56**
0.50	**0.91**	**4.88**
0.25	**0.38**	**5.27**
0.00	**0.00**	**5.00**

c) Restore the expiry to 1 year and complete the table below showing the price of an OTM and an ITM up-and-out call for different volatility levels.

Volatility	OTM: Spot price = $95	ITM: Spot price = $105
10.00	1.50	3.56
5.00	0.37	5.15
0.00	0.00	4.85

Explanation

Volatility	OTM: Spot price = $95	ITM: Spot price = $105
10.00	**1.50**	**3.56**
5.00	**0.37**	**5.15**
0.00	**0.00**	**4.85**

d) Which of the following statements is/are true about a position that is long an ITM up-and-out call?

- [x] It may be short or long theta
- [x] It may be long or short vega
- [x] It may be long or short gamma
- [x] It may be long or short delta

Explanation
They are all true!

Question 5

In this exercise, we explore the price behaviour of various multi-asset options using an Excel-based model. When you have launched the models, please select the **Multi-asset** worksheet and ensure the following data is correctly specified:

Asset 1

Spot price	50.00
Number of units	2.0
Strike	50.00
Yield	3.00%
Volatility	12.00%

Asset 2

Spot price	100.00
Number of units	1.0
Strike	100.00
Yield	3.00%
Volatility	8.00%

Option

Type	**Basket**
Expiry	1.00
Funding rate	3.00%
Correlation	0.00

The basket is made up of:

- 2 units of Asset 1, currently trading at $50 each
- 1 unit of Asset 2, currently trading at $100

The current market value of the basket is therefore $200, 50% of which is allocated to Asset 1 and 50% to Asset 2, and the option is ATM. The model calculates to the nearest 10 cents the price of this option as a percentage of the contract size.

Contract size = Strike of Asset 1 x Units of Asset 1
+ Strike of Asset 2 x Units of Asset 2

a) What is the price of the basket option? Enter the result in the box below and validate.

Please check your calculator settings, above, if your answer does not match!

2.80

b) Complete the table below, showing the price of this option assuming different correlations between the assets.

Correlation	Premium price (%)
+0.80	3.70
0.00	2.80
- 0.80	1.40

Explanation

Correlation	Premium price (%)
+0.80	**3.70**
0.00	**2.80**
- 0.80	**1.40**

c) Which of the following statements is/are true of a long position in a basket call option?

- [x] It is long delta in any of its constituent assets
- [x] It is long the correlation vega
- [x] It is long vega in any of its constituent assets
- [ ] It is long theta

Explanation

- It is long vega in any of its constituent assets
- It is long delta in any of its constituent assets
- It is long the correlation vega

Question 6

Please change the fields indicated in bold below in the **Multi-asset** worksheet of the pricing model.

Asset 1

Spot price	50.00
Number of units	2.0
Strike	50.00
Yield	3.00%
Volatility	12.00%

Asset 2

Spot price	100.00
Number of units	1.0
Strike	100.00
Yield	3.00%
Volatility	8.00%

Option

Type	**Exchange**
Expiry	1.00
Funding rate	3.00%
Correlation	**0.00**

This exchange option gives the holder the right to:

- Receive 2 units of Asset 1, which is currently trading at $50
- By delivering 1 unit of Asset 2, which is currently trading at $100

The model calculates the price of this option as a percentage of the nominal value of the asset(s) to be delivered.

Contract size = Strike of Asset 2 x Units of Asset 2

a) What is the price of the exchange option, as a percentage of the underlying asset value, to 2 decimal places?

5.60

b) Complete the table below, showing the price of this option assuming different correlations between the assets.

Correlation	Premium price (%)
+0.80	2.90
0.00	5.60
- 0.80	7.40

Explanation

Correlation	Premium price (%)
+0.80	**2.90**
0.00	**5.60**
- 0.80	**7.40**

c) Which of the following statements is/are true of a long position in this exchange option?

- [x] It is short the correlation vega
- [ ] It is long delta in either of its constituent assets
- [x] It is short theta
- [x] It is long delta in the callable asset; short delta in the putable asset

Explanation

- It is short the correlation vega
- It is short theta
- It is long delta in the callable asset; short delta in the putable asset

Question 7

Please change the fields indicated in bold below in the **Multi-asset** worksheet of the pricing model.

Asset 1

Spot price	50.00
Number of units	1.0
Strike	50.00
Yield	3.00%
Volatility	12.00%

Asset 2

Spot price	100.00
Number of units	1.0
Strike	100.00
Yield	3.00%
Volatility	8.00%

Option

Type	**Rainbow**
Expiry	**1.00**
Funding rate	3.00%
Correlation	**0.00**

This rainbow will pay the percentage gain in the best-performing of the two assets (a rainbow call).

Contract size = 1 unit of Asset 1 *or* 1 unit of Asset 2

a) Price of rainbow option (%)

6.70

b) Complete the table below, showing the price of this option assuming different correlations between the two assets.

Correlation	Premium price (%)
+0.80	5.30
0.00	6.70
- 0.80	7.60

Explanation

Correlation	Premium price (%)
+0.80	**5.30**
0.00	**6.70**
- 0.80	**7.60**

c) Which of the following statements is/are true of a long position in a rainbow call option?

- [x] It is short theta
- [x] It is long delta in any of its constituent assets
- [x] It is short the correlation vega
- [ ] It is short vega in any of its constituent assets

Explanation

- It is short theta
- It is long delta in any of its constituent assets
- It is short the correlation vega