ISMA CENTRE - THE BUSINESS SCHOOL OF THE FINANCIAL MARKETS

UNIVERSITY OF READING ENGLAND

IFID Certificate Programme

Rates Trading and Hedging

Introduction to Exotic Options

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1. Definition

Exotic options are contracts:

- Where the terms that are normally found in the vanilla (European or American) option are in some way different; or

- Which have additional terms that are not found in the vanilla option contracts

Recall Vanilla Options

Recall the standard terms of the vanilla option:

Type:	Call or put
Style:	European or American
Strike:	$XXX
Expiry date:	DD/MM/YY
Underlying:	A security, commodity, currency amount, futures or price index
Expiry payoff:	Physical settlement or cash settlement based on the difference between a reference market price and the strike

These terms result in the expiry payoffs that we saw in module Option Concepts - Payoff Geometry and in the risk profiles that we summarised in section *Summary of Greeks*.

With an exotic option either one (or more) of these terms is defined differently or there are additional contractual clauses that are not present in the vanilla contract. As a result:

- The expiry payoffs of exotic options cannot be exactly replicated with combinations of vanilla options, or with other traditional financial instruments

- Their Greeks typically display different behaviours from the ones we summarised for vanilla options, making these instruments in many cases very difficult to delta-hedge

1.1. The Dividing Line

As with all definitions, the distinction between a vanilla and an exotic option sometimes becomes blurred. Some options appear 'exotic' at first sight, but on closer inspection their payoff may be replicated reasonably closely with vanilla products. Notable examples of this are:

- **Bermudan options**: options that may be exercised on specific dates during the contract life. They are also known as mid-Atlantic options because they are a half-way house between American options (which can be exercised on any date) and European options (which can only be exercised at expiry). Bonds that are callable only on their coupon dates are a typical example of a Bermudan option (see Callable Bonds - Structure).

  Although the price of a Bermudan is somewhere in between the price of its equivalent European and American options, its Greeks behave like its relatives and therefore Bermudans are typically not classified as exotic.

- **Combination strategies**: we should not confuse exotic options with packages that are just combinations of conventional options. In Options Strategies we saw how conventional options may be bundled to produce more complex payoff profiles, for example:

  - A long call spread, which reduces the premium cost by capping the upside but is just a combination of long a call and short a further OTM call

  - A long straddle, which allows the investor to profit if the market moves either up or down is just a combination of long a call and long a put

  These packages may be analysed (and priced) as the sum of their generic components, which are conventional options.

- **Deferred-premium contracts**: in these contracts the seller (typically a bank) allows the buyer to pay the option premium:

  - In instalments (see for example Interest Rate Options - Caps, Floors & Collars); or

  - In one lump-sum at the expiry of the options (sometimes called **break-forward contracts**)

  With a deferred-premium contract, the option seller effectively lends the buyer the premium funds for a specified time. There is nothing really exotic about this feature: the cost of a deferred-premium option is simply the spot price of the option plus an interest charge.

- **Chooser options**: give the buyer the right to decide, by a given date (the **choose date**) whether the option is a call or a put. Like straddles (see Options Strategies – Volatility Trading) choosers are attractive to investors who expect higher price volatility but do not yet have a view on future market direction. However:

    - With a straddle the investor retains the right to call or put the underlying right up to the expiry date

    - With a chooser the investor loses one of these rights after the choose date

  The chooser should therefore be cheaper than the straddle - and this is one of its attractions[1].

[1] **How can you price an option when you don't know whether it is a call or a put?**

On closer inspection, it turns out that the chooser's features may be replicated (therefore priced) reasonably well by a combination of two conventional options. The pricing of choosers is outside the scope of the IFID Certificate programme, but for your future reference, the example illustrates how this is done.

Example
An investment bank issues a chooser on the following terms:

Style: European
Strike: \$100 (= ATM forward)
Expiry: 3 months
Choose date: 1 month

The investor has 1 month to decide whether the option is to be a call or a put. Her decision will be based on which of the two options has a higher market value by the choose date. That will depend on whether the underlying price is higher or lower than \$100 at that time.

The investor in a chooser gains nothing by choosing before the last possible choose date, and therefore will delay her decision until then.

Hedging Programme
The bank selling this chooser may therefore hedge its risk as follows:

- Buy a conventional 3 month ATM European call
- Buy a 1 month ATM European put

Consider two possible scenarios on the last choose date:

- **Scenario 1**: the underlying price rises above \$100. Conventional European \$100 calls are therefore more valuable than the equivalent puts, so the investor chooses her option to be a call.

  At that point the 1 month put in the bank's hedge book expires OTM, leaving just the longer dated call which mirrors exactly the investors' choice.

- **Scenario 2**: the underlying price falls below \$100. Conventional \$100 European calls are therefore less valuable than the equivalent puts, so the investor chooses her option to be a put.

At that point the 1 month put in the bank's hedge book expires ITM, creating a short position in the underlying. The combination short the underlying and long the calls gives the bank a synthetic long put position in its hedge book (see Option Pricing - Put-call Parity), which again mirrors exactly the investors' choice!

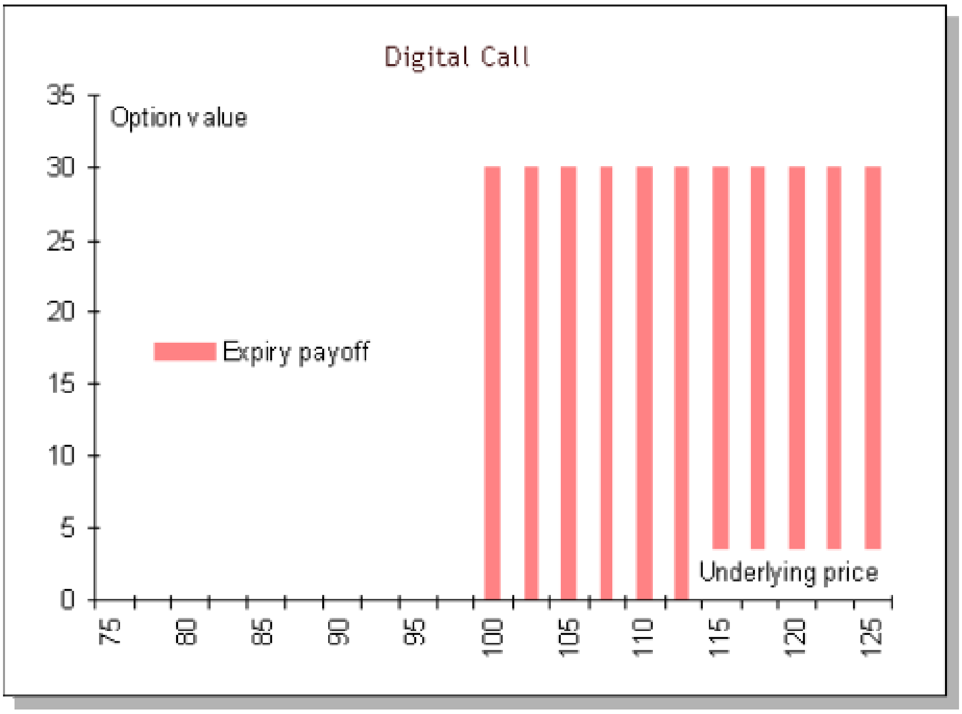
2. Binary Options

Cash-or-nothing binary options pay a fixed amount of cash if they expire in the money, no matter how deeply, otherwise nothing.

Call payoff = { Cash payout if Spot > Strike, else 0 }
Put payoff = { Cash payout if Spot < Strike, else 0 }

Also known as: **Digital options**, **All-or-nothing options**, **Bet options**.

Cash-or-nothing binaries differ from conventional options in that their payoff does not depend on the extent to which the option is ITM, only on whether it is ITM. Binaries are a bet on the market reaching a certain level, with a fixed 'prize' (the cash payout) if the bet is won, otherwise nothing. The figure below illustrates the profile of a digital call with a strike of $100 and a cash payout of $30.



For an equivalent binary put the payout would $30 if the spot is below $100, otherwise zero.

2.1. Applications

Although binaries are traded as standalone products, they are often found embedded in structured securities.

Example - Accrual Note

Situation
Sterling interest rates are gradually converging down to European levels, as the prospects of sterling joining the Euro improve, and sterling investors are searching for ways of maintaining yields in their money market portfolios. A top-name investment bank offers the following sterling note:

Maturity:	12 months (365 days)
Coupon:	12 LIBOR + 0.45%
Interest calculation:	Interest on the note shall accrue daily on an Actual/365 basis provided the daily 12 month LIBOR London fixing remains between 2.00% and 7.00%. Interest will not accrue on any day(s) that the 12 month LIBOR fixing is lower than 2.00% or higher than 7.00%

Analysis

The coupon rate on the note is fixed on the issue date and remains fixed for 12 months. A top-bank normally pays LIBOR flat, so LIBOR + 45 sounds too good to be true. The 'catch' is that every day the issuer will check the 11:00 LIBOR fixing in London, and if this is lower than 2.00% or higher than 7.00% then the investor loses that day's accrued interest.

Note that this is not the same as a **collared note** - i.e. an FRN with a conventional interest rate collar (see Interest Rate Options - Caps & Floors). In an equivalent collared note the investor would accrue interest every day, subject to a floor rate of 2.00% and a cap rate of 7.00%.

Suppose the coupon on this accrual note is fixed at 6.20% - i.e. 12 month LIBOR of 5.75% plus 0.45%. For a client investing GBP 1 million, each day of accrued interest is worth:

Accrued interest = 1,000,000 x 0.062 x 1 / 365
= **GBP 169.86**

This is the value that could be lost on any business day that the London LIBOR fixes outside the specified range. In effect, the investor has:

- Deposited GBP 1 million at LIBOR flat

- Sold the bank a strip of 262 binary calls on the reference LIBOR with a strike of 7.00%, each option expiring on a different business day and paying GBP 169.86 if ITM (options expiring on Mondays pay 3 times this amount)

- Sold the bank 262 binary puts on the same reference LIBOR with a strike of 2.00%, each option expiring on a different day and paying GBP 169.86 if ITM (or 3 times this amount for options which expire on Mondays).

For the 524 options sold (or 262 binary strangles) the investor receives a deferred premium of GBP 4,500 - i.e. 0.45% of the principal - payable at maturity of the note.

Accrual structures may be designed to be contingent on any reference market price - an interest rate, currency rate or equity price. Other examples include:

- **Accrual swaps**, similar to accrual notes, where the fixed side of the swap accrues interest only on days when the floating rate is below or above a specified level (or within or outside a specified range)

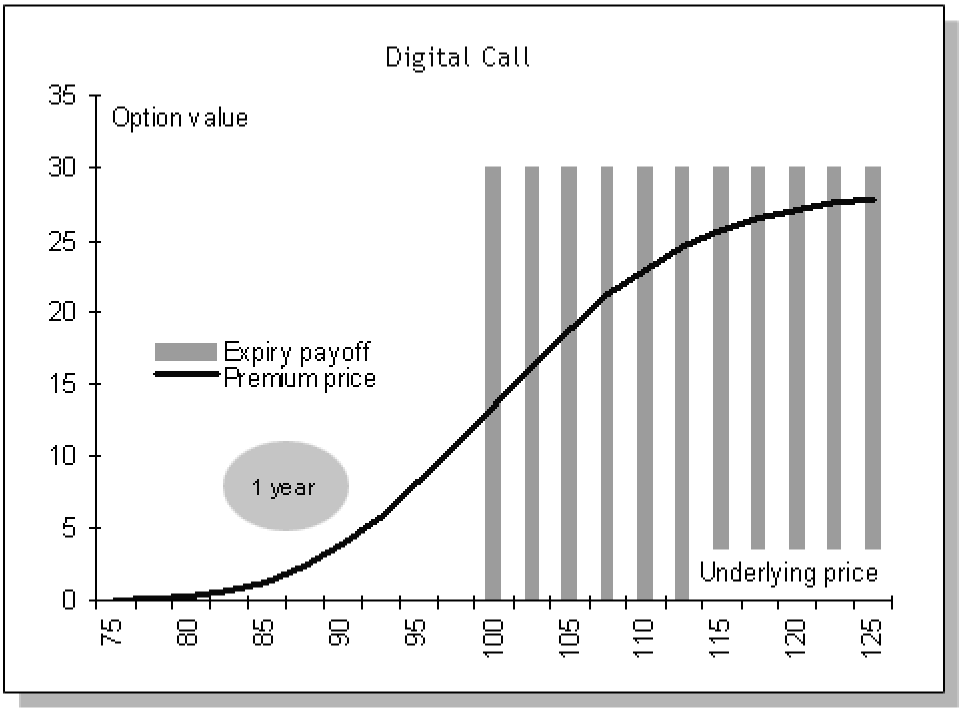
- **Supershares**, equities which pay a fixed bonus, in addition to dividends, if the share price trades above a certain level by some future date.

2.2. Pricing

Mathematically, European style binaries are easier to price than conventional options:

Digital price = PV { Fixed payout x Probability of reaching the strike }

What makes the digital option exotic is its risk profile, in particular the behaviour of its delta. The figure on the next page shows the price of a European digital call for different levels of the underlying, as the option approaches expiry



Risk Profile

The stepped expiry payoff means that the delta of an ATM option converges to infinity as it approaches expiry, while the delta of an OTM or ITM option converges to zero. A close-to-the-money binary that is approaching expiry becomes almost impossible to delta-hedge. Option traders are therefore careful to:

- Limit the amounts of binaries they carry in their books
- Build significant safety margins into their pricing, to allow for the additional risks - a good example of the difference between theoretical pricing, based on a mathematical model, and actual pricing based on the potential costs of hedging the risks in practice

An exotic option is a contract whose 'Greeks' behave differently from those of a conventional option (see Option Risks).

Whereas a conventional long call position is always theta-negative and vega-positive, the equivalent position in a binary may be positive or negative in theta and vega, depending on whether it is OTM or ITM.

In practice, binaries are sometimes priced off (and hedged with) conventional call spreads or put spreads. Although the expiry payoff of the binary call above cannot be replicated exactly using conventional options, its risk profile approximates that of a conventional call spread with strikes very close to $100. For example, the delta profile of a short position in this binary call may be hedged with a long position in six 97.5 - 102.5 call spreads. At expiry:

Maximum payoff on the call spreads = 6 x (102.5 - 97.5)
= **$30**, the same as the binary.

2.3. Variations

There are two varieties of binaries worth noting:

- **American style binary**: as with all American options, its market price may not be lower than its intrinsic value, so the option's price rises even more steeply than the European equivalent as it approaches the strike. As a rule of thumb, the cost of an American OTM binary is approximately twice cost of an equivalent European OTM binary.

  When it is ITM, the American binary has no more time value and should be exercised immediately. In section *Barrier Options* we explain how this product is actually created.

- **Asset-or-nothing binary**:
  - Asset-or-nothing binary call: if the option is ITM, it gives the holder the right to buy the underlying asset at a specified discount to the strike

  - Asset-or-nothing binary put: if the option is ITM, it gives the holder the right to sell the underlying asset at a premium to the strike.

Asset-or-nothing binaries (also known as **gap options**) are a combination of a conventional option and a cash-or-nothing binary.

Example

Style:	European
Strike:	$100
Expiry:	3 months
Delivery:	The buyer has the right to acquire the underlying asset at a price of $70 if the option expires ITM.

This structure is a combination of:

- Long a conventional European call with a strike of $100

- Long a binary call with a strike of $100 and a cash payout of $30; this reduces the net cost of the strike on the conventional option to $70.

3. Barrier Options

The payoff of a conventional option depends only on the price of the underlying relative to the strike at the time of exercise, but there are so-called **path-dependent** options whose payoffs also depend on the history of the underlying price - i.e. where the market has been before expiry. One important class of this type is the barrier option.

- **Knock-out** option: the option contract becomes nulled if the underlying spot or cash price reaches a pre-specified barrier level (the **outstrike**) at any time during the contract's life

- **Knock-in** option: becomes effective only if the underlying spot price reaches a specified barrier level (the **instrike**) during the contract's life

What makes this class of option exotic is the barrier clause that is additional to the terms of an otherwise conventional option contract. An example will help to clarify how it works.

Example

Type: Knock-out call
Strike: $100
Expiry: 3 months
Outstrike: $80

This option pays off just like a conventional option, except that if at any time during its life the cash market price touches or falls below $80 then the seller is relieved of its contractual obligations, even if the option subsequently expires ITM.

3.1. Contract Types

In all, there are 8 basic variations of barrier option - 4 for calls and 4 for puts:

Option	Name	Barrier
Call	Knock-out (KO)	Outstrike is OTM
	Kick-out (or reverse KO)	Outstrike is ITM
	Knock-in (KI)	Instrike is OTM
	Kick-in (or reverse KI)	Instrike is ITM
Put	Knock-out (KO)	Outstrike is OTM
	Kick-out (or reverse KO)	Outstrike is ITM
	Knock-in (KI)	Instrike is OTM
	Kick-in (or reverse KI)	Instrike is ITM

Starting from these 8 simple variations there are many other variations possible;
for example:

- Some contracts have more than one barrier - e.g. a **double knock-out** option knocks out if either a higher or a lower outstrike is reached

- Some barrier options knock in or out depending on the performance of a different market. An example of this type is the **soft call provision** embedded in many Euroconvertible bonds, which gives the issuer the right to call the bond if the underlying shares reach a specified threshold level (see Convertible Bonds - Structure)

Example: Knock-out Call with Rebate

Barrier contracts sometimes include a **rebate clause**, as in this example.

Type: Knock-out call
Strike: $100
Expiry: 3 months
Outstrike: $80
Rebate $5

A rebate of $5 means that the option seller would be obliged to pay the buyer a one-off fee of $5 if the option were to be knocked out[2].

? Which of the following do you think is true of a knock-out call, other things being equal?

Question 1

a) Select one or more from this list.

- ☐ The larger the rebate, the more expensive is the contract
- ☐ The larger the rebate the cheaper is the contract
- ☐ The lower the outstrike the cheaper is the option
- ☐ The lower the outstrike the more expensive is the option

3.2. Applications

Unless the rebates are very large, barrier options are typically cheaper than otherwise equivalent conventional options (since a barrier option can never perform better) and this is their main appeal to investors.

Barrier options are one of the most widely-traded classes of exotic option. Knock-outs are more popular than knock-ins, perhaps because investors are reluctant to pay for something which does not yet (and may never) exist. The most popular flavours are knock-out calls and puts. These have the same pay-off as the regular options, except that if the options go sufficiently OTM to hit the outstrike, they immediately expire worthless.

Example 1: Ladder Structures

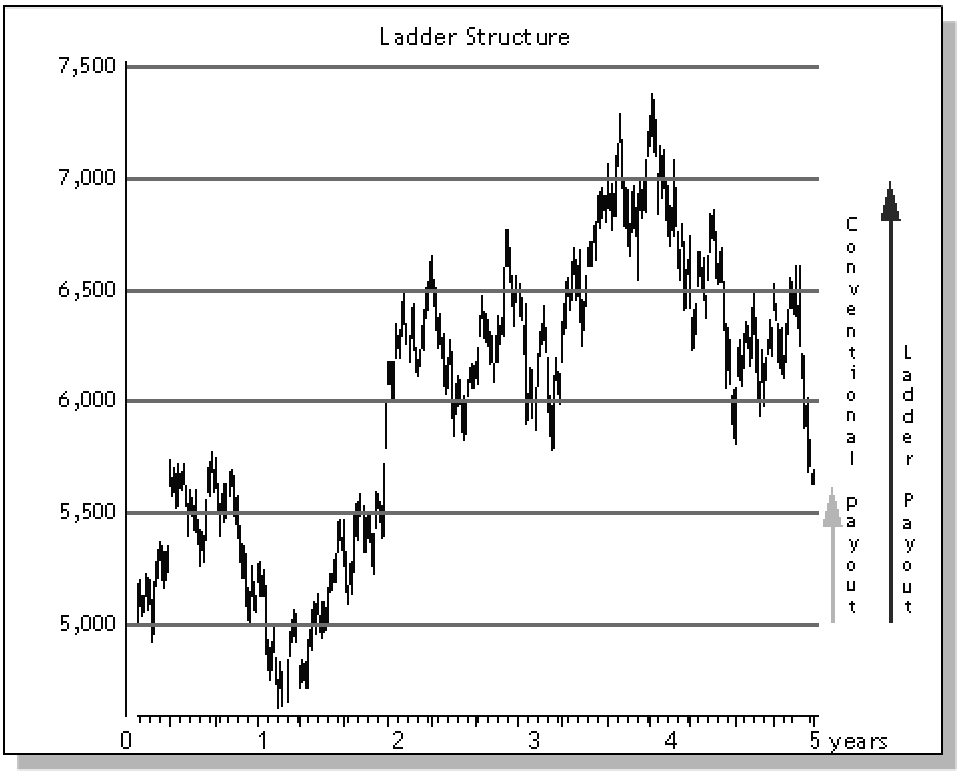
Situation
A client seeks exposure to an equity index through long-dated call options, but is concerned about the possibility of a last-minute market setback, just before the options expire, which could disproportionately reduce any interim gains achieved.

[2] In effect, the rebate clause is just an American style cash-or-nothing digital call added to the simple knock-out structure (see Digital Options - Definition).

Solution
The bank offers its client the following contract:

Type:	Call
Style:	European
Underlying:	The equity index
Strike:	5,000 (= ATM cash)
Expiry:	5 years
Additional protection:	Every 500 points gain in the index achieved during the contract's life will be guaranteed, even if the market subsequently retraces.

This contract features a **ladder structure**, which locks in some of the gains achieved on the option as the market advances. The figure below compares the payoff of the ladder with that of a conventional call in a hypothetical market scenario.



The gains achieved as the market crosses successive 'rungs' are locked in, even if it subsequently falls. The only gain that was not guaranteed in this case was between 7,000 and 7,430 (the all-time high) because the market failed to reach the next rung at 7,500.

- At expiry the ladder call pays 7,000 - 5,000 = 2,500 index points
- A conventional call would have paid only 5,650 - 5,000 = 650 index points.

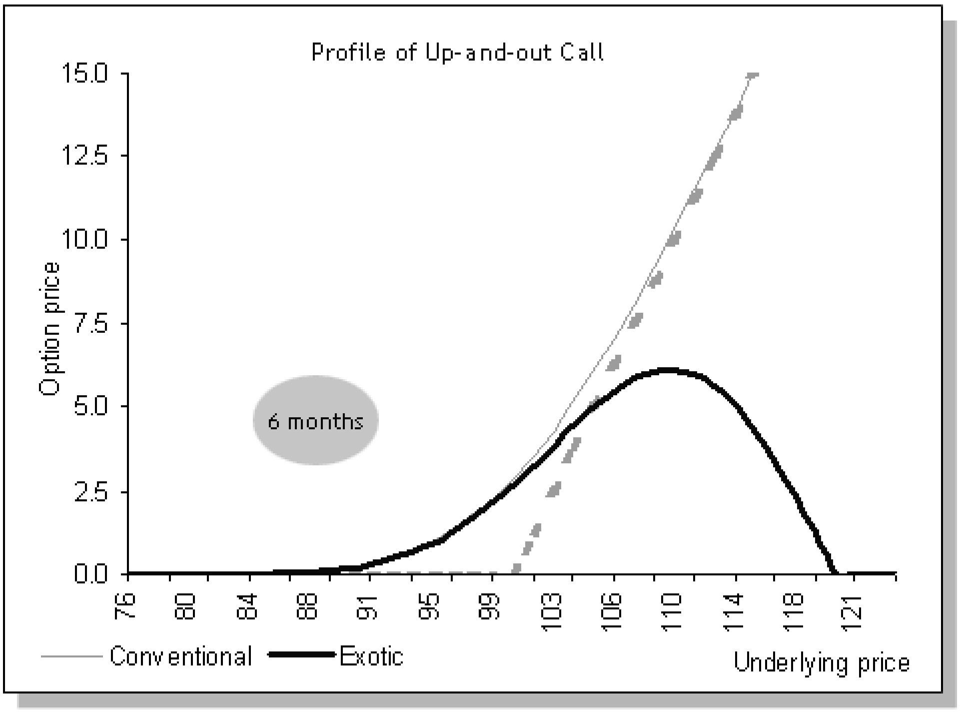
Ladder structures are normally found embedded in the types of investment vehicles described in Structured Option-based Notes – Principal Protected Notes.

Components that make up the ladder structure and costs

The ladder structure is made up of a conventional European call plus a series of barrier options:

- A 5,500 - 5,000 up-and-in put spread with an in level at 5,500
- A 6,000 - 5,500 up-and-in put spread with an in level at 6,000
- A 6,500 - 6,000 up-and-in put spread with an in level at 6,500
- And so on...

Cost of a Ladder Option



As each put spread knocks in, it locks the market level achieved so far. The whole package adds up to quite a number of barrier options, in addition to the conventional call. It must be more expensive than a conventional call because it reduces the downside risk to the investor, without diminishing the upside.

Ladder structures are often found embedded in principal protected notes (see *Structured Option-based Notes - Principal Protected Notes*). These are equity-linked securities that offer investors upside exposure to a selected market, with the guarantee that the principal invested will be repaid in full at maturity, even if the market perform poorly. Typically, these notes have maturities of 5 years or longer, so investors value the comfort of knowing that not all the gains achieved over the term could be lost if the market fell sharply at the eleventh hour.

Example 2
Hedging with knock-out puts.

Situation
An equity investor remains fundamentally bullish, although short-term he perceives some risk of a temporary market setback, if the forthcoming trade figures prove disappointing. The investor would like to protect his equity portfolio with conventional index options, but finds their premium cost prohibitive.

Solution
Knock-out puts might be a lower-cost alternative in this scenario: if the markets continued to rally the puts might cease to exist, but then the risk of a market setback might have receded too.

Example 3
Touch options.

Consider the following contract:

Type: Call spread
Strike: \$100
Touch level: \$120
Exercise date: 3 months

Analysis
This **touch option** - also known as a **capped** or **exploding spread** option - works like this:

- If the cash or spot price reaches the specified touch level at any time during the life of the contract, the holder is entitled to the difference between the touch level and the strike, even if the underlying price subsequently pulls back. The locked-in spread may be deferred until the option's final expiry or paid immediately (**instant one touch**).

- If the touch level is not reached, then the option pays at expiry the difference between the underlying price and the strike (if positive), just like a conventional call spread.

This structure is simply a reverse knock-out call with:

- A strike of $100
- An outstrike of $120
- A rebate of $20

If the touch level of $120 is reached, then the option knocks out but pays the fixed rebate of $20.

An American-style digital option is a knock-out barrier option with positive rebate and a strike equal to its outstrike.

In this example, if the outstrike is set at $100 then either the option expires OTM or pays $20 if the $100 strike is ever hit.

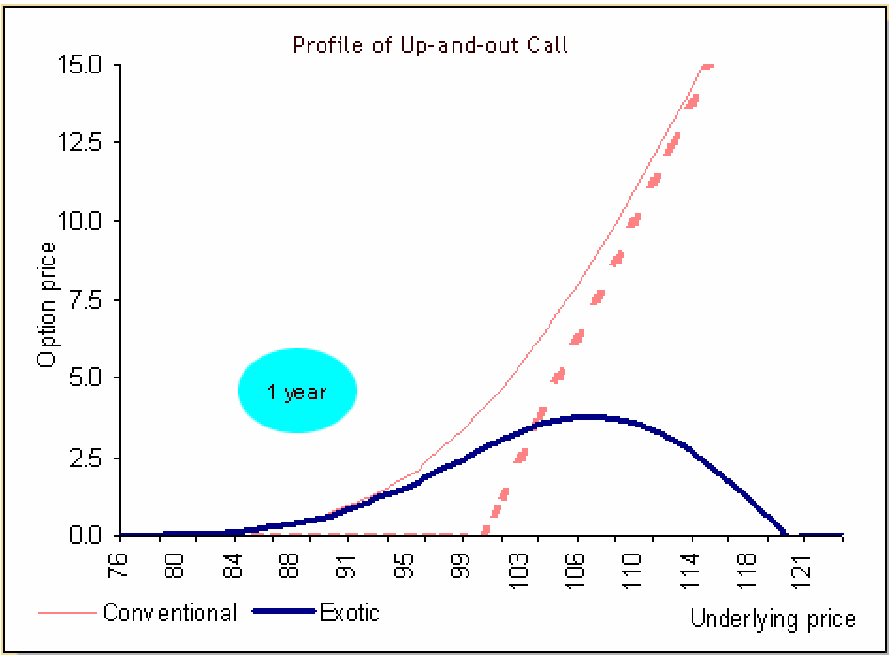
3.3. Pricing

Versions of the pricing models introduced in module Option Pricing have been developed to price barrier options, but a description of these lies outside the scope of the IFID Certificate syllabus.

What is important to note here is some of the 'eccentric' pricing behaviour of these options, especially when the underlying price is close to an instrike or outstrike. As with digital options, what is exotic about these contracts is the behaviour of their Greeks, especially delta. The examples on the next pages illustrate this point.

Example 1: reverse knock-out call

The figure below compares the risk profile of a reverse knock-out call (strike $100 and outstrike $120) with the profile of an equivalent conventional call.



Initially, the price of the kick-out call rises as the option moves into the money. However, as the outstrike is approached the option begins to lose value. The position is delta-positive, if the call is OTM, but delta-negative closer to the outstrike level. Moreover, the value of delta becomes very large close to the outstrike.

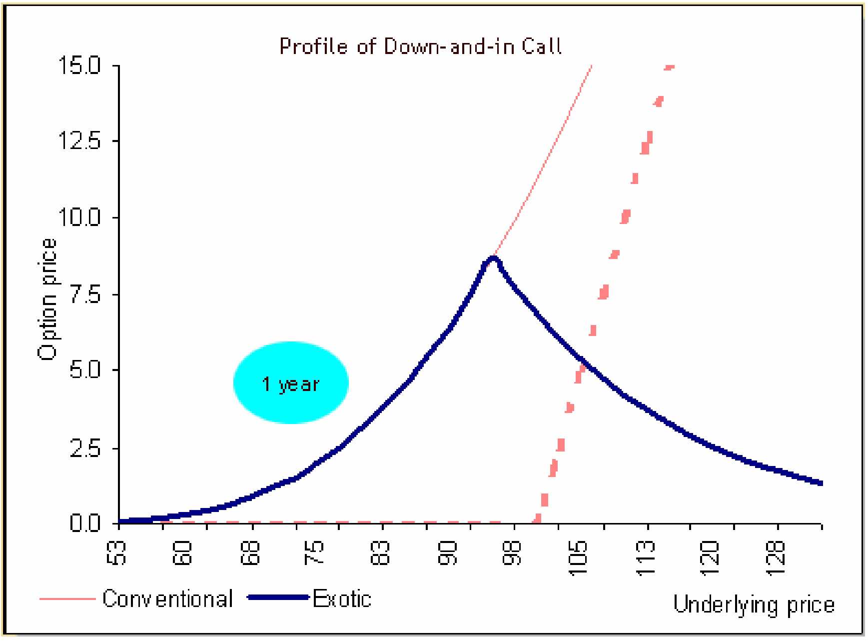
For a trader, such potentially large swings in delta become impossible to hedge dynamically, so it is wise to:

- Build a significant profit margin into the quoted option premium
- Never take on very large positions in such options

Whereas a conventional long call position is always theta-negative and vega-positive, the equivalent position in this barrier call may be positive or negative in theta and vega, depending on how close the underlying price is to the out level.

Example 2: reverse knock-in call

Another notable example of a so-called **malignant barrier** is the reverse knock-in call, shown below.



The strike of this call is $100 and the instrike is $95. The option's price rises as it moves OTM and the knock-in level is approached. In fact, until the in barrier is reached the position is delta-negative and the profile looks more like that of a put than a call. Once the instrike is touched, the option becomes a conventional call and its delta swings dramatically to positive.

Therefore, a trader delta-hedging a short position in this option starts off initially with a short position in the underlying but may have to turn the hedge very quickly into a long position, if the barrier is hit. Again, it is wise not to take on sizeable positions in such options as it may be very difficult, in practice, to turn the hedge around so quickly.

Not all barrier options have malignant barriers. For a knock-in call (where instrike > strike) the 'graduation' from a barrier option into a conventional one, as the instrike is touched, results in a much smaller change in delta, as both the strike and the instrike pull the option's price in the same direction. This is an example of a so-called **benign barrier**.

Contractual Issues

As well as the additional risks involved, which may be significant, there are two contractual issues that anyone trading barrier options should consider carefully:

? How frequently will the barrier be tested: hourly, daily or monthly?

A barrier that is tested monthly is less risky than one that is tested hourly, as it is possible that the underlying price may move through the barrier but retrace before the next look-up date.

? What is the reference market price against which the barrier will be tested: the price quoted by one market maker, an official market price or a broad market index?

Obviously, testing the barrier against an official price or broad index makes the contract less vulnerable to temporary market distortions or abuse.

3.4. Exercise

Question 2

In this exercise, we compare the expiry payoff of a knock-out put option with that of a conventional European put with the same strike and expiry. The details of the knock-out put are as follows:

Type: Knock-out put
Strike: $100
Outstrike: $120
Rebate: None

a) Complete the table below, showing the payoff of the two options in different market scenarios.

Underlying Price at expiry	Highest Underlying price	European Put	Knock-out Put
110	125		
90	110		
90	125		

b) Which of the following statements is/are true about a long position in a knock-out put option that has no rebate clause?

- ☐ The knock-out put is always cheaper than the European put
- ☐ The knock-out put is always more expensive than the European put
- ☐ The knock-out put is riskier than the European put
- ☐ The knock-out put is less risky than the European put

4. Asian Options

Asian options flavours:

- **Average price**: at expiry the option pays the difference between the strike and an average of the underlying price achieved during a specified **averaging period** in the option's term

- **Average strike**: the strike of the option is an average of the underlying price over the specified averaging period, and at expiry the option pays the difference between this strike and the underlying market price

Of the two types, the average price option is by far the most common.

4.1. Applications

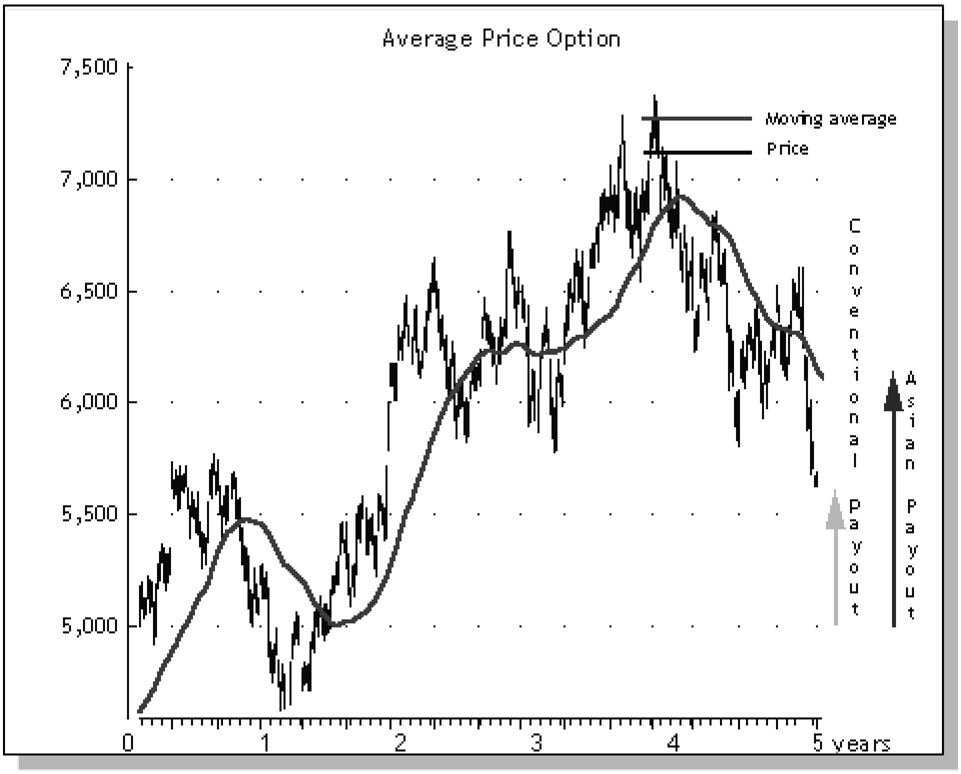
Example - Principal Protected Notes

Average price options are frequently found embedded in **principal-protected notes** (see Structured Option-based Notes - Principal Protected Notes). They offer another way of protecting the investor against a last-minute fall in the market, just before the option's expiry, which could disproportionately reduce any interim gains achieved.

Consider the following contract:

Type:	Call
Style:	European
Underlying:	An equity index
Strike:	5,000 (= ATM cash)
Expiry:	5 years
Expiry payoff:	The seller shall pay the difference between the strike and the arithmetic average of the index level achieved during the final 6 months of the option's term (the **averaging period**).

The figure below compares this option's expiry payoff with that of an equivalent conventional European call in one hypothetical market scenario.



The benefit to the option buyer is that any last-minute market setback is averaged up; the drawback, of course, is that any last-minute market advance would likewise be averaged down!

Average price options also appeal to investors in thinly traded markets, since manipulation of the underlying price close to the expiry date will have little effect on the average.

The terms of an Asian option must specify:

- **The averaging period** - e.g. the last six months, the entire term of the option or the closing prices at month-ends
- **The sampling frequency** - e.g. the daily, weekly or monthly closing prices during the averaging period
- **The averaging method** - e.g. a simple arithmetic average, a geometric average or some weighted average.

4.2. Pricing

There are three main approaches to pricing Asian options:

1. European-style options based on geometric averages can be priced by adapting the models discussed in Option Pricing - Analytic Models. This is because if the underlying price is assumed to be log-normally distributed then its geometric average is also log-normal.
2. There is no equivalent solution for options based on arithmetic averages, because even if the underlying price is log-normally distributed, the arithmetic average is not. However, various analytic approximations have been developed which work reasonably well.
3. Any Asian option, no matter what its style or averaging method, may be priced by Monte Carlo simulation, but this is computationally much more intensive (see Option Pricing - Monte Carlo Model).

The volatility of an average is always less than the volatility of the price itself; the longer the averaging period, the lower is its volatility.

Whatever pricing method is used, average price options come out very much cheaper than conventional ones. If you compare the price and the moving average curves in the PPN example on the previous page, you can see that the moving average (the 'underlying' in an Asian option) smooths out spikes in the market price, so the option's price is correspondingly reduced.

5. Forward Options

In a forward option the strike is set at some specified future date, rather than on the effective date (i.e. when the premium is payable)

Also known as: **Forward-start options**, **Delayed-start options**

The actual strike will not be known until the future **effective date**, but the contract does specify in advance what the strike will be relative to the underlying market at the time it is set - e.g. ATM, or 10% OTM.

In other words, the contract specifies the option's future **parity ratio** (= Spot / Strike) at the outset. As we shall see, this provides the key to pricing these options.

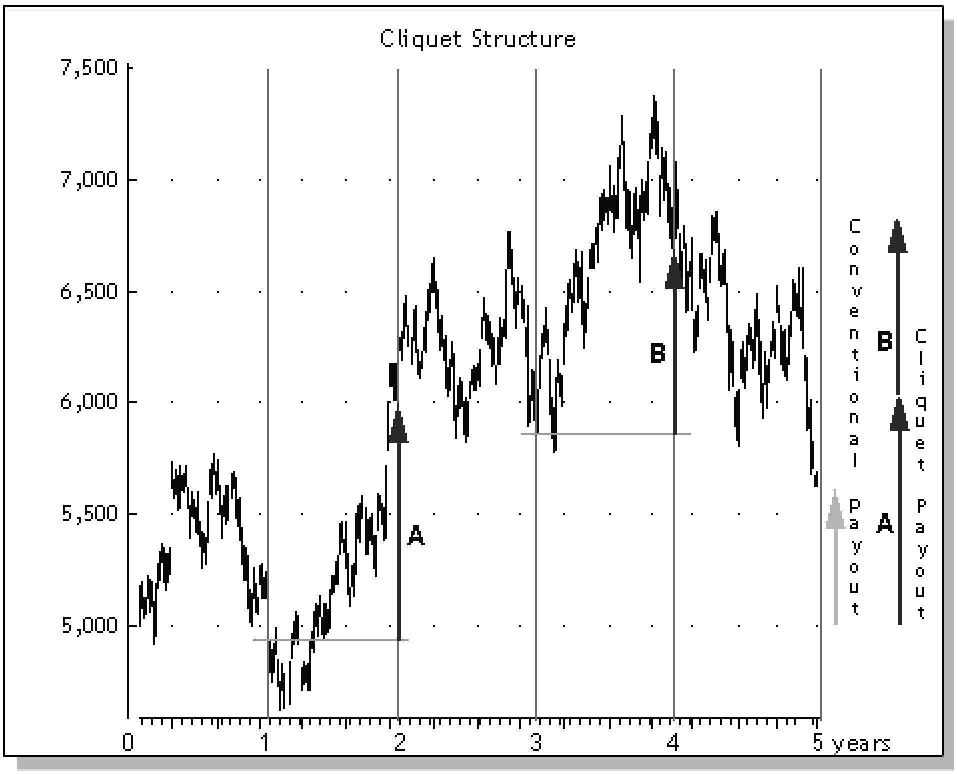
5.1. Applications

Example – Cliquets

Forward options are found in structures known as **cliquets**[3], **resetting options** or **moving strike options**. As the name suggests, the structure locks in any gains achieved as a result of favourable market movements by resetting the strike up or down at regular intervals, in line with the underlying market.

Like ladder structures and Asian options, cliquets are sometimes embedded in principal-protected notes to shelter investors from possible last-minute market setbacks (see Structured Option-based Notes - Principal-protected Notes).

The figure below shows the payoff of a 5 year cliquet call on an equity index, where the strike is reset ATM every year.



- Initially, the option is struck at the current index level of 5,000, but in year 1 the market finishes down at 4,900, so the strike is moved to this new level.

[3] Cliquet is French for ratchet.

- In year 2 the market closes at 6,000, so the investor locks in a gain:

  A = 6,000 - 4,900
  = 1,100 index points.

- At the start of year 3 the strike is raised to 6,000 but during the year the market closes down at 5,800, so the strike is moved back to this level without any loss to the investor.

- In year 4 the market rises again to close at 6,650, so the investor locks in a gain:

  B = 6,650 - 5,800
  = 850 index points.

- In year 5 the market finishes sharply down at 5,650, but the investor keeps all the gains achieved previously:

  Cliquet payoff
  = 1,100 + 850
  = **1,950 index points**.

The cliquet performed significantly better than an otherwise equivalent conventional call in the same scenario, which would have paid only 650 index points!

Analysis
The cliquet structure in this example is made up of:

- A conventional 1 year European ATM call
- A series of forward-start ATM calls (parity ratio = 1), each of which becomes effective on an anniversary date and expires a year later.

Payment on the cliquet, if any, is typically made at each reset date, but it may be deferred until the maturity of the whole structure.

5.2. Pricing

The buyer of a forward option pays a premium today for an option whose strike will be determined at some future date.

? How can you price an option whose strike you do not yet know?

The trick to this pricing problem relies on the fact that the option's parity ratio is specified in advance.

For a given parity ratio, an option's premium is proportional to the underlying spot price (and strike).

In other words, other things being equal, if you double both the spot and the strike, while keeping the parity ratio constant, then the option premium doubles as well. This makes the profit/loss on a forward option position proportional to the underlying price and suggests a simple programme for hedging its risks until the effective date, when the strike is set.

Once you know how to hedge the derivative then you know how to price it!

Example

A client asks you to quote for the following contract:

Type:	Call
Style:	European
Strike:	ATM
Expiry:	1 year
Effective date:	6 months

At the time, the breakeven price for a vanilla 1 year ATM call (i.e. effective spot) is $3.50 at an implied volatility of 10%.

Solution
Assuming that in 6 months' time the 1 year ATM call will still be trading at 10% implied, then the forward option may also be quoted at $3.50 on a breakeven basis.

The hedging programme is as follows: if the client buys the contract at the quoted price, then the trader hedges her risks by using all the premium received to fund a long position in the underlying.

The outcome

- **Scenario 1**: the underlying price rose 50% so on the effective date the option's strike is set at $150. At this point, if the 1 year ATM call trades with 10% volatility, then its premium price will also be 50% higher - i.e. $5.25.

  In this scenario, the client made a profit of $1.75 on the trade (= 5.25 - 3.50) and the trader's option position will show a corresponding loss. However, the trader spent $3.50 to buy the underlying when it was trading at $100 and now the hedge is worth 50% more - a gain of $1.75 that covers exactly the losses made on the option!

- **Scenario 2**: the underlying price fell 50%, so on the effective date the option's strike is set at $50. If the 1 year ATM call trades with 10% volatility, then its premium price would also be 50% lower - i.e. $1.75.

  In this scenario, the client made a loss of $1.75 on the trade (= 1.75 - 3.50) and the trader's option position will show a corresponding gain. However, the trader spent $3.50 to buy the underlying when it was trading at $100 and now the hedge is worth 50% less - a loss of $1.75 that matches exactly the gains made on the option!

The hedging programme for a short forward put position is the same as that for a short forward call.

In both cases the trader would lose out if the price of the underlying were to rise, as both types of option would become more expensive.

On the option's effective date, the strike is set and the trader replaces this static hedge with a conventional dynamic delta-hedging programme such as we discussed in module Conventional Options - Delta.

The key assumption: forward volatility

Conceptually, a forward option may be priced as if it was a conventional (spot-start) contract. What makes the forward option exotic is the fact that the trader does not know, on the trade date, whether the option she is pricing will be trading with the same implied volatility as the conventional option does today.

In our example (in the previous section) we assumed that this was indeed the case, but the chances are that it will not be. So the main risk with forward options is getting the assumed volatility wrong; there may be significant vega risk in pricing these contracts.

It is possible to derive a forward volatility from the volatility curve, just as we can derive a forward yield from the yield curve, and it is also possible to lock into forward volatilities today using combinations of vanilla options. However, these techniques lie outside the scope of the IFID Certificate syllabus and will therefore not be covered here.

6. Multi-asset Options

Multi-asset options are options whose payoffs at exercise depend on the performance of more than one underlying market.

Below we describe the 4 major types of multi-asset option.

1. Basket Option

The underlying is a portfolio of different assets, typically in the same class, in pre-defined amounts. Bond and equity index options are basket options, but many OTC basket options are specially-designed for investors seeking exposure to specific market sectors - e.g. a basket of sterling investment grade corporate bonds.

2. Rainbow Option

Also known as a **"better of" option**, this entitles the holder to receive the gain on the best performing of two or more assets. A typical payoff might be:

Exercise payoff = MAX { A1, A2, A3, ... An, 0 }

Where A1 ... An are the gains in the prices of the n assets represented in an **n-coloured rainbow**, typically expressed in percentage terms.

Here the payoff is based on the absolute performance of the best-performing asset, if positive, rather than on its performance relative to the other assets, as in the spread option.

3. Exchange Option

Also known as an **out-performance option** or **spread option**, this gives the holder the right to receive one asset (A1) in exchange for another asset (A2) according to a specified **conversion ratio**, or the right to receive the amount by which the price of one asset exceeds that of another. A typical payoff might be:

Exercise payoff = MAX { A1 - A2, 0 }

Where A1 and A2 are the market values of the two underlying assets on the exercise date.

Here, the payoff depends on the extent to which the performance of A1 exceeds that of A2, irrespective of whether both asset prices have moved up or down (whereas in the case of the rainbow option the payoff depends on absolute best performance). This is equivalent to an option to exchange the lower-valued asset for the higher-valued one, or to go long the higher-valued asset and pay for it by going short the lower-valued one - i.e. an option to enter into a spread position.

6.1. Applications

Multi-asset options are ideal for implementing relative value plays. Typical underlying assets might include:

- Two different market indices in the same asset class - e.g. the relative performance of a US corporate bond index against that of a European index

- A single security and its market index - e.g. the performance of the 3.75% General Motors of 2010 relative to the Lehman Brothers US corporate bond index

- Prices or indices in different asset classes - e.g. the performance of a Government bond index relative to that of the main equity index for that country

The examples on the following pages illustrate typical applications of multi-asset options.

Example 1: Convergence Plays

Situation
An investment client believes that sterling interest rates will gradually converge down to European levels, as the prospects of sterling joining the Euro improve, and would like to profit from this view. An investment bank offers the client the following contract:

Expiry: 12 months
Reference rate: GBP 5 year swap rate - EUR 5 year swap rate
Expiry payoff: MAX { 1.25% - Reference rate, 0 }

Analysis
This is a spread option with a strike of 1.25%, which may be the current difference between the 1 into 5 years forward GBP swap rate and the equivalent forward EUR rate - i.e. the option is ATM forward (for a description of forward swaps see Interest Rate Swaps - Forward Swaps).

Example 2: The 20/20 Option

Rainbow options are particularly useful as asset allocation tools, ensuring that the investor always ends up in the best-performing asset or sector.

Situation
A fund manager believes the US and some European bond markets have considerable upside potential, although there are also risks of possible setbacks.

The manager would like to profit from this anticipated scenario but the rules of the fund do not allow him to gear up the fund by placing bets on all markets at the same time. An investment bank offers the investor the following 3-coloured rainbow option with a strike of 0:

Expiry: 12 months
Payoff: 0.75 x MAX { %Rise in Lehman Brothers US Treasury Bond Index,
%Rise in JP Morgan German Government Bond Index,
%Rise in FT-Actuaries UK Government Bond Index,
0% }

Example 3: Convertible Bonds

A **convertible bond** gives its holder the right to exchange the principal of the bond certificate, plus all its remaining unpaid coupons, for a specified number of alternative securities from the same issuer - typically equity.

An **exchangeable bond** gives its holder the right to exchange the principal of the bond certificate for a specified number of alternative securities from a different issuer.

In Convertible Bonds - Structure we examine the key structural features of convertible and exchangeable bonds and we look at various ways in which the market prices these securities. Here, we shall explain how the conversion feature may be interpreted and priced as an exchange option.

Situation
Consider the following issue:

Issuer	HH Capital Inc.
Coupon	3.75% (annual, 30/360 basis)
Issue date	25 Sep 2002
Maturity	25 Sep 2012
Denomination	USD 5,000
First interest date	25 Sep 2002
Convertible from	10 Oct 2002 - 11 Sep 2012
Conversion ratio	76.9231
Call Features	None

Analysis
There are two assets embedded in this security:

- 76 ordinary shares of HH Capital - let's call them A1

- A straight 3.75% coupon 15-year bond certificate with a face value of USD 5,000 - let's call this A2

Since the investor has the right to exercise at any time during the conversion period, the convertible should be worth at least as much as the most valuable of these two assets in the current market:

Convertible's price $\geq$ MAX { A1, A2 }
$\geq$ A2 + MAX { A1 - A2, 0 }

This expression suggests one way to price a convertible:

- Estimate what would be the price of a straight 3.75% coupon 15-year bond issued by HH in the current market - i.e. price a security like A2 to yield the same as any other straight bond issued by the same type of credit

- Add this to the calculated value of an exchange option that pays MAX {A1 - A2, 0}

Convertible's price = Price of straight bond + Price of exchange option

6.2. Pricing

All of the options described in this section may be priced using variations of the analytical models discussed in Option Pricing.

A detailed description of multi-asset option pricing models is outside the scope of the IFID Certificate syllabus. However, you should be able to explain the way that correlation between the prices of the underlying assets in a multi-asset option affect its market value.

1. Basket Option

In a basket option the underlying is simply a portfolio of assets and the option can be priced using a standard model, provided we know what is the volatility of the basket.

Where the basket options are widely traded - e.g. in index options - the volatility of the basket is already implied in the market premium prices, but where this is not the case it can be derived from the volatilities and correlations of the constituent assets using the standard portfolio risk formula that we shall introduce in module Portfolio Construction - Risk & Return.

How to calculate the volatility of a basket

For a two-asset basket of securities **A** and **B**:

Volatility of basket (σ_P)

$$= \sqrt{ \{ (w_A \times \sigma_A)^2 + (w_B \times \sigma_B)^2 + (2 \times w_A \times w_B \times \sigma_A \times \sigma_B \times \rho_{AB}) \} }$$

Where:
w_A = Proportion of the portfolio invested in asset A - the ' weight' of A
w_B = Proportion of the portfolio invested in asset B - the ' weight' of B
σ_A = Standard deviation ('risk') of the return on asset A (in percent)
σ_B = Standard deviation of the return on asset B
ρ_{AB} = Correlation coefficient of the returns on assets A and B

Basket options are normally cheaper than options on single assets, as the volatility of the basket is typically lower than that of its components. This is one reason why basket options appeal to investors.

The higher the correlation between the assets, the higher is the volatility of the basket, hence the price of the basket option.

2. Rainbow Option

Rainbow options are very complex products typically priced by Monte Carlo simulation and a discussion of these models lies outside the scope of the IFID Certificate syllabus. However, as you would expect rainbows are:

- More expensive than conventional options on any one of the assets represented in the rainbow, because there is a higher probability that at least one of the assets will perform well

- More expensive than a basket option on the same assets, because the performance of the worst-performing assets is included in the value of the basket, whereas it is excluded from the rainbow's payoff

- Cheaper than a portfolio of calls on each of the assets, because only the best-performing asset is considered in the rainbow; the other ones are eliminated from the payoff calculation, even if they perform well.

The higher the correlation between the assets, the lower is the price of the rainbow option.

High positive correlation means that if the best-performing asset performed well, then the other assets are also likely to have performed well. Since only the best-performing asset is considered in the payoff calculation, the opportunity cost of exercising into this asset is higher. Put in a different way, with negative correlation it is more likely that at least one of the assets will perform well, whereas with positive correlation it is more likely that none of the assets will perform at all.

3. Exchange Option

Compare the payoff of a conventional call with that of an exchange option:

- When an investor exercises a conventional call, she receives the underlying asset (A1) in exchange for payment of the strike which has a fixed cash value:

  Conventional call payoff = MAX{ A1 – Strike, 0}

- When the investor exercises an exchange option she receives one of the assets (A1) in exchange for payment in another asset (A2):

  Exchange call payoff = MAX{ A1 – A2, 0}

  So the strike in an exchange option (A2) is also a variable with an uncertain future value

? How can you price an option whose strike is uncertain?

One solution, proposed by William Margrabe, relies on a key property of option pricing models that we already saw in section *Forward Options*:

For a given parity ratio, an option's premium is proportional to the underlying spot price (and strike).

In other words, other things being equal if you double both the spot and the strike while keeping the parity ratio constant, then the option premium doubles as well.

This makes it possible to transform the standard option pricing formula as follows:

Call price
= Function of { A1, Strike, Expiry, Volatility, Yield, Funding rate }
= Strike x Function of { A1/Strike, 1, Expiry, Volatility, Yield, Funding rate }

Applying this transformation to an exchange option (where Strike = A2):

Exchange option price = Price of A2 x Price of call on A1/A2 with strike of $1

Now we can use a standard option pricing model to price a call on the variable A1/A2 with a strike of 1. The volatility of A1/A2 is the same as that of a spread position - long A1 and short A2 - and can be estimated from the volatilities of these two assets in question and the correlation between their prices.

The higher the correlation between the assets prices (or yields) the lower is the price of an exchange option.

High correlation means there is a low probability that the two asset prices will diverge significantly, so the volatility of the spread is low.

6.3. Correlation Risk

Correlation risk: the risk that the market value of a derivative may change as a result of changes in the price correlation between its various underlying instruments.

$$\textbf{Correlation vega} = \frac{\textbf{Change in option price}}{\textbf{1\% point change in each of the correlation coefficients}}$$

NB: there are n-1 correlation vegas for n assets in a multi-asset option.

Correlations are Unstable

As we have seen, a key factor in pricing multi-asset options is the assumed price correlation between the different assets represented, as well as their respective volatilities.

Since correlations are notoriously unstable over time - even more so than volatilities - these options are difficult to price with confidence. This, more than anything else, is what makes multi-asset options exotic.

7. Exercise

7.1. Question 1

Question 3

You are NOT required for the IFID Certificate exam to perform valuations on exotic options.

However, this exercise involving the use of some exotic valuation models may help you gain a closer understanding of the ways in which some of the structures introduced in this module respond to changes in underlying market conditions.

In this first question we explore the price behaviour of European cash-or-nothing digital options. When you have launched the model, please select the **Digital** worksheet and ensure the following data is correctly specified:

Type	Call
Spot price	100.00
Strike	100.00
Expiry	1.00
Cash payout	10.00
Funding rate	3.00%
Yield	3.00%
Volatility	10.00%

a) What is the premium price of this option? Enter the result in the box and validate.

Please check your calculator settings, above, if your answer does not match!

b) Complete the table below, showing the evolution of two digital calls over time: one with a strike of $100 and the other with a strike of $90.

Expiry	Strike = 110	Strike = 90
1.00		
0.50		
0.25		
0.00		

c) Which of the following statements is/are true?

- ☐ A position long an OTM call is short theta
- ☐ A position short an ITM call is long theta
- ☐ A position short an OTM call is short theta
- ☐ A position long an ITM call is long theta

d) Restore the expiry to 1 year and complete the table below showing the price of the two digital calls (the $110 and the $90 strikes) for different volatility levels.

Volatility	Strike = 110	Strike = 90
10.00		
5.00		
0.00		

e) Which of the following statements is/are true?

- ☐ A position long an ITM call is short vega
- ☐ A position short an OTM call is short vega
- ☐ A position short an ITM call is short vega
- ☐ A position long an OTM call is long vega

f) Restore the volatility to 10%, the strike to $100 and change the expiry to 0.08 years (i.e. approximately 1 month). Complete the table below, showing the sensitivity of the price of this call to changes in the underlying price.

Spot price	Call price	Change in price (i.e. Delta)	Change in delta (i.e. Gamma)
85.00		-----	-----
90.00			-----
95.00			
100.00			
105.00			
110.00			
115.00			

g) Which of the following statements is/are true?

- ☐ A position long an OTM call is long gamma
- ☐ A position long an ITM call is short gamma
- ☐ A position long an OTM call is short gamma
- ☐ A position long an ITM call is long gamma

7.2. Question 2

Question 4

In this question, we explore the price behaviour of European barrier options using an Excel-based model. When you have launched the models, please select the **Barrier** worksheet and ensure the following data is correctly specified:

Flavour	Up-and-out-call
Spot price	95.00
Strike	100.00
Barrier	120.00
Rebate	0.00
Expiry	1.00
Funding rate	3.00%
Yield	3.00%
Volatility	10.00%

a) What is the premium price of this option, and what is the price of an equivalent conventional European call? Enter the result in the box and validate.

Premium up-and-out call

Premium European call

Please check your calculator settings, above, if your answer does not match!

b) Complete the table below, showing the evolution of an OTM and an ITM up-and-out call over time.

Expiry	OTM: Spot price = $95	ITM: Spot price = $105
1.00		
0.50		
0.25		
0.00		

c) Restore the expiry to 1 year and complete the table below showing the price of an OTM and an ITM up-and-out call for different volatility levels.

Volatility	OTM: Spot price = $95	ITM: Spot price = $105
10.00		
5.00		
0.00		

d) Which of the following statements is/are true about a position that is long an ITM up-and-out call?

- ☐ It may be short or long theta
- ☐ It may be long or short vega
- ☐ It may be long or short gamma
- ☐ It may be long or short delta

7.3. Question 3

Question 5

In this exercise, we explore the price behaviour of various multi-asset options using an Excel-based model. When you have launched the Exotic Option Pricing models spreadsheet, please select the **Multi-asset** worksheet and ensure the following data is correctly specified:

Asset 1

Spot price	50.00
Number of units	2.0
Strike	50.00
Yield	3.00%
Volatility	12.00%

Asset 2

Spot price	100.00
Number of units	1.0
Strike	100.00
Yield	3.00%
Volatility	8.00%

Option

Type	**Basket**
Expiry	1.00
Funding rate	3.00%
Correlation	0.00

The basket is made up of:

- 2 units of Asset 1, currently trading at $50 each
- 1 unit of Asset 2, currently trading at $100

The current market value of the basket is therefore $200, 50% of which is allocated to Asset 1 and 50% to Asset 2, and the option is ATM. The model calculates to the nearest 10 cents the price of this option as a percentage of the contract size.

Contract size = Strike of Asset 1 x Units of Asset 1
+ Strike of Asset 2 x Units of Asset 2

a) What is the price of the basket option? Enter the result in the box below and validate.

Please check your calculator settings, above, if your answer does not match!

b) Complete the table below, showing the price of this option assuming different correlations between the assets.

Correlation	Premium price (%)
+0.80	
0.00	
- 0.80	

c) Which of the following statements is/are true of a long position in a basket call option?

- [ ] It is long delta in any of its constituent assets
- [ ] It is long the correlation vega
- [ ] It is long vega in any of its constituent assets
- [ ] It is long theta

7.4. Question 4

Question 6

Please change the fields indicated in bold below in the **Multi-asset** worksheet of the pricing model.

Asset 1

Spot price	50.00
Number of units	2.0
Strike	50.00
Yield	3.00%
Volatility	12.00%

Asset 2

Spot price	100.00
Number of units	1.0
Strike	100.00
Yield	3.00%
Volatility	8.00%

Option

Type	**Exchange**
Expiry	1.00
Funding rate	3.00%
Correlation	**0.00**

This exchange option gives the holder the right to:

- Receive 2 units of Asset 1, which is currently trading at $50
- By delivering 1 unit of Asset 2, which is currently trading at $100

The model calculates the price of this option as a percentage of the nominal value of the asset(s) to be delivered.

Contract size = Strike of Asset 2 x Units of Asset 2

a) What is the price of the exchange option, as a percentage of the underlying asset value, to 2 decimal places?

b) Complete the table below, showing the price of this option assuming different correlations between the assets.

Correlation	Premium price (%)
+0.80	
0.00	
- 0.80	

c) Which of the following statements is/are true of a long position in this exchange option?

- [ ] It is short the correlation vega
- [ ] It is long delta in either of its constituent assets
- [ ] It is short theta
- [ ] It is long delta in the callable asset; short delta in the putable asset

7.5. Question 5

Question 7

Please change the fields indicated in bold below in the **Multi-asset** worksheet of the pricing model.

Asset 1

Spot price	50.00
Number of units	1.0
Strike	50.00
Yield	3.00%
Volatility	12.00%

Asset 2

Spot price	100.00
Number of units	1.0
Strike	100.00
Yield	3.00%
Volatility	8.00%

Option

Type	**Rainbow**
Expiry	**1.00**
Funding rate	3.00%
Correlation	**0.00**

This rainbow will pay the percentage gain in the best-performing of the two assets (a rainbow call).

Contract size = 1 unit of Asset 1 *or* 1 unit of Asset 2

a) Price of rainbow option (%)

b) Complete the table below, showing the price of this option assuming different correlations between the two assets.

Correlation	Premium price (%)
+0.80	
0.00	
- 0.80	

c) Which of the following statements is/are true of a long position in a rainbow call option?

- ☐ It is short theta
- ☐ It is long delta in any of its constituent assets
- ☐ It is short the correlation vega
- ☐ It is short vega in any of its constituent assets