ISMA CENTRE - THE BUSINESS SCHOOL OF THE FINANCIAL MARKETS

UNIVERSITY OF READING ENGLAND

IFID Certificate Programme

Fixed Income Analysis

Answers to Exercises

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1. Time Value Of Money

Question 1

In the table below, please enter your answers in percent to 3 decimal places.

a) What is the effective interest rate payable on a 1 year loan at 5% per annum nominal, when the rate is compounded:

Compounding period	Effective Rate
Annually	5.000
Semi-anually	5.063
Quarterly	5.095

Explanation

Compounding period	**Effective Rate**	
Annually	$(1 + 0.05) - 1$	= 0.05000 or **5.000%**
Semi-anually	$[1 + (0.05/2)]^2$	= 1.05063 or **5.063%**
Quarterly	$[1 + (0.05/4)]^4$	= 1.05095 or **5.095%**

Using the HP17 calculator

Select FIN and ICNV from the main menu and then PERiodic compounding.

Input:

NOM%:	5	5	5
P:	1	2	4

Solve for:

EFF%: **5.000% 5.063% 5.095%**

Using the BAII Plus calculator

To calculate the effective annual rate corresponding to a 5% annual percentage rate that is compounded a) annually, b) semi-annually and c) quarterly, proceed as follows:

Annual: 2nd [IConv] 5 ENTER ↓ ↓ 1 ENTER ↑ CPT
= 0.05000 or **5.000%**, rounded.

Semi-annual: 2nd [IConv] 5 ENTER ↓ ↓ 2 ENTER ↑ CPT
= 0.05063 or **5.063%**, rounded.

Quarterly: 2nd [IConv] 5 ENTER ↓ ↓ 4 ENTER ↑ CPT
= 0.05095 or **5.095%**, rounded.

Question 2

Consider the following loan:

Principal:	$100
Nominal rate:	10%
Maturity:	2 years
Compounding period	Daily (1/365 of a year)

a) What is the effective annual rate on this loan? Enter your answer in percent per annum, rounded to 3 decimal places.

Effective rate 10.516

Explanation

Effective rate $= (1 + 0.10/365)^{365} - 1$
$= 0.1051558$ or **10.516%**, rounded.

Using the HP17 calculator

Select FIN and ICNV from the main menu and then PERiodic compounding.

Input:
NOM%: 10
P: 365

Solve for:
EFF%: 10.51558 or **10.516%**, rounded.

Using the BAII Plus calculator

2nd [IConv] 2nd [CLR Work] 10 ENTER ↓ ↓ 365 ENTER ↑ CPT
= 10.51558 or **10.516%**, rounded.

b) Another bank quotes an effective rate of 10.49%, monthly compounded, for the same loan. Which of the two is cheaper?

- ○ The daily compounded one
- ● The monthly compounded one

Explanation
The monthly compounded one!

Effective rates can be directly compared with each other whereas nominal rates cannot, unless the compounding period is the same.

c) What would be the nominal rate on a loan that pays an effective rate of 10.49%, monthly compounded? Enter your answer in percent per annum, rounded to 3 decimal places.

Nominal rate 10.017

Explanation

Nominal rate $[(1 + 0.1049)^{1/12} - 1] \times 12$
$= 0.1001706$ or **10.017%**, rounded.

Using the HP17 calculator

Select FIN and ICNV from the main menu and then PERiodic compounding.

Input:
EFF%: 10.49
P: 12

Solve for:
NOM%: 10.01706 or **10.017%**, rounded.

Using the BAII Plus calculator

2nd [IConv] 2nd [CLR Work].
To enter the effective interest rate press ↓ until EFF is displayed. Type 10.49, press ENTER and then ↓ until C/Y is displayed. Type 12 ENTER and press ↑ until NOM is displayed and finally press CPT
= **10.017%**

Question 3

What is the annually compounded yield on a bond yielding 4%, semi-annually?

a) Enter your answer in percent, rounded to 3 decimal places.
4.040

Question 4

a) Consider the following loan:

Principal:	$100
Nominal rate:	10%
Maturity:	1 year
Compounding period:	Continuous

What is the effective annual rate on this loan? Enter your answer in percent per annum, rounded to 3 decimal places.

Effective rate 10.517

Explanation

Effective rate $= 2.7182818^{0.10} - 1$
$= 0.105170918$ or **10.517%** rounded.

This result is very close to the effective rate that we obtained in the daily compounding example in *Question 1*. In practice daily compounding is a good approximation of continuous compounding.

Using the HP17 calculator

Select FIN and ICNV from the main menu and then CONTinuous compounding.

Input:
NOM%: 10

Solve for:
EFF%: 10.51709 or **10.517%**, rounded.

Using the BAII Plus calculator

0.10 2nd [e^x] - 1
= 0.1051709 or **10.517%**, rounded.

b) A derivatives system developer models the forward price of an underlying asset based on the following continuously compounding formula:

$$\mathbf{X_T = e^{R.T}}$$

Where:

X_T = Value of $1 today in year T

T = Number of years (or fraction of a year)

R = Nominal interest rate (in decimal - i.e. 10% is 0.10)

e = 2.71828182846... (the exponential coefficient)

If the developer takes the simple interest rate of 10% quoted by a broker to calculate the 1 year forward price for the asset, what should be the continuously compounded equivalent nominal rate that he should enter into his model? Enter your answer in percent per annum, rounded to 3 decimal places.

Continuously compounded equivalent rate [9.531]

Explanation

Continuously compounded equivalent rate $= \ln(1 + 0.10)$
$= 0.0953102$ or **9.531%**, rounded.

Using the HP17 calculator

Select FIN and ICNV from the main menu and then CONTinuous compounding.

Input:
EFF%: 10

Solve for:
NOM%: 9.53102 or **9.531%**, rounded.

Using the BAII Plus calculator

(1 + 0.1) LN
= 0.095310 or **9.531%**, rounded.

Question 5

Consider the following loan:

Principal:	$100
Nominal rate:	10%
Maturity:	2 years
Compounding period:	Daily (1/365 of a year)

a) What is its repayment amount at maturity? Enter your answer rounded to the nearest cent.

Repayment amount 122.14

Explanation

Repayment amount $= 100 \times (1 + 0.10/365)^{365 \times 2}$
$= 100 \times [(1 + 0.10/365)^{365}]^2$
$= 100 \times (1 + 0.1051558)^2$
= $122.1369 or **$122.14** rounded.

Using the HP17 calculator

Select FIN and TVM from the main menu.

Input:
N: 730 (i.e. 365 x 2)
I%YR: 10
PV 100
PMT 0
OTHER > P/YR: 365 (END MODE)

Solve for:
FV: = $122.1369 or **$122.14** rounded.

Using the BAII Plus calculator

First set the number of payments per year (P/Y) to 365 and the compounding period (C/Y) also to 365 by pressing: 2nd [P/Y] type 365 ENTER ↓ type 365 and finally 2nd QUIT.

Then type -100 PV, 730 N, 10 I/Y and 0 PMT.

Finally, press CPT FV.
= $122.1369 or **$122.14** rounded. This is the amount of money that the bank will receive when the loan matures.

Note: when using a financial calculator you must remember that each variable must be entered with the proper sign. All cash inflows (money that you receive) must be entered with a positive sign, while all cash outflows (money that you pay) must be entered with a negative sign. Use the +/- key instead of the minus key to change the sign of the variable.

Question 6

What is the future value of USD 100 deposited at 10% interest, annually compounded, for 7 years?

a) Type your answer rounded to 2 decimal places and validate.

USD 194.87

Explanation

Future Value = Present Value x $(1 + \text{Rate})^n$
= $100 \times (1 + 0.10)^7$
= USD **194.87** rounded.

Notice: after 7 years the interest payable on the loan is almost equal to the principal amount invested!

Using the HP17 calculator

Select FIN and TVM from the main menu.

Inputs:

N:	7
I%YR:	10
PV:	100
PMT:	0
OTHER > P/YR:	1

Solve for FV: **194.87**

Using the BAII Plus calculator

Set the number of payments per year equal to 1 and the compounding period equal to 1 by pressing: 2nd [P/Y] type 1 ENTER ↓ type 1. Then 2nd QUIT to quit.

Then type 7 N 10 I/Y 100 +/- PV

Finally, press CPT FV.
=USD **194.87** rounded.

Question 7

In this question, we show how to manipulate the terms of the annuity formula to calculate the amount of an annuity, given its PV and the rate of interest.

a) What is the annual amortisation amount on a $100 loan repayable over 3 years in fixed instalments (principal plus interest), at a rate of interest of 10%?

Enter your answer to 2 decimal places.

USD 40.21

Explanation
This problem requires solving for C (the amortisation amount) in the annuity expression:

$$100 = \frac{C}{(1 + 0.10)} + \frac{C}{(1 + 0.10)^2} + \frac{C}{(1 + 0.10)^3}$$

So far we have used this formula to solve for the present value, given the stream of cash flows. Now we need to find the size of the regular stream of cash flows, C, which have a present value equal to USD 100 (the amount borrowed) when discounted at 10%. It is easier to work with the closed-form version of this formula:

$100 = C \times (D_1 - D_4) / (1 - D_1)$

Therefore:

$$\begin{aligned} C &= 100 \times (1 - D_1) / (D_1 - D_4) \\ &= 100 \times (1 - 0.909091) / (0.909091 - 0.68301) \\ &= \$\mathbf{40.21} \end{aligned}$$

Using the HP17 calculator

Select FIN and TVM from the main menu.

Inputs:

N	3
I%YR	10
PV	100
FV	0
OTHER > P/YR	1

Solve for PMT **40.21**

Using the BAII Plus calculator

Set the number of payments per year equal to 1 and the compounding period equal to 1 by pressing: 2nd [P/Y] type 1 ENTER ↓ type 1 ENTER 2nd QUIT.

Then type 100 +/- PV 3 N 10 I/Y 0 FV.

Finally, press CPT and PMT.
=USD **40.21** rounded.

Question 8

In the table below, express the bond prices in percentages of face value, rounded to 2 decimal places.

a) What is the PV of a 2-year zero coupon bond to yield 10%, when the compounding period is:

Compounding period	PV
1. Annual	82.64
2. Semi-anual	82.27

Explanation

Compounding period	**PV**	
1. Annual	PV = 100% / $(1 + 0.10)^{1.2}$	= **82.64**%
2. Semi-annual	PV = 100% / $(1 + 0.10/2)^{2.2}$	= **82.27**%

Using the HP17 calculator

Select FIN and ICNV from the main menu and then PERiodic compounding.

Inputs:

N:	2	4
OTHER > P/YR:	1	2
I%YR:	10	10
FV:	100	100
PMT:	0	0
Solve for PV:	**82.64%**	**82.27%**

Using the BAII Plus calculator

In the case of annual compounding:
Set the number of payments per year equal to 1 and the compounding period equal to 1 by pressing: 2nd [P/Y] type 1 ENTER ↓ type 1 ENTER 2nd QUIT.

Press -100 FV 0 PMT 2 N 10 I/Y.

Press CPT and PV
=82.64

In the case of semi-annual compounding:
Set the number of payments per year equal to 2 and the compounding period equal to 2 by pressing: 2nd [P/Y] type 2 ENTER ↓ type 2 ENTER 2nd QUIT.

Press 100 FV 0 PMT 4 N 10 I/Y.

Press CPT and PV
=82.27

b) Price of bond 2 above is lower than that of bond 1 because:

○ Bond 1 earns a higher effective yield

○ Bond 2 is in higher demand

○ Bond 1 earns more coupon income

◉ Bond 2 earns a higher effective yield

Explanation
Bond 2 earns a higher effective yield

Bond 2 has a lower PV because a 10% nominal rate, semi-annually compounded, represents a higher effective rate than 10% annually compounded.

Question 9

What is the PV of a 3 year annuity paying USD 100 annually when discounted at 10%?

a) Enter your answer to 2 decimal places.

USD 248.68

Explanation

$$\frac{100}{(1 + 0.10)} + \frac{100}{(1 + 0.10)^2} + \frac{100}{(1 + 0.10)^3} = \mathbf{248.68}$$

Alternatively, we can use the closed form version of the annuity formula presented in section *Present Value*:

Present value = C x (D_1 - D_{txn+1}) / (1 - D_1)

Where:
C = Regular cash flow
D_i = Discount factor = $1/(1 + R/t)^i$
R = Discount rate (the required return on the annuity)
t = number of payments per year (= compounding period)
n = number of years

In this case C = 100, t = 1 and n = 3, so:

Present value = 100 x (D_1 - D_4) / (1 - D_1)

D_1 = 1 / (1 + 0.10) = 0.90909
D_4 = $1 / (1 + 0.10)^4$ = 0.68301

Present value = 100 x (0.90909 - 0.68301) / (1 - 0.90909)
= **USD 248.68**

Using the HP17 calculator

Select FIN and TVM from the main menu.

Inputs:

N:	3
I%YR:	10
PMT:	100
FV:	0
OTHER > P/YR:	1

Solve for PV: **248.68**

Using the BAII Plus calculator

Set the number of payments per year equal to 1 and the compounding period equal to 1 by pressing: 2nd [P/Y] type 1 ENTER ↓ type 1 ENTER 2nd QUIT.

Press -100 PMT 3 N 10 I/Y.

Press CPT and PV
=248.68

Question 10

In this question we compare the PV of a long-dated annuity with the PV of a perpetuity for the same annual cash flow.

a) What is the PV of a 99-year rental lease with an annual payment of $1,000, when discounted at 10%? Round your answers to 2 decimal places.

USD 9999.20

Explanation
Using the closed form version of the annuity formula presented in section *Present Value*:

$D_1 = 1 / (1 + 0.10) = 0.90909091$
$D_{100} = 1 / (1 + 0.10)^{100} = 0.00007257$[1]

PV = 1,000 x (0.90909091 - 0.00007257) / (1 - 0.90909091)
= **$9,999.20**

Using the HP17 calculator

Select FIN and TVM from the main menu.

Inputs:

N:	99
I%YR:	10
PMT:	1000
FV:	0
OTHER > P/YR:	1

Solve for PV: **9,999.20**

Using the BAII Plus calculator

Set the number of payments per year equal to 1 and the compounding period equal to 1 by pressing: 2nd [P/Y] type 1 ENTER ↓ type 1 ENTER 2nd QUIT.

Press 1,000 PMT 99 N 10 I/Y.

Press CPT and PV
=9,999.20

[1] This is a very small number, so remember to keep plenty of precision on it!

b) What is the present value of a straight perpetuity paying $1,000 annually, when discounted at 10%?

USD 10000.00

Explanation
Present value of perpetuity = C / R

Where C = annual cash flow
R = annual rate of discount

In this case C = 1,000 and R = 10%:

Present value = 1,000 / 0.1
= **$10,000.00**

The price of the perpetuity is very close to that of an equivalent very long-dated annuity, because the PVs of the distant cash flows are very small in relation to the PVs of the nearer cash flows. This is why the market price of a 99-year lease tends to be very close to the price of a free-hold!

Question 11

A bank lends $1,400 to a client and ask the client to repay the entire debt, including interest, over 2 years in equal instalments of $375 every 6 months. What is the internal rate of return on this loan? Enter your answer in percentage per annum, rounded to the nearest 3 decimal places.

a) Nominal IRR 5.636
Effective IRR 5.715

Explanation

Nominal IRR **5.636%**
Effective IRR **5.715%**

The nominal IRR is that rate R which satisfies this equation:

$$1{,}400 = \frac{375}{(1 + R/2)} + \frac{375}{(1 + R/2)^2} + \frac{375}{(1 + R/2)^3} + \frac{375}{(1 + R/2)^4}$$

There is no reduced form for this equation, so the answer must be found by iteration.

The effective rate is the annual-equivalent of the nominal, calculated using the annual/semi-annual conversion formula in section *Simple and Compound Interest*, above.

Using the HP17 calculator

First, select FIN and TVM from the main menu.

Inputs:

N	4
PV	-1400
PMT	375
FV	0
OTHER > P/YR	2

Solve for I%YR **5.636**

Don't forget to reverse the sign of either PV or of PMT!

If you don't do that then the calculator will beep and fail to find a solution because (being a good capitalist) it does not understand the concept of a positive PV generating more positive PMTs in the future!

Next, select FIN and ICNV from the main menu and then PERiodic compounding.

Input:
NOM%: 5.636
P: 2

Solve for:
EFF%: **5.715**

Using the BAII Plus calculator

Set the number of payments per year equal to 2 and the compounding period equal to 2 by pressing: 2nd [P/Y] type 2 ENTER ↓ type 2 ENTER 2nd QUIT.

Nominal rate:
Press 375 PMT -1,400 PV 0 FV 4 N.

Press CPT and I/Y
= **5.636%**

Press STO 1 to save this nominal rate.

Effective rate:
Press 2nd [IConv]

Press RCL 1 to recall the nominal rate calculated in (a), above.

Press ENTER ↓ ↓ 2 ENTER ↑ CPT.
= **5.715%**, rounded.

2. Money Market Instruments

Question 1

Security:	SD Fixed Rate CD
Type:	Actual/360
Issuer:	JP Morgan Chase, London
Amount:	USD 100,000.00
Rate:	5.3/4%
Issued:	5 July 2002
Maturity:	7 July 2003
Settlement:	11 February 2003
Price:	5.38%

a) What is the repayment amount on this CD at maturity?

USD 105861.81

Explanation
Actual term of the loan (5 July 2002 - 7 July 2003) = 367[2]

Repayment amount = Principal x (1 + Rate x Tenor / Year Basis)
= 100,000 x (1 + 0.0575 x 367/360)
= **USD 105,861.81**

b) Effectively, the CD pays more than 5.75% interest because:

- (●) The actual number of days is 367 but the basis is 360
- ( ) The borrower pays a yield spread over the coupon rate
- ( ) Interest is paid semi-annually, so it compounds
- ( ) The actual number of days is 360 but the basis is 365

Explanation
The actual number of days is 367 but the basis is 360.

Interest is calculated on the actual number of days on the loan in a 360-day year, so it accrues 5.75% for the first 360 days plus 7 days of interest at the annual rate of 5.75%.

c) What is the bond equivalent yield on this investment, annually compounded, to the nearest 2 decimal places?

5.45

[2] This is a 12 month deposit but 5 July 2003 is a Saturday so the maturity date has been rolled over to the next business day.

d) What is the settlement amount on this trade?

USD 103601.34

Explanation
What is the settlement amount on this trade?

Actual number of days to maturity (11 Feb - 7 July 2003) = 146

$$\text{Settlement amount} = \frac{\text{Repayment amount}}{(1 + \text{Yield x Days / Year Basis})}$$

$$= \frac{105,861.81}{(1 + 0.0538 \times 146/360)}$$

= **USD 103,601.34**

e) You bought this CD at 5.38% for settlement 11 February 2003 and you re-sell it at 5.50% for settlement 21 February. What is the gross profit or loss on the trade:

(i) In USD? 105.67

(ii) As a percentage of capital employed (annualised, actual/360, to 2 decimal places) - i.e. the horizon return?

Return 3.67

Explanation

(i) Actual number of days to maturity (21 Feb - 7 July 2003) = 136

$$\text{Settlement amount} = \frac{\text{Repayment amount}}{(1 + \text{Yield x Days / Year Basis})}$$

$$= \frac{105,861.81}{(1 + 0.0550 \times 136/360)}$$

= USD 103,707.01

Profit/loss = Settlement value on 21 Feb - Settlement value on 11 Feb
= 103,707.01 - 103,601.34
= **USD 105.67**

(ii) $$\text{Horizon return} = \frac{\text{Profit}}{\text{Initial outlay}} \times 100 \times \frac{360}{\text{Holding period}}$$

$$= \frac{105.67}{103,601.34} \times 100 \times \frac{360}{10}$$

= **3.67%**

f) We earned less than the yield on the CD because:

- [ ] Interest is paid net of withholding tax
- [x] Market yield rose, so we made a capital loss
- [ ] Yields rose so we accrued less interest
- [ ] Funding costs were high

Explanation
Market yield rose, so we made a capital loss.

By the time we sold the CD market yields rose to 5.50%, so we made a capital loss. The paper accrues interest over the 10 days, which hides this trading loss. Had we funded this position at 5.75%, however, then we would have shown a net loss of 2.08% (= 3.67 - 5.75)!

Question 2

Security:	USD Bankers Acceptance
Type:	Actual/360
Acceptor:	HSBC, New York
Amount:	USD 100,000.00
Underlying transaction:	Coal exports
Maturity:	3 July 2002
Settlement:	7 February 2002
Price:	5.27%

a) What is the settlement amount on the trade, in USD?

97862.72

b) What is the money market yield on this investment, to the nearest 2 decimal places?

5.38

Question 3

A 91-day US T-bill is quoted at 8.60%.

a) What is its annually compounded bond-equivalent yield, rounded to 2 decimal places?

8.91

3. Bond Pricing and Yield

Question 1

How many days of accrued interest are there on a bond maturing on 15 September 2006, for settlement on 2 November 2004, if the bond is:

a) A US Treasury (Actual/Actual)? Enter your answer in the box below, then validate.

48

b) A US domestic corporate bond (30/360)?

47

Question 2

A 10% sterling corporate bond pays coupons on 21 January and 21 July and is bought for settlement on 6 June 2002. Calculate the amount of accrued interest payable on a GBP 1 million deal under the following day count conventions. Enter your answer rounded to the nearest pence.

a) Accrued interest, Actual/365 (GBP):

37260.27

b) Accrued interest, Actual/Actual (GBP):

37362.64

Question 3

Security:	7.50% Coca-Cola maturing 15 December 2005
Type:	Eurobond, annual, 30E/360
Settlement date:	12 August 2004
Amount dealt:	USD 10 million
Yield:	6.75%

a) What is total accrued interest payable on this trade?

493750.00

b) What is the bond's dirty price, rounded to the nearest 2 decimal places?

105.81

c) What is the clean price on this bond, rounded to the nearest 1/8%? (Enter in decimal).

100.875

d) Assuming you bought the bond at the price calculated in (c), what is the total settlement amount of the transaction?

10581250.(

e) Would you need to pay more or less for this bond if you required a yield of 7%?

- (●) Less
- ( ) More

Explanation
Less.

If we apply a higher discount rate (7%) to the bond pricing formula, this results in a lower present value, so we would pay less. The higher the yield on a bond the lower is its price.

Question 4

This exercise is too complex to perform with a simple calculator. You should use a financial calculator such as the HP 17 (or later model) or the bond pricing model, on the left.

Security:	7½% US Treasury bond maturing 21 October 2009
Type:	semi-annual, actual/actual
Settlement date:	20 April 2004
Yield:	7.00%.

a) What is the bond's clean price, rounded to the nearest 1/32%? (Enter in decimal).

102.25

b) What is its dirty price, rounded to 2 decimal places?

105.98

c) Given the same yield, what would be the bond's clean price for value 22 April 2004, rounded to the nearest 1/32%?

Enter the result in decimal, to 2 decimal places.

102.25

d) What would be its dirty price, rounded to 2 decimal places?

102.27

e) Explain the differences between (b) and (d), above.

- ( ) The clean price changed
- (●) A coupon was paid on 21 April
- ( ) The bond has been 'sold down'
- ( ) The bond's yield rose

Explanation
A coupon was paid on 21 April.

There was no change in the bond's yield and no change in its clean price, only in accrued interest. This is because a coupon was paid on 21 April, so the bond now begins to start accruing again.
This demonstrates the purpose of clean prices: to smooth out sharp fluctuations in the value of the bond that arise from the payment of coupons.

Question 5

This exercise is too complex to perform with a simple calculator. You should use a financial calculator such as the HP 17 (or later model) or the bond pricing model, on the left.

Security:	7½ FNMA bond maturing 21 October 2009
Type:	USD Eurobond, annual 30/360
Settlement date:	20 April 2004
Yield:	7.00%.

a) What is the bond's clean price, rounded to 2 decimal places?

102.16

b) What is its dirty price, rounded to 2 decimal places?

105.89

c) Since the settlement, maturity, coupon and yield on this bond are the same as for the bond in *Question 4*, why are the prices of the two bonds different?

- [ ] This bond has a lower credit rating
- [ ] The two bonds were issued on different rates
- [ ] A semi-annual yield is equivalent to a lower annual yield
- [x] This bond is annual 30/360; the other one is semi-annual Actl/Actl

Explanation
This bond is annual 30/360; the other one is semi-annual Actl/Actl.

This bond pays annual coupons while the one in the previous question pays semi-annually. For the same yield the semi-annual coupon is more valuable as it pays cash flows sooner. Also, the day-count conventions applied are different so the accrued interest should be different although this is not significant to just 2 decimal places.

Question 6

Security:	4½% Japanese government bond maturing 23 Sep 2005
Type:	domestic, semi-annual actual/365
Price:	108.55
Settlement date:	21 June 2002

Calculate:

a) The bond's current yield (CY) in percent. (Enter your answer, to 2 decimal places, in the box below, then validate.)

4.15

b) The bond's adjusted current yield (ACY) rounded to 2 decimal places.

1.73

Question 7

This exercise is too complex to perform with a simple calculator. You should use a financial calculator such as the HP 17 (or later model) or the bond pricing model, on the left.

Security:	4½% Japanese government bond maturing 22 Sep 2005
Type:	Domestic, semi-annual actual/365
Settlement date:	20 June 2002
Price:	108.55

a) Calculate the bond's yield to maturity, rounded to the nearest 2 decimal places.

1.78

Question 8

This exercise is too complex to perform with a simple calculator. You should use a financial calculator such as the HP 17 (or later model) or the bond pricing model, on the left.

Security:	4½% Asian Investment Bank maturing 22 Sep 2005
Type:	JPY Eurobond, annual 30E/360
Settlement date:	20 June 2002
Price:	108.55

a) Calculate the bond's yield to maturity, rounded to the nearest 2 decimal places.

1.77

b) Why is the yield on this bond different from the yield on the bond calculated in *Question 2*, if the two bonds have the same coupon, maturity and price?

- ○ The two bonds were issued on different dates.
- ○ A semi-annual yield is equivalent to a lower annual yield.
- ◉ This bond is annual 30/360; the other one is semi-annual Actl/Actl.
- ○ This bond has a lower credit rating.

Explanation
This bond is annual 30/360; the other one is semi-annual Actl/Actl.

The two yields are different principally because the first one is a semi-annual and the second an annual one. For the same price, the semi-annual bond has a higher yield than the annual bond because it pays cash flows sooner.

In fact, the bond pricing model shows that the equivalent 1 year compounded yield on the first bond is 1.792%. On this basis, the government bond clearly trades cheap relative to the supranational and should be preferred.

Also, the day-count conventions differ between the two bonds.

Question 9

This exercise is too complex to perform with a simple calculator. You should use a financial calculator such as the HP 17 (or later model) or the bond pricing model, on the left.

Security:	7½% Ford Motor Finance, maturing 22 October 2009		
Type:	Eurobond, annual 30E/360		
Call features:	Callable at:	101.00 on 22 October 2007	
		100.50 on 22 October 2008	
Settlement date:	19 April 2002		
Price:	102¾		

a) What is the bond's yield to maturity, rounded to the nearest 2 decimal places?

7.01

b) What is the bond's yield to worst, rounded to 2 decimal places?

7.01

c) Without doing the calculations, could you have predicted which was the worst call date?

- (•) No
- ( ) Yes

Explanation

No, because the bond is callable at a premium to par. If it was called at par then, since the bond currently trades at a premium to par, we would know that the yield to the first call date would be the worst: the 2.75% capital loss would have to be amortised over the shortest period.

Question 10

This exercise is too complex to perform with a simple calculator. You should use a financial calculator such as the HP 17 (or later model) or the bond pricing model, on the left.

Security	6¼% British government bond (UK Gilt) maturing 20 October 2010
Type:	Semi-annual, Actual/actual
Price:	102.34375
Settlement:	19 April 2002

What is the bond's yield to maturity:

a) On a semi-annual basis, in percent, rounded to the nearest 3 decimal places?

5.896

b) On an annualised basis (to 2 decimal places)?

5.98

Question 11

This exercise is too complex to perform with a simple calculator. You should use the bond pricing mode on the left.

Security: 8% Eurobond maturing 10 October 2001
Type: Annual, 30/360
Settlement date: 5 January 1998
Price: 93.516 (clean)
Horizon date: 10 October 2001 (the maturity date)
Horizon price: 100.00

a) What is the yield to maturity and the horizon yield on this bond, assuming a reinvestment rate of 10.134% (annual, 30/360)? Enter your answers in percent per annum to 3 decimal places.

Yield to maturity	10.134
Horizon yield	10.134

Explanation

Yield to maturity **10.134%**
Horizon yield **10.134%**

The horizon yield in this case is the same as the bond's yield to maturity. The example demonstrates the crucial assumption behind yield to maturity: that the coupons are all reinvested at the same yield, which of course is not always the case!

Manual calculation

Below we calculate the horizon yield on this bond manually. Please note that this is just for your reference only, if it helps you to understand the concept better:

You are not expected to perform this complex type of yield calculation in the IFID exam!

Number of Eurobond days since last coupon
(10 October 1997 - 5 January1998) = 85
Fractional coupon period = 85/360 = 0.23611

Accrued interest = 8 x 0.23611 = 1.889
Dirty price = 93.516 + 1.889 = 95.405

Terms used in the horizon return formula:
a = 1 – 0.23611 = 0.76389
b = 0
m = 3 (there are 3 whole coupon periods to the horizon date)

$$\text{Future value of coupons} = \frac{8 \times [\,(1 + 0.10134)^4 - 1\,]}{0.10134}$$

$$= 37.2013$$

Horizon cash flow = 100.00 + 37.2013
= 137.2013

$$\text{Horizon return} = \left\{ \frac{137.2013}{95.405} \right\}^{1/3.76389} - 1$$

$$= 0.10134 \text{ or } \mathbf{10.134\%}$$

4. Spot and Forward Yields

Question 1

The table below shows the first 3 points on an annual par curve. Calculate the corresponding spot yields and discount factors.

a) Enter your answers in each box below, then validate. Yields should be in percent, rounded to 2 decimal places, and discount factors should be rounded to 5 decimal places.

Year	**1**	**2**	**3**
Par Yield	6.00	6.70	6.85
Spot yield	6.00	6.72	6.88
Discount factor	0.94340	0.87797	0.81913

Explanation

Year	**1**	**2**	**3**
Par Yield	6.00	6.70	6.85
Spot yield	**6.00**	**6.72**	**6.88**
Discount factor	**0.94340**	**0.87797**	**0.81913**

The spot yield for year 1 ($R_{0,1}$) is the same as the 1 year par yield (Y_1) - the first coupon is payable after one year. For the subsequent years follow the following sequence:

1. Calculate the discount factor for year 1 from the 1 year par yield (Y_1):

$$D_{0,1} = \frac{1}{(1 + Y_1)}$$

$$= 1 / 1.0600$$

$$= \mathbf{0.94340}$$

2. Calculate the discount factor and the spot yield for year 2 using the formula developed in section *Discount Factors*:

$$\mathbf{D_{0,2} = (1 - Y_2 \times D_{0,1}) / (1 + Y_2)}$$

$$= (1 - 0.0670 \times 0.94340) / 1.0670$$

$$= \mathbf{0.87797}$$

$$R_{0,2} = (1 / D_{0,2})^{(1/2)} - 1$$

$$= (1 / 0.87797)^{(1/2)} - 1$$

$$= 0.0672 \text{ or } \mathbf{6.72\%}$$

3. Calculate the discount factor and the spot yield for year 3:

$$D_{0,3} = [1 - Y_3 \times (D_{0,1} + D_{0,2})] / (1 + Y_3)$$

$$= [1 - 0.0685 \times (0.94340 + 0.87797)] / 1.0685$$

$$= \mathbf{0.81913}$$

$$R_{0,3} = (1 / D_{0,3})^{(1/3)} - 1$$

$$= (1 / 0.81913)^{(1/3)} - 1$$

$$= 0.0688 \text{ or } \mathbf{6.88\%}$$

You can program these formulas on a spreadsheet, as the sample spreadsheet on the left illustrates.

b) In (a) spot yields are higher than par yields because:

- (selected) The yield curve is positive
- The yield curve is inverted
- Demand for zero coupon instruments is weaker
- Zero coupon instruments have higher credit risk

Explanation
The yield curve is positive.

Question 2

The table below shows the first 3 points on an annual par curve, together with their corresponding spot yields and discount factors. Calculate the annual forward yields from this data.

a) Enter your answers in each box below, in percent, rounded to 2 decimal places, and validate.

Year	**1**	**2**	**3**
Par Yield	6.00000	6.70000	6.85000
Spot yield	6.00000	6.72361	6.87670
Discount factor	0.94340	0.87797	0.81913
Forward yield	---	7.45	7.19

Explanation

Year	**1**	**2**	**3**
Par Yield	6.00000	6.70000	6.85000
Spot yield	6.00000	6.72361	6.87670
Discount factor	0.94340	0.87797	0.81913
Forward yield	---	**7.45**	**7.19**

We calculate the annual forward yields using the formula developed in section *Derivation*:

$R_{1,2} = D_{0,1} / D_{0,2} - 1$
$= 0.94340 / 0.87797 - 1$
$= 0.0745$ or **7.45%**

$R_{2,3} = D_{0,2} / D_{0,3} - 1$
$= 0.87797 / 0.81913 - 1$
$= 0.0718$ or **7.18%**

You can program these formulas on a spreadsheet, as the example on the left illustrates.

Question 3

The table below shows one-year forward yields effective in years 1, 2 and 3. Calculate the 1, 2 and 3 year spot and par yields using the formulas developed in sections *Spot from Forward Yields* and *Par from Forward Yields*.

a) Yields should be in percent, rounded to 2 decimal places, and discount factors should be rounded to 5 decimal places.

Year	**1**	**2**	**3**
Forward yield	7.50	6.06	5.85
Spot yield	7.50	6.78	6.47
Discount factor	0.93023	0.87708	0.82861
Par yield	7.50	6.80	6.50

Explanation

Year	**1**	**2**	**3**
Forward yield	7.50	6.06	5.85
Spot yield	7.50	**6.78**	**6.47**
Discount factor	**0.93023**	**0.87708**	**0.82861**
Par yield	**7.50**	**6.80**	**6.50**

Spot Yields

In section *Spot Yields from Forward Yields* we showed the fundamental relationship between a spot yield and its corresponding strip of forward yields:

$$(1 + R_T / t)^T = (1 + R_{S+1} / t) \times (1 + R_{S+2} / t) \times \ldots \times (1 + R_L / t)$$

$$= \Pi (1 + R_i / t)$$

Where:
R_T = Zero coupon rate for total period T
i = Interest period; i = S … L
R_i = Forward rate for period i
D_i = Discount factor for period i
t = Compounding period (1 = annual, 2 = semi-annual, etc.)
T = Total number of interest periods = (L – S) x t

In this case, for the 2 year spot yield t = 1, S = 0 and L = 2, so:

$$(1 + R_{0,2})^2 = (1 + R_{0,1}) \times (1 + R_{1,2})$$

$$R_{0,2} = [(1 + R_{0,1}) \times (1 + R_{1,2})]^{1/2} - 1$$

$$= [1.075 \times 1.0606]^{1/2} - 1$$

$$= 0.067776 \text{ or } \mathbf{6.78\%}$$

And for the 3 year spot yield:

$$(1 + R_{0,3})^3 = (1 + R_{0,1}) \times (1 + R_{1,2}) \times (1 + R_{2,3})$$

$$R_{0,3} = [(1 + R_{0,1}) \times (1 + R_{1,2}) \times (1 + R_{2,3})]^{1/3} - 1$$

$$= [1.075 \times 1.0606 \times 1.0585]^{1/3} - 1$$

$$= 0.064675 \text{ or } \mathbf{6.47\%}$$

Par Yields

To calculate the par yields we use the formula developed in section *Par from Forward Yields*:

$$\mathbf{C_{0,L} = \left(\frac{1 - D_{0,L}}{\Sigma D_{0,i}} \right) \times t}$$

for i = 1 to L

Where $C_{0,L}$ = Par yield effective year 0 for L coupon periods
t = Compounding period
$D_{0,i}$ = Discount factor on the spot yield to period i

$= 1 / (1 + R_i / t)^i$

Here t = 1 (annual compounding), so using the spot yields calculated above:

$D_{0,1} = 1 / 1.075000 = \mathbf{0.93023}$
$D_{0,2} = 1 / 1.067776^2 = \mathbf{0.87708}$
$D_{0,3} = 1 / 1.064675^3 = \mathbf{0.82861}$

Therefore:

$C_{0,2} = (1 - D_{0,2}) / \Sigma D_{0,i}$ for i = 1 to 2
$= (1 - 0.87708) / (0.93023 + 0.87708)$
$= 0.068013$ or **6.80%**

$C_{0,3} = (1 - D_{0,3}) / \Sigma D_{0,i}$ for i = 1 to 3
$= (1 - 0.82861) / (0.93023 + 0.87708 + 0.82861)$
$= 0.065021$ or **6.50%**

You can program the formulas used in sections (a) and (b) on a spreadsheet, as the example illustrates.

b) What is the rate for a loan commencing in 1 year's time and ending in 3 years (i.e. a 1x3 year forward yield), in percent to 3 decimal places:

(i) On a zero coupon basis? 5.955

(ii) On a coupon (i.e. par) basis? 5.958

Explanation
On a zero coupon basis:

$$(1 + R_{1,3})^2 = (1 + R_{1,2}) \times (1 + R_{2,3})$$

$$R_{1,3} = [(1 + R_{1,2}) \times (1 + R_{2,3})]^{1/2} - 1$$

$$= [1.0606 \times 1.0585]^{1/2} - 1$$
$$= 0.059549 \text{ or } \mathbf{5.955\%}$$

On a coupon (or par) basis:

$$C_{1,3} = (D_{0,1} - D_{0,3}) \ / \ \Sigma D_{0,i} \qquad \text{for } i = 2 \text{ to } 3$$

$$= (0.93023 – 0.82861) / (0.87708 + 0.82861)$$
$$= 0.059577 \text{ or } \mathbf{5.958\%}$$

You can program the formulas used in sections (a) and (b) on a spreadsheet, as the example illustrates.

c) Which (one or more) of the statements below explain(s) why in this example spot yields are lower than par yields?

- [ ] Zero coupon instruments have low credit risk
- [ ] Demand for zero coupon instruments is stronger
- [x] The yield curve is inverted
- [x] Forward yields are lower than par yields

Explanation
- **Forward yields are lower than par yields**
- **The yield curve is inverted**

As we explained in section *Derivation*, if the curve is inverted then forward yields are lower than yields to maturity. Therefore, an investor in a par bond is able to lock in lower reinvestment rates on their future coupons than is implied in the bond's yield to maturity.

5. Interest Rate Risk

Question 1

Try to estimate the Macaulay duration of each of the securities below just by visualising it on the Macaulay 'see-saw' (see section *Macaulay Duration*). Enter your answers in the boxes below, then validate.

a) Non-interest bearing demand deposit:

0

b) Six months Eurodeposit (in months):

6

c) Ten-year floating rate note where the coupon rate is reset semi-annually, in line with the 6 month LIBOR (in months):

6

d) Ten-year zero coupon bond (in years):

10

e) A 10-year 10% annual coupon bond (rounded to the whole number of years):

- ○ 5 years
- ◉ 7 years
- ○ 10 years

Explanation
Approximately: **7 years.**

Question 2

Settlement date:	16 April 2002
Security:	12¼% US T-Bond maturing on 4 January 2008
Type:	Semi-annual, Actl/Actl
Price:	124-11+[3]
Accrued:	3.4517%
Yield:	7.00%
Duration:	4.32 years

a) Using the bond's Macaulay duration, calculate its modified duration to 4 decimal places.

4.1739

b) Using the result in (a), calculate the bond's BPV, per USD 100 nominal, to 5 decimal places.

0.05335

c) Using the result calculated in (b), estimate the expected change in the bond's price if its yield were to rise by 10 basis points, from 7.00% to 7.10% (your answer to 2 decimal places).

0.53

[3] US Treasuries are quoted to the nearest 1/32% of par value, so 124-11 means 124 + 11/32%. Half a tick (i.e. 1/64%) is represented with a '+', so 124-11+ means 124 + 11.5/32%, or 124 + 23/64%.

Question 3

Consider the following securities:

- 12 month T-bill
- 14 year 8% US Treasury bond yielding 12%
- 6 month CD
- 16 year 15% US Treasury bond yielding 12%

a) Rank these securities in order of price risk (basis point value) by entering the appropriate number (1 = lowest risk, 4 = highest risk) in each box below.

12 month T-bill	2
14 year 8% US Treasury bond yielding 12%	3
6 month CD	1
16 year 15% US Treasury bond yielding 12%	4

Explanation

12 month T-bill	**2**
14 year 8% US Treasury bond yielding 12%	**3**
6 month CD	**1**
16 year 15% US Treasury bond yielding 12%	**4**

The T-bill has twice the risk of the CD because it has twice the duration (1 year instead of 1/2 year). Clearly, both the T-bonds have higher duration than the CD or the T-bill. How do we know that the 15% coupon bond riskier than the 4% bond? We need to:

- **Either** calculate their durations, and then their BPVs
- **Or** estimate their BPVs directly by repricing these bonds for a 0.01% change in yield.

Both these methods are illustrated below.

Method 1: calculate durations, then BPVs

Macaulay duration is a complex formula. Like the bond pricing formula, you are not expected to perform such calculations by hand! Use bond pricing model supplied or a financial calculator with a duration function.

Once you have calculated the Macaulay duration:

$$\text{Modified duration} = \frac{\text{Macaulay duration}}{(1 + \text{Yield} / t)}$$

$$\text{BPV} = \frac{\text{Modified duration}}{100} \times \frac{\text{Dirty Price}}{100}$$

	Price	Duration	Modified	BPV
14 year 8%	73.19	7.720 yrs	$\frac{7.720}{(1+0.12/2)}$ = 7.283%	$\frac{7.283}{100} \times \frac{73.19}{100}$ = **0.05330**
16 year 15%	121.13	7.192 yrs	$\frac{7.192}{(1+ 0.12/2)}$ = 6.785%	$\frac{6.785}{100} \times \frac{121.13}{100}$ = **0.08219**

These calculations show an interesting result:

- In terms of duration (Macaulay or modified) the 8% bond comes out riskier: it has a shorter maturity than the 15% but a much lower coupon
- In terms of BPV - i.e. in absolute price change - the 15% is riskier: it trades at a high premium to par, so its absolute price risk is higher.

Method 2: estimate BPVs by repricing

If you are given the Macaulay duration of a bond, it is a straightforward step to calculating modified duration and hence BPV. But if you don't know the duration (or your financial calculator does not have this function) then it is easier to calculate BPV directly by repricing the bond for a 1 basis point change in yield.

Using the HP17 calculator

Input
TYPE: Semi-annual, A/A
SETT: 1.012000 (any arbitrary date)
MAT: 1.012014 (Settlement plus 14 years)
CPN%: 8
YLD%: 12

Solve for:
PRICE: 73.18767

Press:
[STO] [1] (save the calculated price to memory)
[RCL] [YLD%] (recall the yield)
[+] 0.01 [YLD%] (add 1 basis point to it and re-input it)

Solve for:
PRICE: 73.13440

Press
[-] [RCL] [1] (subtract from it the original bond price)
[=] (result is the **BPV: 0.05327**)

If you repeat the same sequence of steps for the 15% bond, this should come out as **0.08213**.

Using the BAII Plus calculator

Access the Bond worksheet pressing 2nd [BOND] and clear it by pressing 2nd [CLR WORK]. Enter an arbitrary date such as 01.0100 ENTER ↓ 8 ENTER ↓ 01.0114 ENTER ↓ ↓ . To select the Actual/Actual day-count method, press repeatedly 2nd [SET] until ACT is displayed. Press ↓ and select the coupon frequency by press 2nd [SET] until 2/Y is displayed. Press ↓ until YLD is displayed and key in the value 12 and press ENTER.

Press ↓ to display PRI and then press CPT .
=**73.18767** (clean price)

Now we store this result in memory 1 by pressing STO 1.
We proceed using the arrow ↑ until YLD is displayed. To this value of 12.000 we add 0.01 by pressing + and typing 0.01 ENTER. Press ↓ until PRI is displayed and then press CPT. The screen should display a price of 73.13440, which we can subtract from the original price, stored in memory 1, as follows:
Type - RCL 1 =
The net result is **-0.05327**.

If you repeat the same sequence of steps for the 15% bond, this should come out as **0.08213**.

Question 4

You currently hold the following:

Security:	10% bond maturing in 10 years
Type:	Annual, E30/360
Price:	103.00
Macaulay duration:	6.809 years
Portfolio held:	USD 10 million.

a) If the yield on this bond were to rise by 100 basis points, what percentage of your capital would you lose? In other words, calculate the bond's modified duration.
Using a financial calculator or the bond pricing model supplied:

1. Calculate the yield on the bond at its current price
2. Add 1% to the calculated yield and re-price the bond[4]
3. Take the difference between the two prices
4. Divide this difference by the bond's original dirty price

Enter your answer in percent to 3 decimal places.

5.957

b) Using the analytical formula developed in section *Modified Duration*, calculate the bond's modified duration.

$$\textbf{Modified duration} = \frac{\textbf{Macaulay duration (in years)}}{\textbf{(1 + Yield/Nr Coupons per year)}}$$

Enter your answer in percent to 3 decimal places.

6.217

c) Why do the results calculated in (a) and (b) differ?

- ○ Because of rounding errors
- ◉ Because the bond has convexity
- ○ Because the bond has credit risk, as well as market risk
- ○ Because of specific supply and demand factors

Explanation
Because the bond has convexity.

A large rise in yields results in a smaller actual price fall than the analytical formula would predict. Conversely, a fall in yields would result in a larger actual price increase than the analytical formula would predict.

[4] Take care to work with your calculator's full precision – i.e. do not to round your result until you reach the final calculation.