ISMA CENTRE - THE BUSINESS SCHOOL OF THE FINANCIAL MARKETS

UNIVERSITY OF READING
ENGLAND

IFID Certificate Programme

Fixed Income Analysis

Bond Pricing and Yield

Table of contents

1. Overview

In this module we look at the formula for pricing straight bonds based on the principles developed in *Time Value of Money – Present Value*.

- We show how the formula handles the pricing of a bond part-way into a coupon period
- We distinguish between a bond's:
  - Clean price, which is net of accrued interest, and
  - Dirty price, which includes accrued interest

We also look at the concept of return or **yield** on a bond, which arises from:

- The receipt of periodic coupons
- Interest earned from the reinvestment of any coupons received
- A capital gain or loss realised when the bond is subsequently sold in the secondary market, or redeemed at maturity

The different measures of yield that we define in this module either take only some of these factors into account or make simplifying assumptions about them, and we shall highlight the strengths and weaknesses of each measure in turn.

Learning Objectives

By the end of this module you will be able to:

1. Calculate the dirty price, accrued interest and clean price on straight bonds using different compounding and day-count conventions
2. Using a financial calculator, compute and interpret the following measures of bond yield:
   - Current yield
   - Yield to maturity
3. Explain the significance of reinvestment income in the calculation of yield to maturity
4. Explain the limitation of yield to maturity as a measure of a bond's market value
5. Calculate equivalences between yields to maturity based on different compounding periods
6. Define the concept of the yield curve
7. Interpret the following measures of bond yield:
   - Yield to call / yield to put
   - Horizon yield
8. Define the discount margin on an FRN and describe a methodology for calculating it
9. Identify the main factors that cause price sensitivity in an FRN
10. Describe the generic structure of an inflation-linked bond and outline the so-called Canadian pricing methodology
11. Calculate the real (inflation-adjusted) yield on a bond, given its nominal yield, compounding period and inflation rate
12. Explain in outline how breakeven inflation rates are derived from nominal yields and the market prices of inflation-linked government bonds
13. Calculate the net (after-tax) yield on a taxable security and the gross or tax-equivalent yield of a tax-paid security

2. Valuation Formula

2.1. Pricing on a Coupon Date

The DCF technique introduced in module Time Value of Money may be used to price a straight bond.

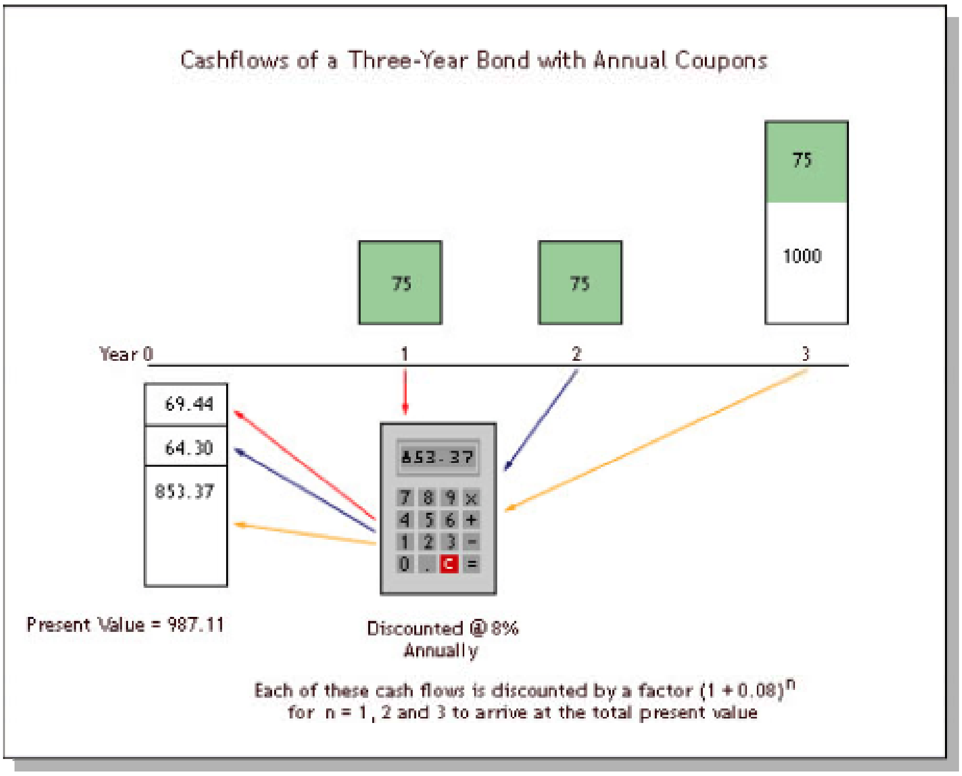
Example

Name: Coca Cola 7½% maturing 19 July 2005
Type: Eurobond, annual coupons
Settlement date: 19 July 2002
Denomination: USD 1,000

? How much should you pay for this bond to achieve an internal rate of return (IRR, or yield) of 8%?

Analysis
The figure below displays on a time-line the three cash flows that the bond will generate:

1. Coupon: USD 75 (7½% of USD 1,000) at the end of year 1, on 19 July 2003
2. Coupon: another USD 75 at the end of year 2
3. Principal repayment plus final coupon: USD 1,075 at the end of year 3



The sum of the present values of the expected future cash flows using an annual discount rate of 8% is $987.1, so this is what the bond should be worth.

2.2. The Pricing Formula

Generalising what we did in the example on the previous page to any maturity gives the formula for the price of a bond shown below.

$$\text{Present value} = \frac{C/t}{(1 + R/t)} + \frac{C/t}{(1 + R/t)^2} + \frac{C/t}{(1 + R/t)^3} + \ldots + \frac{\text{Principal} + C/t}{(1 + R/t)^{nxt}}$$

Where:
C = Annual coupon rate
R = IRR on the investment (the yield)
t = Number of coupon payments per year (= compounding period)
n = Number of years

Equivalent formulas:

$$\text{Present value} = C/t \times \Sigma \frac{1}{(1 + R/t)^i} + \frac{\text{Principal}}{(1 + R/t)^{nxt}}$$

for i = 0 … nxt

$$= C/t \times \Sigma D_i + \text{Principal} \times D_{nxt}$$

Where D_i = Discount factor = $1/(1 + R/t)^i$

Properties

In our example we priced the bond on a coupon date, so the next coupon is exactly one coupon period away. In the next section we shall adapt this basic formula to handle situations when a bond may be traded part-way through a coupon period, so the next coupon may not be exactly one coupon period away.

Two important points follow from the pricing formula developed here:

1. The higher the required yield on the bond - i.e. the higher the rate used to discount its future cash flows - the lower will be its market price. This illustrates:

   Bond Market Law No 1: bond price varies inversely with yield

2. The convention in the bond market is to apply the same rate to discount all of the bond's future cash flows.

   In reality we know that the cost of one-year money is rarely the same as the cost of two-year money, or the cost of three-year money. So we could apply a different discount rate to present value each cash flow. This is the technique used in the derivatives markets. The market value of a bond may be different when its cash flows are traded separately - i.e. as a strip of zero-coupon bonds - than it is when it is traded whole, as a family of cash flows.

2.3. Pricing off a Coupon Date

? What happens if we trade the bond part-way through a coupon period?

The formula is modified so that:

- The first discount factor is raised to a power of **a**, where **a** is the fraction of the coupon period to the next coupon payment
- The second discount factor is raised to the power of **a + 1**
- The third discount factor is raised to the power of **a + 2**
- And so on.

Below is the industry-standard formula for calculating the present value of a bond.

$$\text{Present value} = \frac{C/t}{(1 + R/t)^{a}} + \frac{C/t}{(1 + R/t)^{a+1}} + \frac{C/t}{(1 + R/t)^{a+2}} + \ldots + \frac{\text{Principal} + C/t}{(1 + R/t)^{a+m}}$$

Where:
C = Coupon rate
R = Return on the investment (the yield)
t = Number of coupon payments per year (= compounding period)
m = Number of complete coupon periods to maturity

$$a = \frac{\text{Number of days to next coupon}}{\text{Number of days in current coupon period}}$$

Equivalent formulations:

$$\text{Present value} = C/t \times \Sigma \frac{1}{(1 + R/t)^{a+i}} + \frac{\text{Principal}}{(1 + R/t)^{a+m}}$$

for i = a … a+m

$$= C/t \times \Sigma D_i \quad + \text{Principal} \times D_{a+m}$$

Where D_i = Discount factor = $1/(1 + R/t)^{i}$

Different bond market sectors use different day-count conventions for calculating the **fractional coupon period** - the proportion of the current coupon period already elapsed - hence the variable **a** in the pricing formula, as we shall see in section *Accrued Interest*.

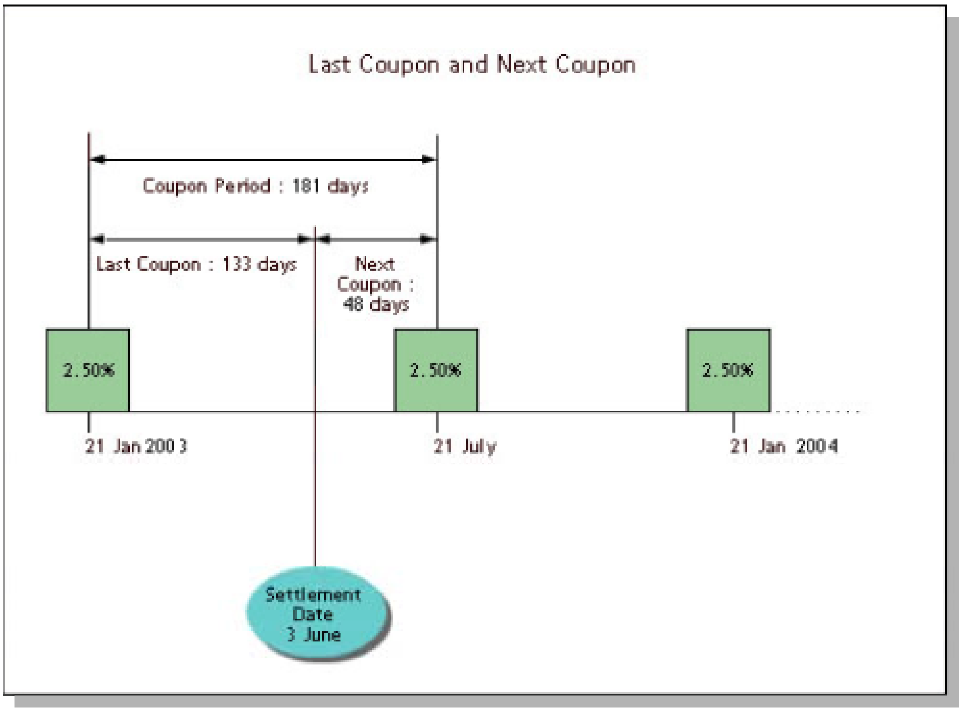
2.4. Pricing Example

Security: 5% US Treasury note maturing 21 January 2005
Type: Semi-annual
Settlement date: 3 June 2003

? What should be the price of this bond to give the investor a return (or yield) 8.00%?

Analysis

With a coupon rate of 5%, the bond will pay a coupon amount equal to 2.50% of its face value on 21 July and 21 January each year.[1] The figure below shows where we are in the current coupon period (21 January - 21 July 2003).



To find the present value of these future cash flows, the convention in this market is to proceed as follows:

1. Calculate the number of days from the settlement date (3 June) to the next coupon date (21 July) as a fraction of the current coupon period:

   48/181 = 0.2652

2. The next coupon is therefore 0.2652 of a coupon period away, so it is discounted by a factor $(1 + 0.08/2)^{0.2652}$.

   The second cash flow is then discounted by a factor $(1 + 0.08/2)^{1.2652}$, the third cash flow by $(1 + 0.08/2)^{2.2652}$, and so on.

[1] Or on the next business day, if the coupon date falls on a week-end or public holiday (the so-called **next business day convention**).

The bond’s present value is the sum of the present values of these cash flows:

Coupon Date	Interest Period	Cashflow	Discounted at 8%	Present Value
21 Jul 2003	0.2652	2.50	$\frac{2.50}{(1 + 0.08/2)^{0.2652}}$	= 2.47
21 Jan 2004	1.2652	2.50	$\frac{2.50}{(1 + 0.08/2)^{1.2652}}$	= 2.38
21 Jul 2004	2.2652	2.50	$\frac{2.50}{(1 + 0.08/2)^{2.2652}}$	= 2.29
21 Jan 2005	3.2652	102.50	$\frac{102.50}{(1 + 0.08/2)^{3.2652}}$	= 90.18
Total				**= 97.32**

Discounted at 8% the bond's present value comes to 97.32% of its face value, so a certificate which repays USD 1,000 at maturity would cost USD 973.20.

3. Clean and Dirty Prices

3.1. Effect of a Coupon Payment

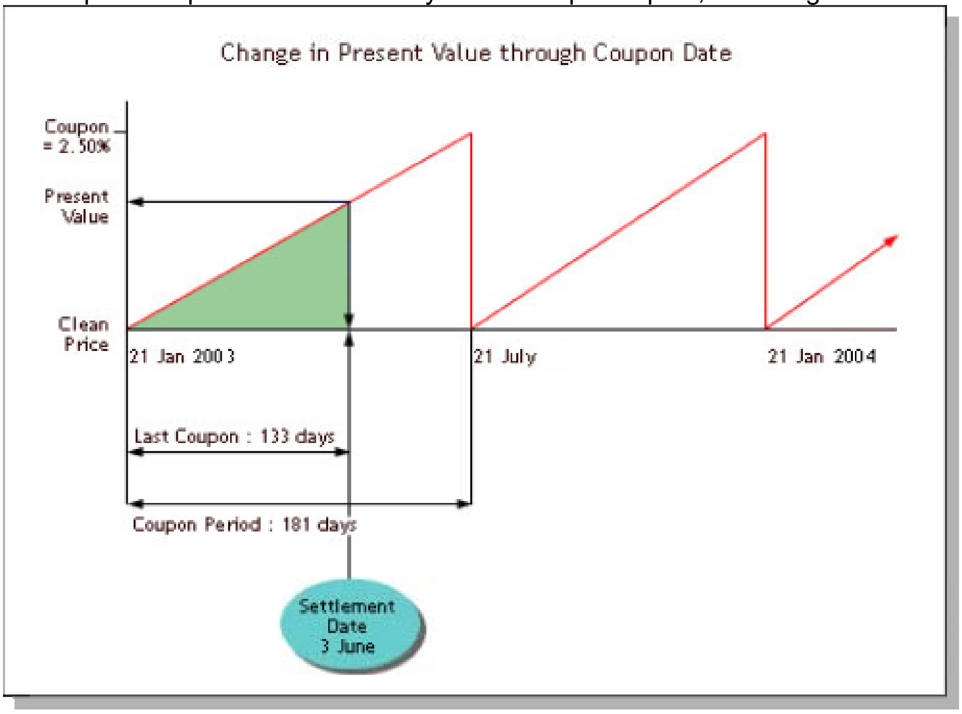
In the example in *Valuation Formula* we priced the following bond:

Security: 5% US Treasury note maturing 21 January 2005
Type: Semi-annual
Settlement date: 3 June 2003
Required yield: 8.00%
Present value: 97.32

Imagine that we hold this bond through the next coupon date, while market conditions remain the same. As the coupon date approaches all the future cash flows get closer, so the present value of the bond rises steadily.

! Then, on or before 21 July, the bond goes ex-coupon and, suddenly, the bond loses 2.5% of its market value!

Similar price drops would occur every time a coupon is paid, as the figure below illustrates.



To eliminate such 'technical' fluctuations the markets quote bond prices on a **clean** (or **flat**) basis: they subtract from the bond's present value the interest amount accrued since the start of the current coupon period (in the figure, the vertical distance between the bond's present value and the horizontal timeline). Thanks to this technique, other things being equal there should be no perceptible change in the bond's quoted price as it goes through a coupon date.

The clean price gives investors a measure of market value that is not affected by the payment of a coupon.

Accrued interest can be calculated directly from the bond's details, so in practice market makers quote bonds prices on a clean basis and leave it to their respective settlement departments to work out the accrued interest, and therefore the dirty price payable by the buyer (i.e. the bond's present value).

Accrued interest and clean price calculation

To calculate the clean price, the procedure is as follows:

Actual number of days since last coupon	(21 Jan - 3 June):	133
Actual number of days in current coupon period	(21 Jan - 21 July):	181

Fractional coupon period = 133/181
= 0.7348

Accrued interest = 0.7348 x 2.50
= 1.837 or **1.84**% rounded.

Subtracting this accrued interest from the bond's present value gives:

Clean price = 97.32 - 1.84
= 95.48[2]

3.2. General Formula

Clean price = Dirty price - Accrued interest

Dirty price = Settlement price
= Present value (as per pricing formula)

Accrued interest = C/t x Fractional coupon period

Where:
C = Coupon rate (%)
t = Number of coupon payments per year (= compounding period)

$$\text{Fractional coupon period} = \frac{\text{Number of days since last coupon}}{\text{Number of days in current coupon period}}$$

The fractional coupon period is calculated differently in different bond market sectors, depending on the day-count convention used:

- Actual/actual - as in the example here
- Actual/365
- 30/360 (for Eurobonds issued prior to January 1st 1999)

We shall examine these conventions in section *Accrued Interest*.

[2] Bond prices in the US Treasury market are quoted in 32nds, rather than in decimal, and are typically rounded to the nearest 64th of a percent. Thus, the 0.48 in the price of 95.48 is equivalent to:

48 x 32 / 100
= 15.36 or 15½ / 32, rounded.

A price like 95.48 would therefore be quoted as **95-15+** or **95*15+** (the "+" signifies the extra ½ tick over 15).

4. Accrued Interest

4.1. Principal Conventions

Not all bond markets calculate accrued interest in the same way. Below are the main day-count conventions used in the debt capital markets.

Debt Capital Market Accruals

$$\text{Accrued interest} = \text{Principal} \times \frac{\text{Coupon rate}}{\text{Nr. Coupons per year}} \times \text{Fractional coupon period}$$

Where Fractional coupon period is calculated in different ways, depending on local market conventions:

- Actual/Actual (e.g. US Treasuries; Eurobonds issued after 31 Dec 1998)
  = Actual number of days since last coupon was paid divided by the actual number of days in the current coupon period

- Actual/365 (e.g. Japanese Government bonds)
  = Actual number of days since last coupon was paid divided by 182.5 (if semi-annual coupons) or 365 (if annual coupons)

- 30/360 (e.g. US corporate bonds; Eurobonds issued before 1 Jan 1999)
  = Number of days since last coupon, assuming every month has 30 days, divided by 180 (semi-annual coupons) or 360 (annual coupons)

See Money Market Cash Instruments - Accrued Interest for a description of the day-count conventions used in the money markets.

The examples below illustrate the differences that can arise, depending on the day-count convention used.

Example 1
Accrued interest and settlement amount calculation with Actual/Actual convention.

Security:	5% British government bond maturing 21 January 2005
Type:	semi-annual
Settlement date:	3 June 2003
Quoted price:	97.32
Trade amount:	GBP 5 million.

Actual number of days since last coupon	(21 Jan - 3 June):	133
Actual number of days in current coupon period	(21 Jan - 21 July) :	181

The number of days in a coupon period could be anything between 181 and 184 days, depending on the dates and whether it's a leap year or not.

Fractional coupon period = 133/181
= 0.734806629834.

Accrued interest = 0.734806629834 x 2.50
= 1.83701657458 or **1.84%** rounded.

$$\text{Settlement amount} = 5{,}000{,}000 \times \left(\frac{97.32 + 1.83701657458}{100}\right)$$

= **4,957,850.83**

Example 2
Accrued interest and settlement amount calculation with Actual/365 convention.

Security:	5% Japanese Government Bond maturing 21 January 2005
Type:	Semi-annual
Settlement date:	3 June 2003
Quoted price:	97.32
Trade amount:	JPY 5 million.

Actual number of days
Since last coupon (21 Jan - 3 June): 133
Number of days in
Current coupon period: 365/2 = 182.5

It is assumed that every year has 365 days - even leap years! - and therefore on a semi-annual coupon bond each coupon period is exactly 182.5 days.

Fractional coupon period = 133/182.5
= 0.72876712329

Accrued interest = 0.72876712329 x 2.50
= 1.82191780822 or **1.82**% rounded.

Settlement amount $= 5{,}000{,}000 \times \left(\frac{97.32 + 1.82191780822}{100} \right)$

= **4,957,095.89**

Example 3
Accrued interest and settlement amount calculation with 30/360 convention.

Security:	5% Coca Cola maturing 21 January 2005
Type:	semi-annual (domestic US corporate)
Settlement date:	3 June 2003
Quoted price:	97.32
Trade amount:	USD 5 million.

Every month is assumed to have 30 days - even February! - and therefore every year has 360 days.

Number of days since last coupon:

21 January - 1 February:		10
1 February - 1 June:	4 x 30 =	120
1 June - 3 June:	-	2
Total		**132**

Number of days in current coupon period: 360/2 = 180

Fractional coupon period = 132/180
= 0.733...

Accrued interest = 0.733... x 2.50
= 1.833 or **1.83%** rounded.

Settlement amount $= 5{,}000{,}000 \times \left(\frac{97.32 + 1.833}{100} \right)$

= **4,957,666.67**

4.2. Selected Markets

The table below summarises the main compounding and accrued interest conventions used in the major bond market sectors.

	Coupon Frequency	Accrued Interest
Government Bonds		
USA	Semi-annual	Actual/Actual
Japan	Semi-annual	Actual/365
UK	Semi-annual	Actual/Actual
France	Annual	Actual/Actual
Germany	Annual	Actual/Actual
Netherlands	Annual	Actual/Actual
Canada	Semi-annual	Actual/365
Australia	Semi-annual	Actual/Actual
Italy	Semi-annual	Actual/Actual
Corporate Bonds		
USA	Annual or Semi-annual	30/360
UK	Semi-annual	Actual/365 or Actual/Actual[3]
Eurobonds		
Issued before 1/1/99	Annual (some Semi-annual)	30E/360[4]
Issued after 31/12/98	Annual (some Semi-annual)	Actual/Actual[5]

[3] For bonds issued after 31 December 1998, accrued interest is calculated on an Actual/Actual basis, instead of the traditional Actual/365.

[4] The 'E' in the 30E/360 (or ISMA) basis is to distinguish this convention from the one that applies in the domestic US corporate bond market, which is also 30/360 but does not include the **end-month rule**. The end-month rule means that the number of days from the 1st to the end of a 31-day month (e.g. 1 May to 31 May) is also counted as 29, rather than 30, which is how it would be counted under the US 30/360 convention. In all other respects the two conventions are identical.

[5] The ISMA accrued interest convention has been changed to Actual/Actual for Eurobonds issued after 1 January 1999, *unless the Eurobond is denominated in US dollars*, in which case accrued interest will continue to be calculated on a 30/360 basis (ISMA Rule 251).

5. Exercise 1

5.1. Question 1

Question 1

How many days of accrued interest are there on a bond maturing on
15 September 2006, for settlement on 2 November 2004, if the bond is:

a) A US Treasury (Actual/Actual)? Enter your answer in the box below, then validate.

b) A US domestic corporate bond (30/360)?

5.2. Question 2

Question 2

A 10% sterling corporate bond pays coupons on 21 January and 21 July and is bought for settlement on 6 June 2002. Calculate the amount of accrued interest payable on a GBP 1 million deal under the following day count conventions. Enter your answer rounded to the nearest pence.

a) Accrued interest, Actual/365 (GBP):

b) Accrued interest, Actual/Actual (GBP):

5.3. Question 3

Question 3

Security:	7.50% Coca-Cola maturing 15 December 2005
Type:	Eurobond, annual, 30E/360
Settlement date:	12 August 2004
Amount dealt:	USD 10 million
Yield:	6.75%

a) What is total accrued interest payable on this trade?

b) What is the bond's dirty price, rounded to the nearest 2 decimal places?

c) What is the clean price on this bond, rounded to the nearest 1/8%? (Enter in decimal).

d) Assuming you bought the bond at the price calculated in (c), what is the total settlement amount of the transaction?

e) Would you need to pay more or less for this bond if you required a yield of 7%?

- Less
- More

5.4. Question 4

Question 4

This exercise is too complex to perform with a simple calculator. You should use a financial calculator such as the HP 17 (or later model) or the bond pricing model spreadsheet.

Security:	7½% US Treasury bond maturing 21 October 2009
Type:	semi-annual, actual/actual
Settlement date:	20 April 2004
Yield:	7.00%.

a) What is the bond's clean price, rounded to the nearest 1/32%? (Enter in decimal).

b) What is its dirty price, rounded to 2 decimal places?

c) Given the same yield, what would be the bond's clean price for value 22 April 2004, rounded to the nearest 1/32%?

Enter the result in decimal, to 2 decimal places.

d) What would be its dirty price, rounded to 2 decimal places?

e) Explain the differences between (b) and (d), above.

- The clean price changed
- A coupon was paid on 21 April
- The bond has been 'sold down'
- The bond's yield rose

5.5. Question 5

Question 5

This exercise is too complex to perform with a simple calculator. You should use a financial calculator such as the HP 17 (or later model) or the bond pricing model spreadsheet.

Security:	7½ FNMA bond maturing 21 October 2009
Type:	USD Eurobond, annual 30/360
Settlement date:	20 April 2004
Yield:	7.00%.

a) What is the bond's clean price, rounded to 2 decimal places?

b) What is its dirty price, rounded to 2 decimal places?

c) Since the settlement, maturity, coupon and yield on this bond are the same as for the bond in *Question 4*, why are the prices of the two bonds different?

- This bond has a lower credit rating
- The two bonds were issued on different rates
- A semi-annual yield is equivalent to a lower annual yield
- This bond is annual 30/360; the other one is semi-annual Actl/Actl

6. Current Yield

6.1. Definition

$$\text{Current yield} = \frac{\text{Coupon rate}}{\text{Clean price}} \times 100$$

Also known as: **Running yield**; **Income yield**.

Example

Coupon rate: 6%
Clean Price: 95.00

? What is the bond's current yield?

Current yield = Coupon rate / Clean price x 100
= 6 / 95 x 100
= **6.32%**

Bond Price and Current Yield

If a bond trades:	Its CY will be:
At par	Equal to the coupon rate
At a premium to par	Less than the coupon rate
At a discount to par	Greater than the coupon rate

Limitations of Current Yield

Current yield is conceptually similar to the dividend yield on equities: it measures the income-generating capacity of the investment. But it ignores:

- Any interest earned on the reinvestment of the coupons received
- Any capital gain or loss realised when the bond is subsequently sold in the secondary market, or is redeemed at maturity.

If the bond in the example was held to maturity and redeemed at par the investor would also make a guaranteed capital gain of 5% on the asset.

Indeed, current yield is totally unsuitable for calculating the return on a zero coupon bond, since zeros do not pay any coupons. Zeros always trade at a discount to par and the return to the investor is implied in the capital gain made if the bond is held to maturity.

Despite its obvious shortcomings, current yield is often used in the context of equity convertible bonds where investors compare the current yield on the convertible with the dividend yield on the underlying equity. On this measure, convertibles typically yield more than the underlying equities, and this is one of their attractions.

6.2. Adjusted Current Yield

Adjusted Current Yield

$$= \left[\frac{\text{Coupon rate} + (100 - \text{Clean price}) / \text{Yrs. to maturity}}{\text{Clean price}} \right] \times 100$$

$$= \text{Current yield} + \left[\frac{(100 - \text{Clean price}) / \text{Yrs. to maturity}}{\text{Clean price}} \right] \times 100$$

Where:

Yrs. to maturity = Full years + Fraction of a year

$$\text{Fraction of a year} = \frac{\text{Number of days to next coupon}^{[6]}}{\text{Number of days in year}^{[7]}}$$

Also known as: **Japanese simple yield**.

Example

Coupon rate:	6%
Clean Price:	95.00
Maturity:	4.75 years

? What is the bond's adjusted current yield?

Current yield = 6 / 95 x 100
= 6.32%

$$\text{Adjusted current yield (ACY)} = 6.32 + \left[\frac{(100 - 95) / 4.75}{95} \right] \times 100$$

= 6.32 + 1.11
= **7.43%**

As we saw earlier, current yield ignores the fact that a bond bought at 95 will pay 100 at maturity, representing a substantial capital gain. ACY amortises this gain over the life of the bond (in this example at a rate of 1.11% per annum) and adds it to the current yield.

Bond Price and ACY

If a bond trades:	Its ACY will be:
At at par	Equal to the coupon rate
At a premium to par	Less than the coupon rate
At a discount to par	Greater than the coupon rate

Limitations of ACY

ACY is a relatively simple formula which used to be popular in the days before electronic calculators. However, it does have two flaws:

- Like current yield, it ignores any interest earned from the reinvestment of coupons
- It ignores the timing of the bond's cash flows - there is no allowance for the time value of money.

Nevertheless, ACY is useful as a quick reckoner of a bond's return and is still commonly quoted in some markets, notably Japanese government bonds.

[6] Using appropriate day-count convention.

[7] The concept of IRR was introduced in module Time Value of Money – Internal Rate of Return. In this section we review the advantages and disadvantages that yield to maturity shares with IRR as a measure of investment return.

7. Yield to Maturity

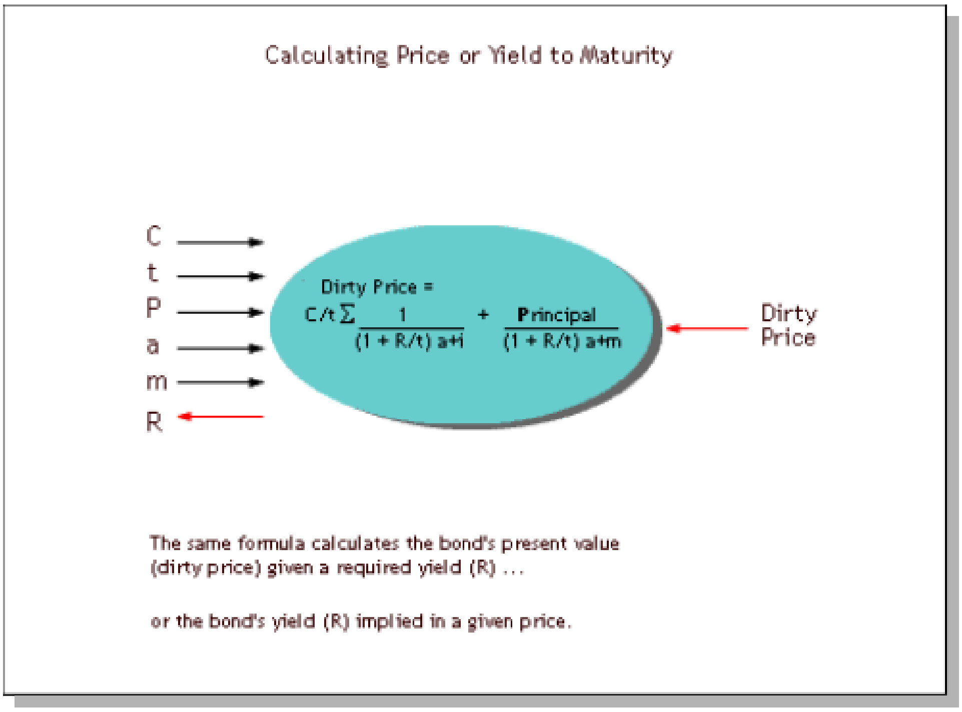
7.1. Definition

Yield to maturity (YTM):

- The uniform discount rate which makes the present value of a bond's future cash flows equal to its quoted dirty price
- The return that would be achieved on the bond if:
    - it is bought at the quoted price
    - *and* it is held until maturity
    - *and* any coupons received are reinvested at the same rate
- The internal rate of return (IRR) on all of the bond's cash flows, including the initial outlay[8]

This is the most widely used measure of return in the bond markets, and in fact when market participants speak of yield they typically mean yield to maturity (YTM).

To calculate the YTM on a bond we use the same pricing model that we developed in Bond Pricing - Valuation Formula, except that this time we use it 'in reverse', as the figure below indicates:



The formula is the same, only the direction of the calculation is different, but the only time when this formula can be reduced to a simple expression for R is in the case of a zero coupon bond.

[8] In module Credit Derivatives we shall see how the credit spread can be interpreted as an option premium for the bond investor and we show a relationship between that 'premium' and the probability of default.

Example

Security: Zero maturing in exactly 12 years
Compounding: Semi-annual
Price: 25.00

? What is the yield to maturity on this zero?

Applying the PV formula for a single cash flow that was introduced in module Time Value of Money – Present Value:

$P = 100 \times (1 + R/t)^{-t.T}$

Where:
P = Market price of zero (given)
R = IRR (the implied return)
t = Number of payments per year (= compounding period)
T = Number of years

Therefore:

$\mathbf{R = [(100 / P)^{1/(t.T)} - 1] \times t}$

$= [(100 / 25)^{1/(2.12)} - 1] \times 2$

= **11.893%**

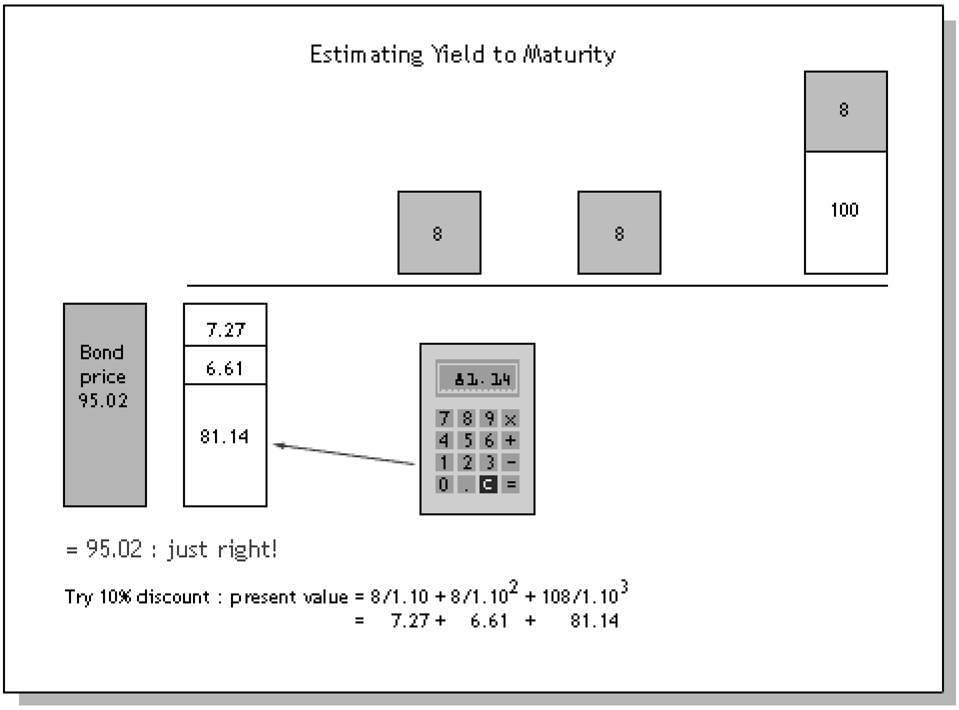
7.2. Yield on Straight Bonds

In the case of straight bonds the yield calculation is difficult to perform because the formula cannot be re-arranged into an expression for **R**. So **R** has to be found by trial-and-error (**iteratively**) by computing present values at different discount rates until we find the one that is equal to the bond's dirty price. Fortunately, dedicated bond calculators can do this in an instant.

Security: 8% bond maturing in exactly 3 years
Type: annual
Price: 95.03 (no accrued)

? What is the yield to maturity on this bond?

The figure on the next page illustrates how the YTM on this bond is arrived at iteratively, by trying different discount rates until we hit on the right one.



Bond price, yield and coupon rate

If a bond trades:	Its YTM will be:
At a par	Equal to the coupon rate
At a premium to par	Less than the coupon rate
At a discount to par	Greater than the coupon rate

Limitations of YTM

YTM takes into account all the three components of return:

- The periodic coupon payments
- Interest earned on the reinvestment of the coupons received
- A capital gain or loss realised when the bond is redeemed

But it makes two fundamental assumptions:

- That the investor actually holds the bond until maturity (there is no guarantee that the bond can be sold at par before maturity)

- That the coupons received will all be reinvested at the bond's YTM!

YTM is therefore a theoretical calculation: it does not compute the actual return that an investor will make on the bond, even if it was held to maturity. The actual return will depend on future reinvestment rates achieved.

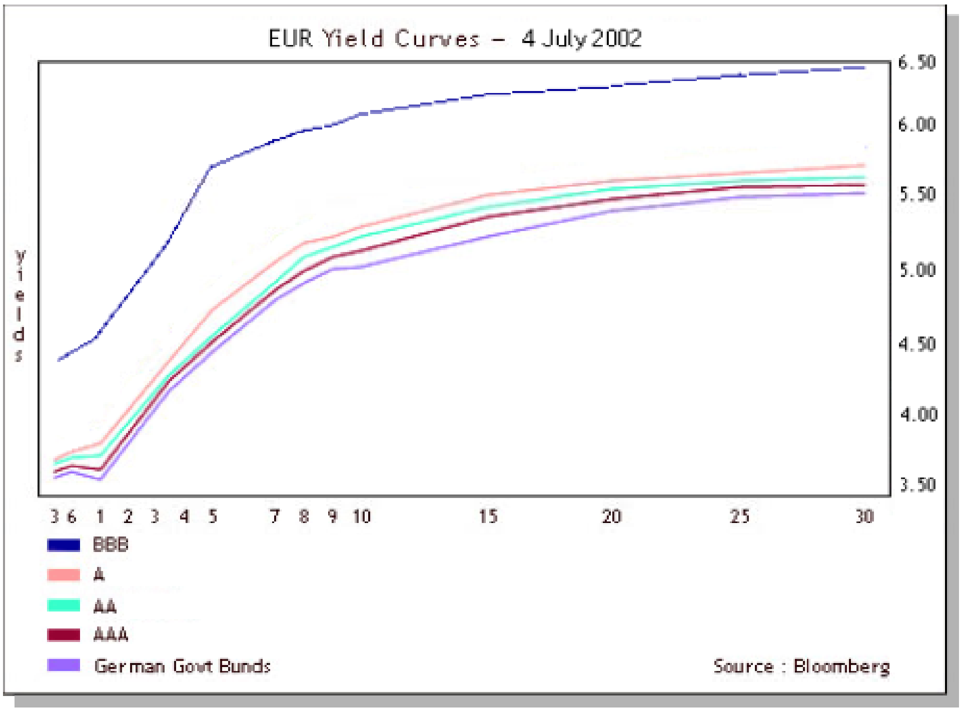
Nevertheless, YTM is useful as a means of comparing the return on bonds with similar maturity and credit quality: a bond may be considered cheap if it yields more than a comparable issue. In this context, assumptions about future reinvestment rates may be less critical, since the same reinvestment rates will apply to all the bonds being compared.

8. The Yield Curve

8.1. Definition

The yield curve displays graphically the relationship between interest rates (or yields) and term to maturity - the term structure of interest rates - for borrowers with comparable credit rating.

The figure below plots a separate yield curve for each class of borrowers with the same credit risk. We can see the pure 'price of waiting' most clearly in the yield curve for government securities (the **treasury curve**), which in most developed markets is assumed to be free of credit risk and is typically AAA-rated.



The yield curve serves a number of functions, summarised below:

A measure of the price of waiting

It is clear from the shape of these curves that lenders require a different price for locking debt capital into investments of different maturities. The cost of three or six-month money is lower than the cost of, say, two- or ten-year money.

A benchmark for pricing securities with different credit risks

Bonds with similar structural characteristics (coupon and maturity) and credit risk should trade at similar yields. New bond issues are typically priced at a **credit spread** to a comparable maturity treasury bond.

Credit spread: the difference between the yield on a bond and the yield on a risk-free security with the same maturity.

The credit spread on bonds represents the investor's credit risk premium for holding an investment that has credit risk – i.e. where there is a risk of the issuer defaulting.

For each rating, the credit spread reflects the market's view on the probability of the issuer defaulting and the likely loss to the investor in such a scenario; the higher the probability of default and/or the loss on default, the higher is the credit spread[9].

A reflection of market expectations about future rates

The shape of the yield curve contains important information about the expected or forward path of short term interest rates. In module Forward Yield - Derivation, we shall see how we can derive the forward rates implied in the current shape of the yield curve.

It also provides information about future inflation expectations when the shape of he nominal yield curve is compared with that of the real yield curve, as we explain in section *Real Yield*, below.

We shall continue our discussion of the factors that drive the shape of the yield curves in module Yield Curve Dynamics below.

[9] For bonds issued after January 1999, accrued interest is calculated on an Actual/Actual basis, instead of the traditional Actual/365.

9. Yield to Call/Put

9.1. Yield to Worst

? So far we have considered the yield to maturity on bullet bonds, but how do you calculate the yield on bonds with uncertain maturity dates, such as callable bonds?

The traditional approach is to calculate the yield to each call date, as well as the yield to maturity, and then take the worst of these yields as the basis for the investment decision.

A yield to call is similar to a yield to maturity, except that in the pricing model we use:

- The call date instead of the maturity date
- The call price, rather than par, as the principal repayment amount.

As with yield to maturity, yield to call assumes that any coupons received will be reinvested at the same rate.

9.2. Example

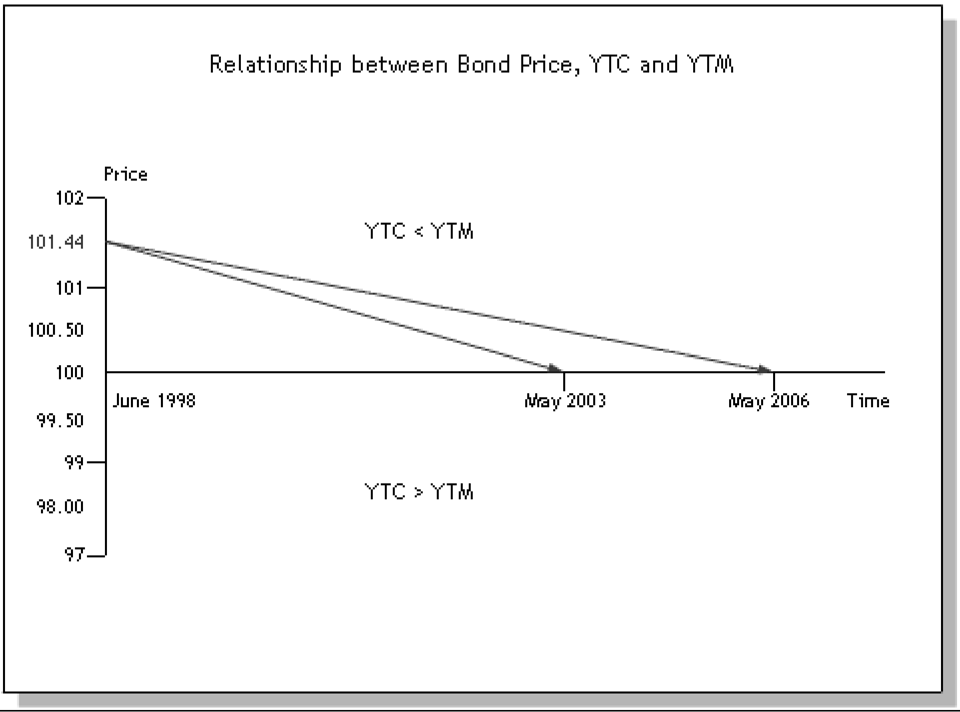
Security: 8% UK government maturing 5 May 2006
Type: domestic **double-dated gilt**,
semi-annual, actual/actual

Call features: callable at par on 5 May 2003
Settlement: 18 June 2002
Price: 101.44 (decimal)

Analysis
Yield to maturity (YTM on 5 May 2006): 7.56%
Yield to call (YTC on 5 May 2003): 6.28%

In the current market, yields are lower than the 8% coupon rate, so the bond trades at a premium to par and there is a built-in capital loss of 1.44% if the bond is redeemed at par, either in 2003 or in 2006. If the bond is called in 2003, the capital loss would have to be amortised over a shorter period, and that is why in this case YTC is lower than YTM.

In a different scenario, if yields were higher than 8% the bond would trade at a discount to par and its YTC would be higher than its YTM.



If the bond trades at a premium, the worst-case scenario is that it is called in 2003. In this case the investor earns 6.28%. If the investor finds this yield attractive enough, then clearly the bond is worth buying because in any other scenario (e.g. the bond is not called) the return would be higher.

9.3. Valuing the Embedded Option

Focusing on the worst-case scenario makes sense, because with a callable bond that is also the most likely scenario:

- If the bond trades at a premium (i.e. market yields are low), it is quite probable that the issuer will call the bond, financing the repurchase through the issue of a new bond carrying a lower coupon. The investor is therefore likely to end up earning the YTC, which in this scenario is lower than the YTM.

- On the other hand, if the bond trades at a discount it is unlikely that the issuer will call the bond at or above par. The investor is more likely to end up earning the YTM, which in this scenario is lower than the YTC.

But focusing exclusively on the yield to worst is an ultra-conservative approach to investment, because we are always assuming that the worst *will* happen, without considering the possibility that it may not.

A more sophisticated approach is to value the embedded option in the callable bond and then calculate its **option-adjusted yield** as a weighted average of YTM and YTC, where the weights are proportional to the probability of the embedded option being exercised (see Structured Securities – Callable Bonds - Pricing). However, this approach is significantly more complex than a simple yield to worst calculation, so many investors still use yield to worst as their benchmark.

9.4. Yield to Best

A similar approach may be used for valuing putable bonds. Here it is the investor who has the option to obtain early repayment and therefore can be assured the most favourable yield. The method is as follows:

- Calculate the yield(s) to put:
    - using the put date(s) instead of the maturity date
    - and the corresponding put price(s) instead of par

- Calculate the YTM

- Take the best of these as the basis for the investment decision.

10. Yield Conversions

10.1. Annual & Semi-annual Yields

Yields on fixed income securities are not always comparable because of the different compounding and accrued day-count conventions used in various markets.

The formulas for converting between money market rates and bond equivalent yields are discussed in *Money Market Cash Instruments - Yield Conversions*. In this section we look at the conversions necessary to compare yields on bonds with different coupon periods.

We saw in *Time Value of Money - Simple and Compound Interest* that the effective - as opposed to the nominal - interest rate depends on the compounding frequency: the shorter the compounding period, the higher is the effective rate. The example below shows that quoted yields on annual and semi-annual coupons are not directly comparable.
The example below shows that quoted yields on annual and semi-annual coupons are not directly comparable.

Example

Security 1:	8% US Treasury bond maturing 5 May 2010
Type	Domestic, semi-annual, actual/actual
Settlement	18 June 2002
Quoted yield	7.75% (semi-annual)
Price:	101.44 (decimal)

Security 2:	8% AAA-rated bond maturing 5 May 2010
Type	Eurodollar bond, annual, 30/360
Settlement	18 June 2002
Quoted yield	7.75% (annual)
Price	101.40 (decimal)

? Why does the US Treasury bond have a higher price than the USD Eurobond, if both have the same credit rating and have identical coupon rates, maturities and quoted yields?

Because the US Treasury bond pays coupons semi-annually, so the investor receives one half of the next coupon rate earlier (on 5 November) and the balance on 5 May 2003, whereas the Eurobond does not pay anything until 5 May 2003. The Treasury bond's cash flows have a higher present value than the Eurobond's.

Put a different way, for the same price the Treasury bond's effective (annual) yield would be higher than the Eurobond's.

10.2. The General Formula

We can use the interest rate conversion formula developed in Time Value of Money - Simple and Compound Interest to convert between semi-annual and annual-equivalent yields.

Annual and Semi-annual Yield Equivalences
(IFID exam)

$$\text{Annual yield} = \text{Effective semi-annual yield}$$

$$(1 + \text{Annual yield}) = (1 + \text{Semi-annual yield}/2)^2$$

Therefore:

$$\text{Annual yield} = (1 + \text{Semi-annual yield}/2)^2 - 1$$

$$\text{Semi-annual yield} = [\sqrt{(1 + \text{Annual yield})} - 1] \times 2$$

In our example, the annual yield equivalent on the US Treasury bond would be:

$= (1 + 0.0775 / 2)^2 - 1$
= 0.0790 or **7.90%**

The 15 basis points difference with the yield on the Eurobond is clearly significant: the unwary investor might interpret the gilt as being overpriced, whereas in fact it is not.

The higher the yield level, the wider is the gap between annual and semi-annual yields.

10.3. Selected Markets

The table below summarises the accrued interest and yield conventions used in the major fixed income markets.

	Coupon Frequency	Accrued Interest	Yield Convention
Government Bonds			
USA	Semi-annual	Actual/Actual	YTM, Semi-annual
Japan	Semi-annual	Actual/365	ACY, Semi-annual
UK	Semi-annual	Actual/Actual	YTM, Semi-annual
France	Annual	Actual/Actual	YTM, Annual
Germany	Annual	Actual/Actual	YTM, Annual
Netherlands	Annual	Actual/Actual	YTM, Annual
Canada	Semi-annual	Actual/365	YTM, Semi-annual
Australia	Semi-annual	Actual/Actual	YTM, Semi-annual
Italy	Semi-annual	Actual/Actual	YTM, Annual
Corporate Bonds			
USA	Annual or Semi-annual	30/360	YTM, Semi-annual
UK	Semi-annual	Actual/365 or Actual/Actual[10]	YTM, Semi-annual
Eurobonds			
Issued before 1/1/99	Annual or Semi-annual	30E/360[11]	YTM, Annual
Issued after 31/12/98	Annual or Semi-annual	Actual/Actual[12]	YTM, Annual
Money Markets			
Fixed deposits, CDs	Bullet, but periodic if longer than 12 mths.	Actual/360 or Actual/365	Money market yield
T-bills, BAs, CP	No coupons	Actual/360 or Actual/365	Discount rate

[10] See modules Interest Rate Swaps – Asset Swaps and Credit Derivatives – Credit Default Swaps

.

[11] The 'E' in the 30E/360 (or ISMA) basis is to distinguish this convention from the one that applies in the domestic US corporate bond market, which is also 30/360 but does not include the **end-month rule**. The end-month rule means that the number of days from the 1st to the end of a 31-day month (e.g. 1 May to 31 May) is also counted as 29, rather than 30, which is how it would be counted under the US 30/360 convention. In all other respects the two conventions are identical.

[12] *Unless the Eurobond is denominated in US dollars*, in which case accrued interest will continue to be calculated on a 30/360 basis (ISMA Rule 251).

11. FRN Discount Margin

11.1. FRN Structure

The floating rate note (FRN) is an interest bearing security whose coupons are paid on a regular basis at a given margin (the **quoted margin** or **given margin**) relative to a money market interest rate index, such as the 3- or 6-month Euribor or LIBOR.

So far in this module we have focused exclusively on straight bonds. The FRN is a hybrid between a debt capital market and a money market security:

- Its original maturity typically exceeds 12 months (indeed most FRNs have longer maturities than straight corporate bonds) and its price is quoted as a percentage of face value, like a bond

- It replicates a string of bank deposits with automatic rollovers at a constant spread over the index

As we shall discuss in module Interest Rate Risk, unlike a straight bond the FRN's price is not very sensitive to changes in market rates because its coupons are adjusted periodically in line with the market. However, its price is very sensitive to changes in the issuer's credit quality. As such, the FRN is closely related to the **asset swap** and the **credit default swap** markets, which we shall examine later in this programme[13].

The FRN is primarily a credit risk instrument.

Example

An FRN rated single-A pays LIBOR + 45 basis points and is issued at par.

If the debt of the issuer was to be downgraded to BBB/Baa, then this note should trade at a significant discount to par in the secondary market, because a BBB note should perhaps pay LIBOR +145.

Someone buying the note below par and holding it to maturity will earn the quoted spread over LIBOR on the note plus a capital gain at maturity, which together should make up to an effective LIBOR + 145 basis points per annum.

11.2. Simple and Discount Margins

? How do you measure the effective spread (or margin) over the index on an FRN that is not trading at par?

Simple margin
A simple way of expressing the effective margin that an investor would earn if the FRN is held until maturity, taking into account the quoted margin and any capital gain or loss at redemption, is as follows:

Simple margin

$$= \left[\frac{(100 - \text{Clean price})}{\text{Maturity in years}} + \text{Quoted margin} \right] \times \frac{100}{\text{Clean price}}$$

Also known as: **Spread for life**

[13] As we shall see in module Interest rate Swaps – Pricing, the swap rate is effectively a single rate of interest that represents a strip of forward LIBORs, so this is the justification for fixing all the future LIBORs at the current market swap rate.

The capital gain or loss made or the note is spread over its remaining life on a straight-line basis (similar to the way we calculate the adjusted current yield on a straight bond): without taking any time value of money into account.

A better and nowadays generally used measure is the **discount margin**.

Discount margin:

- The spread over the index at which the note effectively trades, given its current market price
- The spread over index which should be paid on the FRN in order to make its market price equal to par

Also known as: **Effective LIBOR spread**.

Finding the discount margin involves present valuing the FRN's future cash flows similar to the way we do on a straight bond. In order to do this, however, we need to somehow 'fix' all the future LIBORs on the note and this is done by setting all the future LIBORs at the current market rate for an interest rate swap with the same maturity as the FRN[14].

The exact formula for calculating the discount margin is a bit involved:

$$NP = \frac{(SR + LS) \times d_1}{(1 + Y \times d_0)^1} + \frac{(SR + LS) \times d_2}{(1 + Y \times d_1)^2} + \ldots + \frac{(SR + LS) \times d_n + Par}{(1 + Y \times d_n)^n}$$

Where:

NP = The FRN's **neutral price**
Neutral price
= Cash clean price + Net cost of carrying FRN to the next coupon date

d_i = Number of days in coupon period i divided by 360 or 365, depending on the money market accrued interest day-count convention used
(see Money Market Cash Instruments - Accrued Interest)

LS = The note's quoted spread over LIBOR

SR = The quoted market swap rate covering the period from the next LIBOR fixing on the FRN until its maturity

Y = The discount rate that makes the PV of the FRN's future cash flows equal to its neutral price

N = Number of complete coupon periods to maturity

Notice how the formula first carries the FRN forward to the next coupon date and only then it calculates the discount margin. The neutral price is calculated from the note's given cash price in the same way as we calculate a breakeven forward price[15] for a bond.

Once we have calculated the neutral price from the cash price, we then solve for Y by a process of **iteration** as we do with bond yields - trying different values for Y until we arrive at the one that equates the PV of the FRN's future cash flows with its neutral price. Having calculated Y, we then back out of it the discount margin:

Discount margin = Y - SR

The discount margin gives a more accurate valuation of the note's effective yield spread over the index, but unless the yield curve is very steep the pricing differences between the two methods of . Whichever method you use:

If the note trades at par: Discount margin = LIBOR spread
If the note trades at a discount to par: Discount margin > LIBOR spread
If the note trades at a premium to par: Discount margin < LIBOR spread

[14] Using the day-count convention appropriate to the bond being analysed.
[15] We shall calculate forward bond prices in module Bond Futures – Pricing.

11.3. Example

Security:	Asian Development Bank GBP FRN maturing 29 October 2010
Rating:	Single A
Coupon rate:	6 month LIBOR + 0.15%
Day-count:	Act/365
Settlement:	15 April 2002
Swap rate to 29 Oct 2010:	5.25% (semi-annual, Act/365)
Neutral price:	98.75

? What is the discount margin on this security?

Analysis

The first step is to calculate the coupon rate on the FRN by 'fixing' all the LIBORs at the current market swap rate:

Coupon = 5.25 + 0.15
= 5.40%

Warning: when calculating a discount margin, always be careful to use a swap rate quoted on the same settlement frequency and day-count basis as the coupons on the FRN.

Here, both the spread on the FRN and the swap rate are quoted on the same basis (semi-annual, Act/365), so we can add the two figures directly. If the swap rate had been quoted on a different basis to the FRN spread, then we would first have to convert it onto a semi-annual, Act/365 equivalent before adding the figures. The conversions required were explained in section *Yield Conversions* above.

Having fixed the LIBORs, the second step is to compute the yield to maturity on this 5.40% 'bond', given its price of 98.75. This you can do using either a financial calculator or the bond pricing model provided.

Calculated yield (semi-annual, actual/365) = **5.586%**

Finally, we calculate the discount margin as the difference between this calculated yield and the swap rate used:

Discount margin = 5.586% - 5.250%
= **0.336%**

In other words, if the note paid LIBOR + 34 basis points, instead of LIBOR + 15, then it would trade at par. An investor expecting to earn LIBOR + 40 on this single-A rated paper would therefore consider this note to be trading rich. Using the same formula as above, the investor could calculate what price he should pay for this note in order to earn the discount margin of LIBOR + 40.

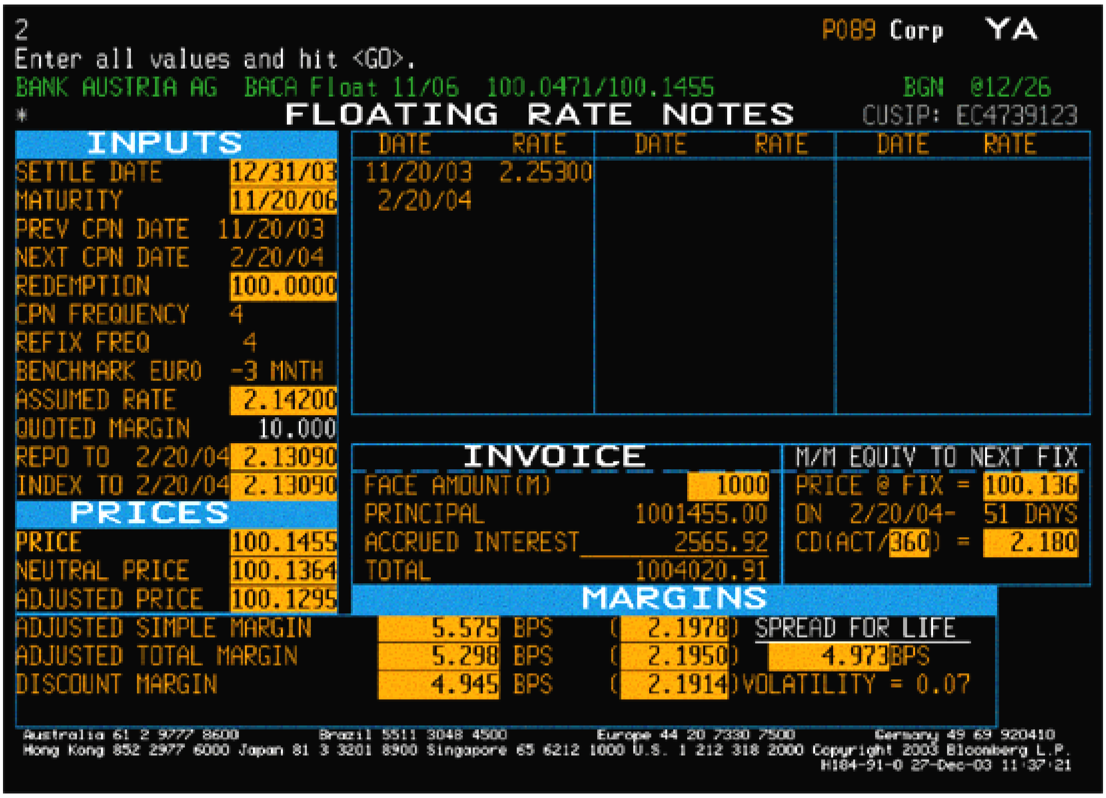
Analytic systems

Examples of Bloomberg and Reuters FRN price analysis functions

Below are sample screens from two widely-used providers of market information and analytics.

These examples are for illustration purposes only and do not form part of the IFID Certificate syllabus.

Bloomberg FRN analysis

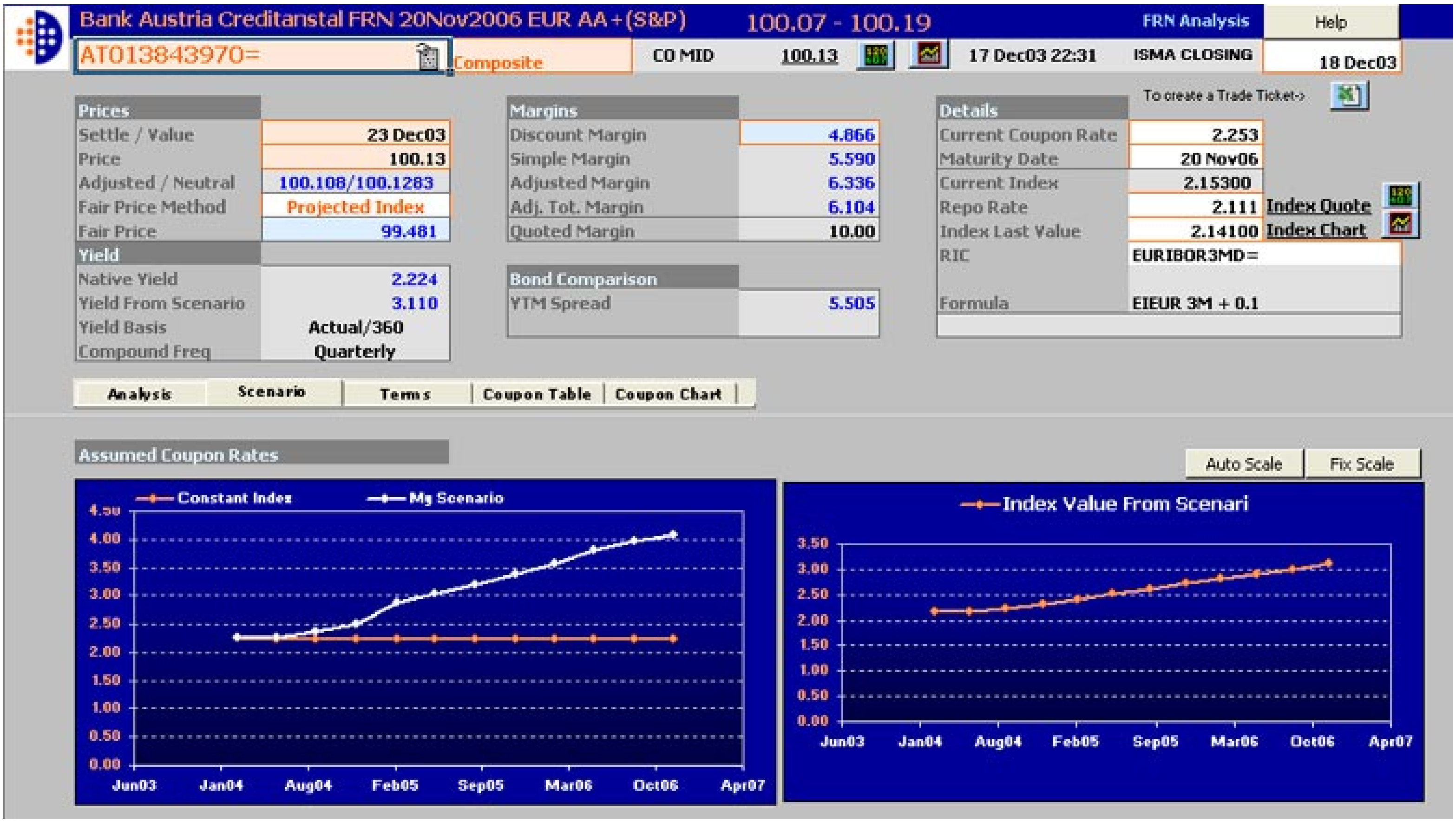


Notes

Using this function, you can calculate either the price payable on a FRN, given a margin that you specify, or any one of three measures of margin, given a specified price. Each of the three calculated margins is shown on the page in basis points, with its effective current valuation (the calculated margin plus the current index) in brackets.

The function calculates two variations of the simple margin concept explained in this section (the **adjusted simple margin** and the **adjusted total margin**) in addition to the discount margin. Neither of those measures of effective spread are nowadays commonly used and they are therefore not included in the IFID Certificate syllabus.

Reuters FRN analysis



Notes

- The page calculates the discount margin on the selected FRN, given a cash price or the **fair price** for the note given a discount margin
- It also displays other measures of margin that are not included in the IFID Certificate syllabus because they are not commonly used.

12. Horizon Yield

12.1. YTM and Horizon Yield Compared

We have seen how yield to maturity (YTM) is limited by the assumptions that:

- The bond will be held until maturity
- And any coupons received will be reinvested at the same rate (the YTM).

Horizon return or yield (HY) is a method of calculating the actual return that will be earned on the bond, taking into account:

- A given holding period (or horizon date), which may be earlier than maturity
- *And* any desired reinvestment rate assumption.

Horizon return, or yield (HY), takes into account:

- A given **holding period** (or **horizon date**), which may be earlier than maturity
- And any desired reinvestment rate assumption.

The example below walks you through the calculation, just to give you an idea of the steps involved.

Example 1

Settlement date:	12 March 2002
Security:	8% Eurobond maturing 12 March 2012
Type:	annual, 30/360
Price:	90.00

Calculated yield to maturity: **9.60%**

What is the horizon yield if the investor expects to:

- Hold the bond for 3 years, at which point it is expected to trade at 93.00 (for a YTM of 9.41%)

- And reinvest all the coupons received at 7% (annual 30/360)?

Analysis
The procedure is as follows:

Step 1: calculate the **future value** of all the coupons plus interest on these at 7%.

There are 3 coupon payments payable during the holding period: the first one next year which will be reinvested for 2 years, so its future value will be $8 \times (1 + 0.07)^2$.

The next coupon is payable in 2 years and will be reinvested for one year, so its future value will be 8 x (1 + 0.07). The final coupon is payable at the end of year 3 and will not be reinvested.

The future value of the coupons, including reinvestment income, will therefore be:

$$= 8 \times (1 + 0.07)^2 \quad + \quad 8 \times (1 + 0.07)^1 \quad + \quad 8$$
$$= 8 \times [\ (1 + 0.07)^2 \quad + \quad (1 + 0.07)^1 \quad + \quad 1\ ]$$
$$= 8 \times [\ 1.1449 \quad + \quad 1.07 \quad + \quad 1\ ]$$
$$= 25.7192\%$$

Step 2: add the future value of the coupons to the estimated future price of the bond, to arrive at the expected **horizon cash flow** - i.e. its total future value:

Horizon cash flow
= 25.7192 + 93.00
= 118.7192

Step 3: use the present value formula developed in Time Value of Money - Present & Future Value to calculate the yield implied in the two cash flows: payment of 90.00 today for a return of 118.7192 in 3 years.

$$\text{Present value} = \frac{\text{Horizon cash flow}}{(1 + \text{Horizon yield} / t)^{n \times t}}$$

Where:
t = Compounding frequency (in this case 1)
n = Number of years

In this case:

$$90.00 = \frac{118.7192}{(1 + \text{Horizon yield})^3}$$

$$\text{Horizon yield} = \left\{ \frac{118.7192}{90.00} \right\}^{1/3} - 1$$

$$= 0.09671 \text{ or } \mathbf{9.67\%}$$

Conclusion
Calculated yield to maturity: **9.60%**
Calculated horizon yield: **9.67%**

By making his own assumptions, the investor has arrived at a yield estimate that is different from the bond's yield to maturity. Not surprisingly, this is higher than the yield to maturity. Of course, there is no guarantee that the assumptions made will be borne out.

You can verify these results using the bond pricing spreadsheet.

The general formula

Calculating horizon yield is complex enough when the settlement and the horizon dates fall on coupon dates. The general formula below, which is presented for reference purposes only, can also handle situations when this is not the case.

$$\text{Horizon return} = \left[\left\{ \frac{\text{Horizon cash flow}}{\text{Cash dirty price}} \right\}^{1/(a+m+b)} - 1 \right] \times t$$

Where:
Horizon cash flow = Horizon dirty price + Future value of coupons

$$\text{FV of coupons} = C/t \times [(1 + r/t)^{m+b} + (1 + r/t)^{m+b-1} + \ldots + (1 + r/t)^{b}]$$

$$= C/t \times (1 + r/t)^{b} \times [(1 + r/t)^{m} + (1 + r/t)^{m-1} + \ldots + 1]$$

$$\text{Which reduces to} = \frac{C/t \times (1 + r/t)^{b} \times [(1 + r/t)^{m+1} - 1]}{r/t}$$

C = Coupon rate
t = Number of coupon payments per year (= compounding period)
r = Reinvestment rate on the coupons
m = Number of complete coupon periods to horizon date
a = (1 - Current fractional coupon period)
b = Fractional coupon period at horizon date

$$\text{Fractional coupon period} = \frac{\text{Number of days since previous coupon}^{16}}{\text{Number of days in coupon period}^{17}}$$

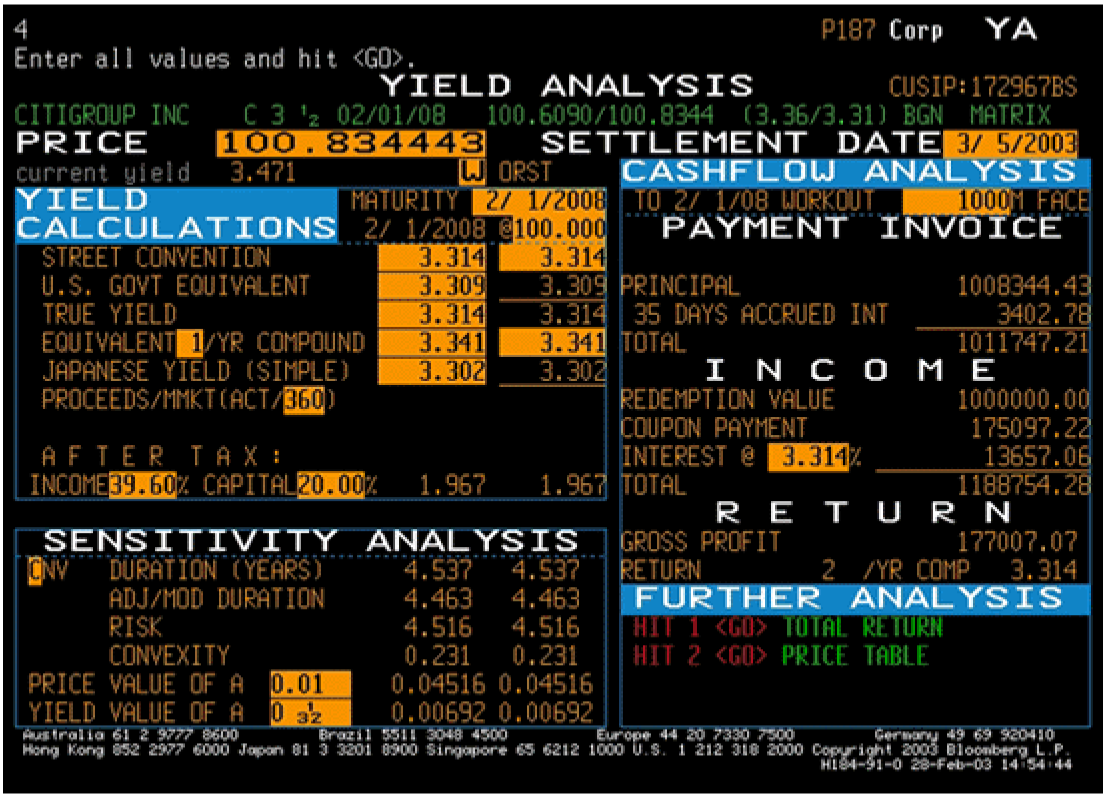
Analytic systems

Examples of Bloomberg and Reuters bond yield analysis functions

Below are sample screens from two widely-used providers of market information and analytics.

These examples are for illustration purposes only and do not form part of the IFID Certificate syllabus.

Bloomberg Yield Analysis



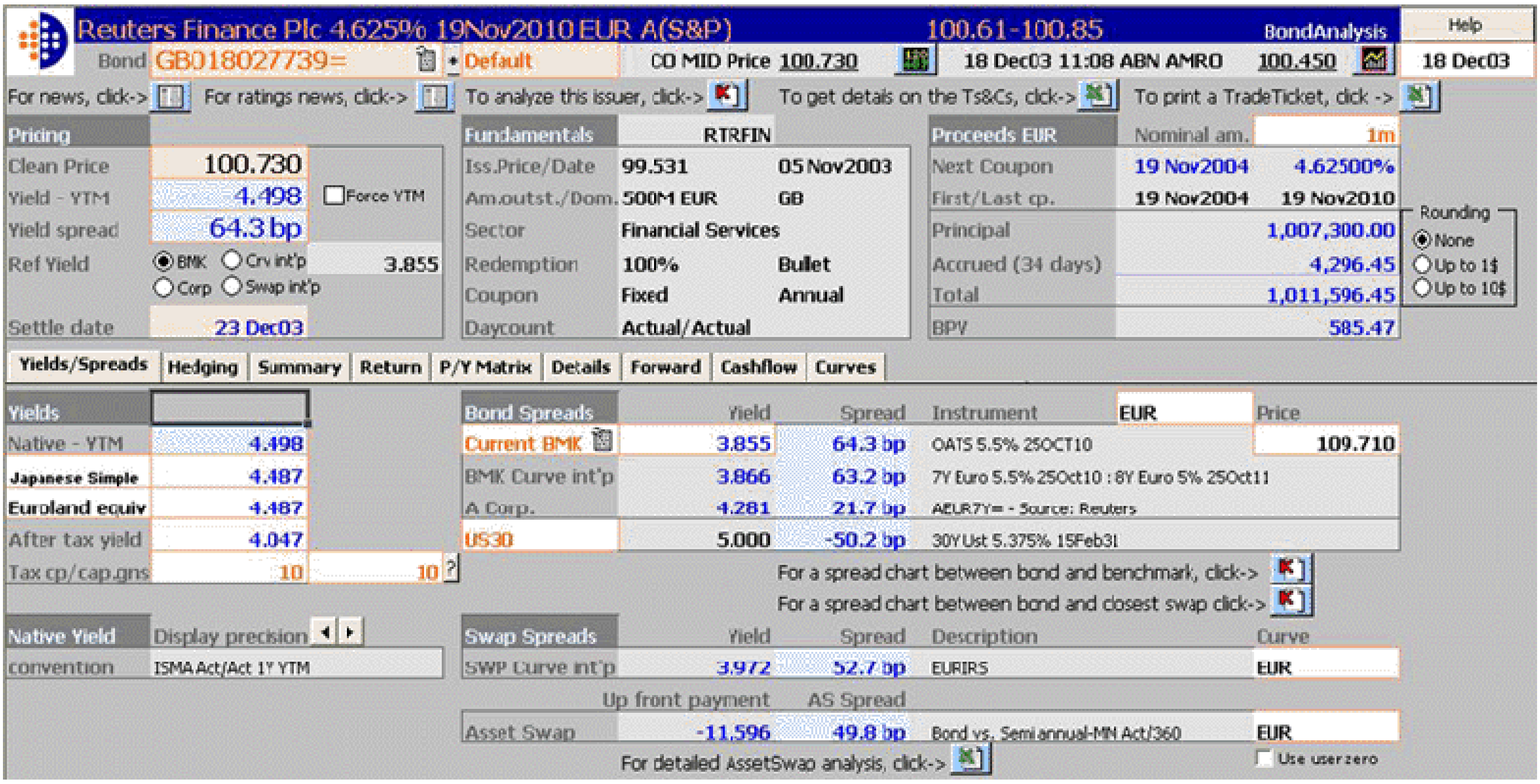
[16] This list is by no means comprehensive as governments in many developing countries are also regular ILB issuers, notably Brazil, Colombia, Czech Republic, Mexico, Hungary and Israel, among others. Some supra-national entities such as the European Investment Bank have also been significant issuers of ILBs in recent years.

[17] The UK Treasury was one of the first OECD countries to issue ILBs back in 1981 and the methodology that it developed to price them suffered from being a first prototype for the linker market. At the time of writing the UK government is considering adopting the Canadian model as well.

Notes

- **Street convention** is the yield calculated using the compounding and accrued day-count conventions of the market in which the selected security is traded (in this case, semi-annual 30US/360)
- **US government equivalent** is the yield calculated on a semi-annual Act/Act basis
- **True yield** is a semi-annual Act/Act, in other words taking into account the exact dates when the coupons and the principal will be paid, moving the dates forward whenever they fall on a week-end or public holiday
- **Japanese simple yield** is the adjusted current yield
- The *Cashflow Analysis* panel allows you to calculate the horizon return on the bond to an assumed horizon (or **workout**) date, bond resale price and reinvestment interest rate on any coupons received during the holding period. (In this case the specified horizon date is the bond's maturity and its reinvestment rate is its YTM so, as you would expect, the horizon return is the same as the YTM!)
- Please refer to module Market Risk – Convexity for a description of the information given under the *Sensitivity Analysis* panel on this screen

Reuters Bond Analysis



Notes

- **Native YTM** is the yield calculated using the compounding and accrued day-count conventions of the market in which the selected security is traded (in this case, annual Act/Act)
- **Japanese simple yield** is the adjusted current yield
- This being a EUR denominated bond, its yield spread is measured here over that of a 5.50% French government bond (**OAT**) maturing in 2010, but notice the calculation of the yield spread over other benchmarks:
  - An interpolated benchmark curve
  - The A-rated corporate yield curve
  - The 30 year US Treasury
  - The swap curve (i.e. the yield on this bond on an **asset swap** basis – see module Interest Rate Swaps – Asset Swaps)

13. Inflation-linked Bonds

Inflation-linked bonds (ILBs) are bonds whose cash flows are linked to an inflation index.

Also known as: **Index Linked Bonds** (UK); **Treasury Inflation Protected Securities – TIPS** (US).

ILBs pay coupons at fixed intervals and return the principal at maturity, like any conventional bond. However, in the most common structure known as the **capital indexed bond**, both the coupons and the principal are adjusted in line with the rate of inflation over the bond's life. Therefore, the bond returns a real (constant purchasing power) set of coupons and principal amount.

Benefits

ILBs are attractive instruments for investors and issuers alike:

- Investors are hedged against inflation risk, as the cash flows on their investments are adjusted in line with inflation
- Commercial borrowers can continue issue debt in high inflation environments in the confidence that the coupons on their ILBs will fall once inflation returns back to normal, which would of course not be the case if they had issued straight bonds
- ILBs reduce the incentive for the government to 'print money' in order to fund its fiscal deficit, which is inflationary, as this would also inflate the borrowing costs on its ILBs
- ILBs may help monetary authorities to obtain useful information about market expectations on future inflation (see section *Real Yield*, below)

Typical commercial issuers of inflation bonds are public utilities, public sector contractors or real estate companies whose revenues are contractually in some way linked to inflation, in some cases by law – e.g. household utilities, train and bus operators, public buildings and facilities maintenance companies, social housing corporations, etc.

Government Issuers

The table below lists some of the main issuers of ILBs in the government sector[18]:

Issuer	**Name**	**Reference index**
Australia	Treasury Indexed Bonds (TIBs)	CPI[19]
Britain	Index Linked Treasuries (ILTs)	RPIX[20]
Canada	Real Return Bonds (RRBs)	CPI
France	OAT*i*	French CPI excluding tobacco
	OAT*ei*	Eurozone HICP[21], ex tobacco
USA	Treasury Inflation Protected Securities (TIPS)	CPI-U[22]
South Africa	CPI linked bonds	CPI – all items for the metropolitan areas
Sweden	Inflation-linked bonds	CPI

[18] The concept of inflation risk premium is analogous to the liquidity risk premium in the liquidity preference theory of interest rates which we introduced in section *The Yield Curve*, above. The theory explains the difference between short and long-term nominal yields as the result of two factors:
1. The market expectation of future rates
2. A liquidity risk premium to compensate the investor for the higher market risk on long-dated bonds

[19] Consumer price index

[20] Retail price index excluding mortgage interest payments

[21] Harmonised index of consumer prices

[22] CPI for urban consumers

13.1. Inflation Hedging in Practice

ILBs are designed to protect the investor against inflation risk.

In reality, ILBs can never fully eliminate inflation risk for the investor for two main reasons:

Lags in the publication of inflation data

To assure full inflation protection, all the cash flows on the bond should be adjusted for the amount of inflation that has occurred from the date when the bond was issued right up to the moment when these are paid.

In reality, there is always a lag between the time when the most recent inflation statistic is published and the time when a cash flow is paid (as we shall see in the next section, below). In most countries it takes approximately 2 months for the inflation statistics to be compiled and published.

This means that any inflation adjustment to the cash flows on the bond must necessarily be based on data that may not reflect the inflation reality at the time they are paid. As we show below, issuers of ILBs attempt to bridge this information gap in different ways, but the fact remains that some element of inflation risk remains with the investor; in particular, the investor loses out in an accelerating inflation scenario.

Basis risk on the reference inflation index

ILBs are typically indexed on commonly-used official indices of inflation, such as the **consumer price index** (CPI), but some OECD countries nowadays compile more than one CPI index, each with some subtle variations in the items that make up the consumer basket.

For example, in the UK the headline inflation index is traditionally the retail price index (RPI) but this includes the cost of mortgage payments which in most other countries is excluded, so in recent years the government has also published an RPIX index which excludes mortgage payments.

In any case, whichever inflation index is used, it will never mirror exactly the true cost of living rises of every investor in ILBs, so for each investor there will remain an element of **basis risk** between their individual exposure to inflation and the reference index on which the ILB's payouts are based.

13.2. Pricing and Settlement

The Canadian Model

In 1991 the Canadian Treasury issued its first ILB, called the Real Return Bond (RRB). These bonds had a much simpler methodology for the indexation of their cash flows than what had until then been used elsewhere (e.g. in the UK[23]) and the so-called **Canadian model** since been adopted with only minor variations by most government ILB issuers, including:

- US TIPS
- French OAT*i* and OAT*ei*
- Sweden's inflation-linked bonds
- South Africa's CPI linked bonds

In the Canadian model, the ILB 'lives entirely in real space'.

[23] The UK Treasury was one of the first OECD countries to issue ILBs back in 1981 and the methodology that it developed to price them suffered from being a first prototype for the linker market. At the time of writing the UK government is considering adopting the Canadian model as well.

This means that the bond trades on a real clean price basis and its yield is also quoted on a real (i.e. inflation-adjusted) basis. The real price is related to its real yield through the standard bond pricing formula that we introduced earlier in section *Valuation Formula*:

$$\text{Real dirty price} = \frac{C/t}{(1+R/t)^{a}} + \frac{C/t}{(1+R/t)^{a+1}} + \frac{C/t}{(1+R/t)^{a+2}} + \ldots + \frac{\text{Principal} + C/t}{(1+R/t)^{a+m}}$$

Real clean price = Real dirty price - Real accrued interest
= Real dirty price - (1-a) x C/t

Where:
C = Real coupon rate
R = Real yield
t = Number of coupon payments per year (= compounding period)
m = Number of complete coupon periods to maturity

$$a = \frac{\text{Number of days to next coupon}}{\text{Number of days in current coupon period}}$$

It is only on the settlement date for any of the bond's cash flows – a coupon, the principal amount or even its dirty settlement price – that the real cash flow is converted into a nominal value by multiplying it by an **indexation factor** that is applicable for that date.

Indexation factor: the ratio of the reference consumer price index (CPI) applicable on the date when a cash flow is settled (CPI_i) divided by the reference index that was applicable on the date when the bond was issued (CPI_B), known as the **base reference index**.

Also known as: **Index ratio**.

Indexation factor = CPI_i / CPI_B

In the Canadian model, the CPI_i applied on the date when the bond's cash flow is settled is based on the CPIs that were published 3 and 2 months prior to that date (CPI_{t-3} and CPI_{t-2}) and is calculated as follows:

Cash flow settlement date	Reference index
1st day of month	CPI_{t-3}
Last day of the month (m)	CPI_{t-2}
dth day of the month (d < m)	$CPI_{t-3} + \frac{(d-1)}{m} \times (CPI_{t-2} - CPI_{t-3})$

In effect, the formula does a linear extrapolation of the CPI to the settlement date, based on the inflation trend that can be observed from the two most recent statistics. The same method is used to calculate the base reference index for the bond, CPI_B, and normally both CPI_i and CPI_B are rounded to 5 decimal places.

The nominal value of the cash flow payable is then calculated as:

Cash flow x CPI_i/CPI_B

If that cash flow is the bond's settlement price, then:

Settlement price = (Real clean price + Real accrued interest) x CPI_i/CPI_B

In most ILB structures the principal amount is protected in case of deflation, even though the coupons or the current settlement price on the bond are typically not.

The deflation floor: when the cash flow to be adjusted is the principal of the bond, then the indexation factor applied is subject to a minimum value of 1.

Worked example

An investor has the following position in an ILB:

Issue	France 3.4% OAT*i* 2029
Maturity	14 Dec 2029
Coupon	Semi-annual, Act/Act
Amount	EUR 1 million
Issued	14 Dec 1999
CPI_B	127.65098
Settlement date	3 Dec 2003
Yield	2.05%

This market trades and settles according to the Canadian model and the following reference inflation indices have already been published:

September CPI (CPI_{t-3}) = 139.97

October CPI (CPI_{t-2}) = 140.15

(a) What is the bond's quoted price, to the nearest two decimal places?

Entering the bond details and its real yield into a standard bond pricing calculator, we find the real clean price of this bond to be **127.12%** with 1.59781420765% of accrued.

(b) What is its settlement value of this position?

Reference CPI for settlement on 3 Dec 2003:

$$CPI_i = CPI_{t-3} + \left(\frac{d-1}{M}\right) \times (CPI_{t-2} - CPI_{t-3})$$

$$= 139.97 + \left(\frac{3-1}{31}\right) \times (140.15 - 139.97)$$

= 139.98161, rounded to 5 decimal places

Settlement price = (Real clean price + Real accrued interest) x CPI_i/CPI_B
= (127.12 + 1. 59781420765) x 139.98161/127.65098
= 141.151496593

Settlement value = 141.151496593/100 x 1,000,000
= **EUR 1,411,514.97**

(c) What is the amount of coupon that will next be payable on this position?

Reference CPI for settlement on 14 Dec 2003:

$$CPI_i = 139.97 + \left(\frac{14-1}{31}\right) \times (140.15 - 139.97)$$

= 140.05129

Coupon payment = Principal x Coupon rate/2 x CPI_i/CPI_B
= 1,000,000 x 0.034/2 x 140.05129/127.65098
= **EUR 18,651.42**

14. Real Yields and Inflation

14.1. Implied Inflation

The nominal yield on a straight bond is known to the investor, whereas its real yield is uncertain because it depends on future rates of inflation.

Exactly the opposite is the case with an ILB: its real yield is known (because its future cash flows will be adjusted in line with inflation) but its nominal yield is uncertain.

This suggests a relationship between the quoted nominal yields for straight bonds and the real yields quoted on ILBs with the same maturities and credit risk.

The Fisher Equation

$$(1 + R_n / t)^t = (1 + R_r / t)^t \times (1 + Inf) \times (1 + Pr)$$

Where:

R_n = Nominal yield on straight bond

R_r = Real yield on equivalent inflation-linked bond

Inf = Expected average inflation rate to the maturity of the ILB

Pr = Inflation risk premium

t = Number of compounding periods per year

The equation states that there are two factors behind the observed difference between the nominal yield on a bond and the real yield on its equivalent ILB:

1. An expected future inflation rate
2. An **inflation risk premium** which the investor in a straight bond requires to compensate her for being exposed to inflation risk (i.e. the risk that actual inflation may turn out to be different from what was expected)[24]

In other words, investors in ILBs are protected from inflation risk in exchange for earning a slightly lower real yield than what investors in straight bonds may earn in the long run, even after adjusting for inflation.

Other things being equal, the formula suggests that when inflation expectations worsen we should expect the market to adjust nominal yields up in line. This is because:

- The demand for straight bonds decreases, as its real yields are perceived to fall
- The real cost straight debt decreases, so borrowers have an incentive to issue more straight bonds

As a result of this so-called **Fisher effect**, nominal yields should rise while real yields should remain unchanged. In reality, we tend to find that when the inflation climate worsens nominal yields change in the same direction as inflation but not by as much, so real yields tend to fall at least initially.

[24] The concept of inflation risk premium is analogous to the liquidity risk premium in the liquidity preference theory of interest rates which we introduced in section *The Yield Curve*, above. The theory explains the difference between short and long-term nominal yields as the result of two factors:

1. The market expectation of future rates
2. A liquidity risk premium to compensate the investor for the higher market risk on long-dated bonds

14.2. Estimating Implied Inflation

If the inflation risk premium is relatively constant over time, changes in the gap between real and nominal yields should largely reflect changes in expected inflation and a liquid market for ILBs should provide reasonably good estimates of market expectations of future inflation rates over different time horizons.

In practice, this may not always be the case because:

1. Inflation risk premiums do change, as the market becomes more/less certain about future inflation prospects
2. The length of the indexation lag in an ILB introduces an element of uncertainty to the real yield on the bond (as discussed in section *Inflation-linked Bonds*)
3. Nominal yields for different maturities may not only reflect expected future changes in nominal rates, as a result of inflation, but also liquidity premiums (as discussed in section *The Yield Curve*, above)
4. A limited number of ILB issues plus liquidity shortages do not allow investors to cover all required maturities at fair prices
5. Strictly speaking, it is not correct to plot on the same curve the nominal yields on bonds with different coupon rates, as the yield on a bond is not entirely independent of its coupon rate (see Spot Yields – Coupon Stripping)

In practice, inferring inflation expectations out of differences between nominal and real yields is a bit more difficult than a simple subtraction!

14.3. Break-even Inflation

The main problem with the Fisher equation is that only the nominal and the real yields are observable, therefore it is very difficult to disentangle the inflation rate that the market expects from the inflation risk premium that it charges. Instead, the market simplifies the Fisher equation to calculate a **break-even inflation rate**.

Break-even inflation (BEI)
(IFID exam formula)

The BEI is the average inflation rate that gives a straight bond the same real yield as an ILB with the same maturity. Using the same symbols as in the Fisher equation on the previous page:

$$(1 + R_n / t)^t = (1 + R_r / t)^t \times (1 + BEI)$$

$$BEI = \frac{(1 + R_n / t)^t}{(1 + R_r / t)^t} - 1$$

Also known as: **Implied inflation rate**

For relatively low inflation rates, the following equation gives a good approximation to the modified Fisher equation:

$$R_n \approx R_r + BEI$$

In words, the arithmetic difference between nominal yields on straight bonds and real yields on ILBs give an indication of the future inflation rates that the market anticipates[25].

[25] Keep in mind that this rule of thumb doesn't hold in countries where inflation is very high and the full formula should be used instead.

Example1

An investor is planning to buy and hold a 10-year straight bond with a yield to maturity of 7%. If the expected inflation rate over the next 10 years is 3%, then:

Expected real yield $\approx$ 7%- 3%
= **4%**

Example2

A 1 year 4% (annual) straight Treasury bond trades at 99.05 and the real yield on a 1 year ILB is 1.5%. What is the exact 1 year BEI in this market?

Nominal yield = (104 / 99.05) - 1
= 0.04997 or 5.00% rounded.

$$\text{BEI} = \frac{(1 + 0.050)}{(1 + 0.015)} - 1$$

= 0.03448
or **3.45%**, rounded.

This means that risk-neutral investors who expect a 3.45% rate of inflation in the next 12 months would be indifferent between holding the straight Treasury or an ILB with the same maturity. Those who expect higher inflation should invest in the ILB, while those who expect lower inflation should invest in the straight bond.

In practice, even if the market expects a 3.45% inflation rate, investors may still prefer the ILB because its real yield is less uncertain than the real yield on the straight bond.

15. After-tax Yield

Gross yields on taxable assets and gross yields on tax-free investments are not directly comparable: for the investor, what matters is net after-tax yields. Moreover, it is often necessary to compare net yields on assets that are subject to different tax regimes.

The concept of net or **after-tax yield** is simple enough to implement by making sure that one inserts the net after-tax cash flows on a bond, rather than its gross cash flows into the standard pricing formula presented in section *Valuation Formula*, above.

A commonly-used approximation to the after-tax yield is:

After-tax yield = Gross yield x (1 – Marginal income tax rate)

In this formula, the gross yield (or **tax-equivalent yield**) on the bond is the yield that must be paid on a taxable asset before factoring in taxes, so that the asset pays off a given after-tax yield.

$$\textbf{Tax-equivalent yield} = \frac{\textbf{After-tax yield}}{\textbf{(1 – Marginal income tax)}}$$

This formula works reasonably well as long as the income tax rate on the bond's coupons is the same as the tax rate payable on any capital gains on its price – i.e. both components of the bond's yield are taxed at the same rate.

Worked example

An investor pays par for a bond that pays annual coupons at a rate of 10%. The investor expects an average annual inflation rate of 3% until the bond's maturity and pays a marginal income tax rate of 30%.

? What is the after-tax real annual yield on this bond for the investor?

Using the Fisher equation presented in section *Real Yields*, we first calculate the gross real yield on the bond:

$$\mathbf{R_r = \frac{(1 + R_n / t)^t}{(1 + Inf)} - 1}$$

Where:

R_n = Nominal yield on straight bond

R_r = Real yield on equivalent inflation-linked bond

Inf = Expected average inflation rate to the maturity of the investment

t = Number of compounding periods per year

$$R_r = \frac{(1 + 0.10)}{(1 + 0.03)} - 1$$

= 0.06796
or 6.80%, rounded.

Then we calculate its after-tax yield:

After-tax yield = Gross yield x (1 – Marginal income tax rate)
= 6.80% x (1 – 0.30)
= **4.76**%

Notice how if the bond paid interest semi-annually, rather than annually, this would not affect the calculation. Clearly this formula is just an approximation and you should use it with care.

16. Exercise 2

16.1. Question 1

Question 6

Security: 4½% Japanese government bond maturing 23 Sep 2005
Type: domestic, semi-annual actual/365
Price: 108.55
Settlement date: 21 June 2002

Calculate:

a) The bond's current yield (CY) in percent. (Enter your answer, to 2 decimal places, in the box below, then validate.)

b) The bond's adjusted current yield (ACY) rounded to 2 decimal places.

16.2. Question 2

Question 7

This exercise is too complex to perform with a simple calculator. You should use a financial calculator such as the HP 17 (or later model) or the bond pricing spreadsheet.

Security: 4½% Japanese government bond maturing 22 Sep 2005
Type: Domestic, semi-annual actual/365
Settlement date: 20 June 2002
Price: 108.55

a) Calculate the bond's yield to maturity, rounded to the nearest 2 decimal places.

16.3. Question 3

Question 8

This exercise is too complex to perform with a simple calculator. You should use a financial calculator such as the HP 17 (or later model) or the bond pricing spreadsheet.
Security: 4½% Asian Investment Bank maturing 22 Sep 2005
Type: JPY Eurobond, annual 30E/360
Settlement date: 20 June 2002
Price: 108.55

a) Calculate the bond's yield to maturity, rounded to the nearest 2 decimal places.

b) Why is the yield on this bond different from the yield on the bond calculated in *Question 2*, if the two bonds have the same coupon, maturity and price?

- The two bonds were issued on different dates.
- A semi-annual yield is equivalent to a lower annual yield.
- This bond is annual 30/360; the other one is semi-annual Actl/Actl.
- This bond has a lower credit rating.

16.4. Question 4

Question 9

This exercise is too complex to perform with a simple calculator. You should use a financial calculator such as the HP 17 (or later model) or the bond pricing model spreadsheet.

Security:	7½% Ford Motor Finance, maturing 22 October 2009	
Type:	Eurobond, annual 30E/360	
Call features:	Callable at:	101.00 on 22 October 2007
		100.50 on 22 October 2008
Settlement date:	19 April 2002	
Price:	102¾	

a) What is the bond's yield to maturity, rounded to the nearest 2 decimal places?

b) What is the bond's yield to worst, rounded to 2 decimal places?

c) Without doing the calculations, could you have predicted which was the worst call date?

- No
- Yes

16.5. Question 5

Question 10

This exercise is too complex to perform with a simple calculator. You should use a financial calculator such as the HP 17 (or later model) or the bond pricing spreadsheet.

Security 6¼% British government bond (UK Gilt) maturing 20 October 2010
Type: Semi-annual, Actual/actual
Price: 102.34375
Settlement: 19 April 2002

What is the bond's yield to maturity:

a) On a semi-annual basis, in percent, rounded to the nearest 3 decimal places?

b) On an annualised basis (to 2 decimal places)?

16.6. Question 6

Question 11

This exercise is too complex to perform with a simple calculator. You should use the bond pricing spreadsheet.

Security: 8% Eurobond maturing 10 October 2001
Type: Annual, 30/360
Settlement date: 5 January 1998
Price: 93.516 (clean)
Horizon date: 10 October 2001 (the maturity date)
Horizon price: 100.00

a) What is the yield to maturity and the horizon yield on this bond, assuming a reinvestment rate of 10.134% (annual, 30/360)? Enter your answers in percent per annum to 3 decimal places.

Yield to maturity	
Horizon yield	

