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The Bloomberg CDS Model

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Abstract

The Bloomberg CDS model is available as the B-model on CDSW. We describe credit curve stripping and CDS pricing in the context of this model.

Keywords. CDS, default probability, curve stripping.

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1 Introduction

The Bloomberg CDS model values single name and index credit default swaps as a function of their schedule, currency, deal spread, notional, CDS curve and benchmark yield curve. The model consists of two main components: a credit curve "stripper" and a CDS "pricer". The conventions and assumptions of the model, details of the CDS pricer and the credit curve stripper, and the usage of the pricer in nonstandard valuation cases are described below.

2 Conventions and Assumptions

The conventions, assumptions, and auxiliary market data used in both the CDS pricer and the default probability stripper are identical. The key assumptions employed are constant recovery as a fraction of par, piecewise constant risk neutral hazard rates, and default events being statistically independent of changes in the default free yield curve. As a practical matter, the last assumption allows us to perform all discounting with the current default free discount factors without technically assuming that interest rates are deterministic. Index trades are analyzed as if they were CDS referencing a single name, with the current market convention being to value them with a flat CDS curve.

There are specific conventions applicable to various date attributes in the valuation of a CDS deal, namely:

Curve Date This serves as a reference date for all internal model calculations. By construction, the risk free discount factor and the survival probability of the underlying entity are both unity at the end of Curve Date.

Effective Date Default protection begins on this date (inclusive). The standard ISDA terms for new CDS contracts (as per the April 8, 2009, Auction Hardwiring Supplement to 2003 Credit Derivatives Definitions) is that default protection become effective immediately, and includes any credit event that may have occurred during the preceding 60 calendar days. The Effective Date would usually precede the Curve Date (as in SNAC deals or other seasoned trades) or be the calendar date immediately following Curve Date (new trades). However, the model is capable of accepting later dates up to the maturity of the deal, resulting in a forward starting deal described later.

Accrual Start Date This is the date (inclusive) on which premium accrual begins. It is the most recent business day adjusted IMM date on or before the Curve Date (as is the case for single name trades under the new Standard North American Contract (SNAC) and for the main CDX and Itraxx indices) or otherwise the same as the Effective Date.

Maturity Date This is the date (inclusive) on which default protection and premium accrual end, unless a default event occurred prior to this date, in which case premium accrual ends on the default date (inclusive).

Valuation Date This is the date at the end of which the deal is valued, and lies in between Curve Date and Maturity Date (both dates inclusive). In most situations, it coincides with Curve

Date, but the model can handle any other admissible date to produce a forward valuation scenario described later.

The discount function, which takes as input a date and returns a discount factor to the valuation date, uses standard interest rate derivative market conventions, and the market rates are converted into discount factors by assuming piecewise flat continuously compounded forward rates.

3 The CDS Pricer

The inner routines of the CDS pricer take as input a schedule, a default probability function and a discount function and produce as output the present value of the default leg (also known as protection leg) for a unit loss given default, as well as the present value of a flow of a unit premium until the earlier of default and maturity. The model value of the CDS is then found by scaling the default leg by the notional amount of currency times one minus the assumed recovery rate and subtracting the contractual deal spread (premium rate) times the notional amount of currency times the value of the unit premium leg. (The model can also price deals with amortizing notional as described later.) The required schedule is generated by the model from the maturity date, holiday calendar and the applicable business day roll convention. The default probability function is generated from an input CDS curve by the default probability curve stripper, which is described below.

Symbolically, we can express the present value of a unit premium leg as:

$$V_p = \sum_{i=1}^{I} \delta\left(t_{i-1}, t_i\right) P\left(t_i\right) \left(1 - Q\left(t_i\right)\right) + \int_0^T \alpha(t) P(t + \Delta)\, dQ(t), \tag{1}$$

where $P(t)$ is the default free discount function, Δ is the settlement lag following a default event, $Q(t)$ is the default probability function, giving the cumulative default probability to time t; and t measures time with time 0 being the present and $T = t_I$ the time to maturity of the deal. Here $\delta(t_{i-1}, t_i)$ denotes the fractional year between successive premium payment dates, and the function $\alpha()$ measures the length of time over which premium has accrued since the last premium payment date, both measured in the relevant day count convention (typically ACT/360). Specifically,

$$\alpha(t) = \delta\left(t_{\max\{i:t_i \leq t\}}, t\right).$$

The present value of the default leg on a unit notional can similarly be expressed as:

$$V_d = (1 - R) \int_0^T P(t + \Delta)\, dQ(t), \tag{2}$$

where R is the assumed fractional recovery of par in case of default. The estimated market value of being long protection in a CDS with notional N and premium rate c is then:

$$V\left(N, c\right) = N\left(V_d - cV_p\right).$$

The replacement (par) spread for a CDS is given as the spread that equates the present value of the premium leg with the present value of the default leg, *i.e.* as:

$$S^* = \frac{V_d}{V_p}. \tag{3}$$

In implementation, the integrals above are evaluated analytically piecewise, where each piece is chosen to be small enough to justify locally flat hazard rates and forward interest rates, and in no case longer than the time between scheduled premium payments of the underlying swap.

The pricer also produces interest rate and CDS curve risk measures. Spread and interest rate DV01s represent the change in the number of currency units (the D originally comes from Dollar) in the value of the transaction as a result of a parallel shift of 1 basis point in the CDS curve or interest rate forward curve respectively, holding all other inputs constant. The model implementation more generally supports both additive (*e.g.* +5 basis points over the nominal level) and multiplicative shifts (*e.g.* 1% of the nominal level). Similarly, the model also calculates spread KRRs (key rate risks) representing the sensitivity of the deal value to a shift in each single CDS rate, calculated while holding all other market inputs constant.

The choice of parameterizing the risk neutral hazard rate as a piecewise constant function of time yields two concrete benefits: it allows the analytical solution of the integrals in equations (1) and (2), thus speeding up the model execution, and it ensures that key rate credit spread risks are "local" in nature. Alternative approaches, *e.g.* fitting a spline to describe the hazard rates as a function of time, will almost invariably result in non-local effects, such as the value of a two and a half year CDS having a non-zero key rate risk exposure to the ten year curve point. The downside of the piecewise flat hazard rate modeling assumption is that the jumps in the modeled hazard rate at curve points are economically implausible and can lead to exaggerated changes in valuation around the corresponding IMM roll days.

4 The Curve Stripper

The curve stripper takes a set of standard maturity credit default swap rates as inputs, along with associated schedules, a benchmark yield curve and a recovery rate and produces a risk neutral default probability function as its output. This is, in a sense, the inverse operation as that expressed in the previous section, where a CDS par rate S^* is expressed in terms of the default probability function $Q(t)$. In order to solve this "inverse problem", we need to make some assumptions on the structure of the function $Q(t)$. Specifically, we assume that it is defined in terms of a set of dates and associated parameters $\{\tau_i, \lambda_i\}$, where the dates τ_i are the input CDS maturity dates and the λ_i parameters are the risk neutral hazard rates. For $t \in (t_{i-1}, t_i)$ we have the risk neutral default probability function defined by:

$$Q(t) = 1 - \exp\left\{-\lambda_{n+1}(t - \tau_n) - \sum_{i=1}^{n} \lambda_i(\tau_i - \tau_{i-1})\right\}. \tag{4}$$

We solve for the hazard rates via a forward recursive bootstrapping algorithm. We note that the par CDS rate for a given maturity only depends on the default probability function for times up to that maturity. We can plug in the expression for $Q(t)$ above into (1) and (2) and insert the resulting expressions for V_d and V_p into the right hand side of (3), letting T be the first input CDS maturity date, τ_1, and equating it to the quoted CDS rate for this maturity. The only unknown quantity in the resulting equation will be λ_1. We can then solve this nonlinear equation for λ_1, which is determined so that the CDS pricer will match the par spread of the first input CDS. Given λ_1, λ_2 is found so that the CDS pricer will correctly price the second input CDS, and so on until we have matched all the input CDS quotes.

Note from (4) that the risk neutral probability of default taking place over a short period $(t, t+\varepsilon)$ for $t \in (\tau_n, \tau_{n+1})$, given no default until time t, is approximately given by $\lambda_{n+1}\varepsilon$, so the hazard rate measures the intensity of the default risk at each point in time. Consequently, the hazard rates need to be non-negative and finite. This corresponds to upper and lower limits for the CDS input rate for each maturity, as a function of the default probability function computed from the shorter maturities. When the stripping routine encounters a CDS input rate that violates one of its bounds it will fail, and so will the associated CDS pricing. Note that this will only occur if the input CDS rates are internally inconsistent, *i.e.* implying an arbitrage opportunity in the absence of transaction costs.

In certain cases, CDS contracts are quoted in "points upfront", given a certain fixed coupon level. In this case, the curve stripping proceeds essentially as described above, except that each λ_i is found such that the model value of the fixed coupon CDS is equal to the quoted upfront amount.

5 Nonstandard Valuation Modes

The model is capable of handling other nonstandard valuation modes and deal types (in deal pricing only, not in curve stripping mode).

Amortizing Notional Deals with amortizing notional may be specified with an additional schedule of amortization dates and corresponding notional levels outstanding. The model restricts each amortization date to coincide with some coupon date, but notional levels may be arbitrary, and in particular, non-monotonic. The valuation formulas in (1) and (2) respectively change to

$$\begin{aligned} V_p &= \sum_{i=1}^{I} N(t_i^+)\delta\left(t_{i-1}, t_i\right) P\left(t_i\right)\left(1 - Q\left(t_i\right)\right) + \int_0^T N(t^+)\alpha(t) P(t+\Delta)\, dQ(t) \\ V_d &= (1-R)\int_0^T N(t^+) P(t+\Delta)\, dQ(t), \end{aligned}$$

where $N(t^+)$ denotes the fraction of notional outstanding at time t after any amortization scheduled for time t has been applied.

Forward Start When the Effective Date T_{eff} of the deal is later than the calendar date immediately following the Curve Date, the model treats the deal as a forward starting one. In this case, one must additionally specify the knockout provision in the deal, *i.e.* if the contract terminates on a default event that occurs between the Curve Date (inclusive) and the Effective Date (exclusive). If the deal does not knock out on a default prior to the Effective Date, the default protection starts with Curve Date; otherwise, the default protection starts with Effective Date. In both cases of the knockout provision, the premium leg is the Curve Date value of the risky coupons between Effective Date and Maturity Date. Thus the valuation formulas become

$$
\begin{aligned}
V_p &= \sum_{i=1}^{I} \delta\left(t_{i-1}, t_i\right) P\left(t_i\right)\left(1 - Q\left(t_i\right)\right) + \int_{T_{\text{eff}}}^{T} \alpha(t) P(t+\Delta)\, dQ(t), \\
V_d &= (1-R) \int_{T_0}^{T} P(t+\Delta)\, dQ(t),
\end{aligned}
$$

where $T_0 = 0$ (no-knockout) or $T_0 = T_{\text{eff}}$ (knockout).

Forward Valuation When the Valuation Date is later than the Curve Date, a forward valuation scenario results. In this case, the model produces a forward value for the deal, conditional on the assumptions that the credit and benchmark curves as of the Valuation Date are the same as the corresponding forward curves as of Curve Date, and that the underlying entity has incurred no default up to the Valuation Date (inclusive). In implementation, the model first obtains the Curve Date value of both the default and the premium leg cash flows by replacing the lower limits in the summation and integrals in equations (1) and (2) with the value corresponding to Valuation Date. Then this value as of Curve Date is divided by the product of the discount factor and risk neutral survival probability to the valuation date to produce the forward value.