OpenGamma Documentation

Bond Pricing

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Abstract

The details of the implementation of pricing for fixed coupon bonds and floating rate note are provided. The different day count and yield conventions used by sovereign bonds are described.

Contents

1 Introduction

The computation of the present value of fixed coupon bonds and floating rate notes is not very complex mathematically. Nevertheless, it requires a lot of details and there are many subtleties. This note provides the details of the implementation in the OpenGamma analytical library.

When speaking about bonds, one can make the distinction between a bond security (or issue) and a bond transaction. Here, the bond security represents the official description of the bond, with details about the coupons, nominal payments and conventions. A bond transaction is the purchase (or sale) of a certain quantity of the bond issue on a given date for a given price. The bond figures described here involves both of those concepts.

Fixed coupon bonds are usually quoted as clean prices. The clean price is the relative price to be paid at the standard settlement date in exchange for the bond. Some bonds trade ex-coupons before the coupon payment. The coupon is paid not to the owner of the bond on the payment date but to the owner of the bond on the detachment date. The difference between the two is the ex-coupon period (measured in days). The issues related to those detachments are also described.

When a bond has been purchased but has not settled yet, the proceeds of the payment are still pending. The payment of those proceeds need to be taken into account in the present value calculation of the bond purchase. The amount representing the proceed will have the opposite sign to the quantity of bond purchased. The total present value of a bond which has not settled will generally be very small with respect to the notional.

2 Notation

The pricing described here will be relative to a reference date denoted t_r; the will be, depending of the circumstances described below, the settlement date of a particular transaction, the standard spot date of the bond or today. Only the cash flows *after* that date (subject to ex-coupon adjustments) will be taken into account. The definition of *after* will also depend of the circumstances: for settle;net date it means $t > t_r$ but for today, it means $t \geq 0 = t_r$.

2.1 Fixed coupon bond

The coupon amounts are denoted $(c_i)_{i=1,\ldots,n^c}$ and paid on times (t_i^c); the number of coupon periods in a year is denoted m. Only the coupons to be received are taken into account.

When the ex-coupon period is 0, the coupons taken into account are the coupons after the reference date.

When the ex-coupon period is non-zero we need an extra date which is t_x, the reference date plus the ex-coupon period. The coupons taken into account are the coupons such that $t_i^c > t_x$.

The notional (capital) payments are denoted $(N_i)_{i=1,\ldots,n^N}$ and are paid on (t_i^N). Usually, there is a unique payment of the full notional at the final coupon date $t_{n^c}^c$. But in case of amortising bonds, the notional can be paid in several instalments,

The bond settlement (purchase) is done through the payment of an amount S on t^S. The sign of S is the opposite of the sign of N_i. If the settlement has already occurred, the amount used is $S = 0$ and the time is $t^S = 0$.

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2.2 Floating rate note

The coupon amounts are denoted $(N_i^c)_{i=1,\ldots,n^c}$; the spreads are s_i^c, the accrual fraction are δ_i^c and the coupons are paid on (t_i^c). The coupon pay-off is the Ibor rate plus the margin multiplied by the coupon accrual and notional, i.e. $(L_i^I + s_i^c)\delta_i^c N_i^c$. The Ibor fixing accrual fraction is δ_i^I.

As for fixed coupon bonds, only the coupons to be received are taken into account. It means that if the ex-coupon period is 0, the coupons taken into account are the coupons such that $t_i^c > t_r$. If the ex-coupon period is non-zero, let t_x be the reference date plus the ex-coupon period. The coupons taken into account are the coupons such that $t_i^c > t_x$.

The notional (capital) payments are denoted $(N_i)_{i=1,\ldots,n^N}$ and are paid in (t_i^N). Usually there is a unique payment of the full notional at the final coupon date $t_{n^c}^c$.

The bond settlement (purchase) is done through the payment of an amount S in t^S. If the settlement already occurred, the amount used is $S = 0$ and the time is $t^S = 0$.

3 Conventions

The most used day count conventions for bonds are

3.1 Day count: ACT/ACT ICMA

The rule is described in ICMA.

The accrual factor is

$$\frac{1}{\text{Freq}}\text{Adjustment}$$

where Freq is the number of coupon per year and Adjustment depends of the type of stub period.

None The Adjustment is 1. The second expression reduces to 1 and the coupon is 1/Freq.

Short at start The Adjustment is computed as a ratio. The numerator is the number of days in the period. The denominator is the number of days between the standardised start date, computed as the coupon end date minus the number of month corresponding to the frequency (i.e. 12/Freq), and the end date.

Long at start Two standardised start dates are computed as the coupon end date minus one time and two times the number of month corresponding to the frequency. The numerator is the number of days between the start date and the first standardised start date and the numerator is the number of days between the first and second standardised start date. The Adjustment is the ratio of the numerator by the denominator plus 1.

Short at end The Adjustment is computed as a ratio. The numerator is the number of days in the period. The denominator is the number of days between the start date and the standardised end date, computed as the coupon start date plus the number of month corresponding to the frequency (i.e. 12/Freq).

Long at end Two standardised end dates are computed as the coupon start date plus one time and two times the number of month corresponding to the frequency. The numerator is the number of days between the end date and the first standardised end date and the numerator is the number of days between the second and first standardised end date. The Adjustment is the ratio of the numerator by the denominator plus 1.

3.2 Day count: ACT/365 Fixed

Also called *English Money Market basis*. The accrual factor is

$$\frac{d_2 - d_1}{365}$$

where $d_2 - d_1$ is the number of days between the two dates. The number 365 is used even in a leap year.

3.3 Sovereign bonds conventions

The following countries use the *ACT/ACT ICMA*:

- France
- Germany
- Italy
- Spain
- United Kingdom (since November 1, 1998)
- United States

The following countries use the *Day count: ACT/365*:

- United Kingdom (before November 1, 1998)

3.4 Settlement lag

A conventional settlement lag is associated to each bond. The lag is the delay between trade date and settlement date. It is usually one business day for US treasuries and UK Gilts and three days for Euroland government bonds.

transactions can be entered into at any settlement date agreed by the parties. Nevertheless the quoted price from data providers are prices for standard settlement date. It is important to know the correct convention to use the quoted price information correctly even if one does not use those conventions for actual transactions.

4 Repurchase agreement

A repurchase agreement (repo) is a collateralised deposit which is financially equivalent to selling and buying back the same bond at an agreed price at a later date (even if there are legal differences between the two).

If we take a repo with settlement date today and maturity at a date t_r in the future, the flows are given in Table 1.

On the other side it is possible to sell the bond today with value date today and at the same time buy today with settlement t_r. The flows are also given in Table 1.

The two being equal, the dirty price with forward settlement is $\text{Dirty}(t_r) = \text{Dirty}(0)(1 + \delta R)$, i.e. the forward price should be discounted at the (repo) risk free rate to obtain today's value. We will use this discounting from settlement approach for the bond when we discuss present value computation from quoted price.

Repo			Bond today and forward		
Time	Cash flow	Bond	Time	Cash flow	Bond
0	$+P_0$	-1	0	$+P_0$	-1
t_r	$-P_0(1+\delta R)$	$+1$	t_r	$-P_{t_r}$	$+1$

Table 1: Flow for repo and forward settlement bond transaction.

5 Discounting

The discounting price is the price obtained by discounting each cash flow. Not all cash flows are discounted using the same curve. The following sections describes the details for coupon bonds and floating rate notes.

5.1 Fixed coupon bond

For a coupon bond, all the cash flows are known and directly discounted. The coupons and notional are discounted with the curve including credit risk relevant for the issuer. The settlement amount (when the bond has not settled yet) is discounted with the risk-free curve, as the amount to be paid for settlement is collateralised by the bond (delivery versus payment (DVP)).

The two curves used are

1. Discounting (risk free) : $P^D(t)$.
2. Credit (issuer): $P^C(t)$.

The present value of the security without settlement payment is, for $t_i^c \geq 0$,

$$\mathrm{PV}^{\mathrm{Security}}_{\mathrm{Discounting}} = \sum_{i=1}^{n^c} c_i P^C(t_i^c) + \sum_{i=1}^{n^N} N_i P^C(t_i^N). \tag{1}$$

If the settlement date t_s is after today ($t_s \geq 0$) and the settlement amount is S, the present value of a transaction is given by, for $t_i^c > t_s$,

$$\mathrm{PV}^{\mathrm{Transaction}}_{\mathrm{Discounting}} = \sum_{i=1}^{n^c} c_i P^C(t_i^c) + \sum_{i=1}^{n^N} N_i P^C(t_i^N) + SP^D(t_s) = \mathrm{PV}^{\mathrm{Security}}_{\mathrm{Discounting}} + SP^D(t_s). \tag{2}$$

5.2 Floating rate note (FRN)

In the case of a FRN a third curve is used: the forward curve related to the relevant Ibor. The three curves are:

1. Discounting (risk free) : $P^D(t)$.
2. Credit (issuer): $P^C(t)$.
3. Ibor (forward): $P^I(t)$.

The coupons and notional are discounted with the credit curve. The coupons are estimated with the forward curve. The approach is based on an independence hypothesis between the issuer credit risk and Ibor rates.

The forward estimation is

$$F_i^I = \frac{1}{\delta_i^I}\left(\frac{P^I(s_i)}{P^I(e_i)} - 1\right). \tag{3}$$

The present value is

$$\mathrm{PV}_{\mathrm{Discounting}} = \sum_{i=1}^{n^c} \delta_i N_i^c (F_i^I + s_i^c) P^C(t_i^c) + \sum_{i=1}^{n^N} N_i^N P^C(t_i^N) + SP^D(t_s). \tag{4}$$

6 Clean and dirty price

The stander prices (clean and dirty) are defined only for the bonds with unique notional payment N at the end (bullet bonds).

6.1 Dirty price of a bond security from curves

The dirty price is the relative price to be paid in a fair transaction at the standard settlement date. The dirty price, denoted Dirty, should be such that $\mathrm{PV}_{\mathrm{Discounting}} = 0$ for $S = N \cdot \mathrm{Dirty}$, with t^S the standard settlement date. We obtain for the dirty price

$$\mathrm{Dirty}_{\mathrm{Discounting}} = \mathrm{PV}^{\mathrm{Security}}_{\mathrm{Discounting}} / P^D(t^S)/N.$$

6.2 Fixed coupon bond present value from clean price

The present value from the clean price is approximately the clean price plus accrued interest multiplied by the notional. This is not exact as it does not take into account the discounting between settlement and today and any coupon that may be due in between.

We start with the price for a bond with standard settlement date. The settlement date is denoted t_0 and the present value of a synthetic transaction with standard settlement date is denoted P^{Standard}. Its present value is

$$\mathrm{PV}^{\mathrm{Standard}}_{\mathrm{Price}} = (P^{\mathrm{Quoted}} + A) N P^C(t_0).$$

For the coupon that are added or subtracted from the standard one, we use the curves

$$\mathrm{PV}^{\mathrm{Transaction}}_{\mathrm{Price}} = (\mathrm{PV}^{\mathrm{Transaction}}_{\mathrm{Discounting}} - \mathrm{PV}^{\mathrm{Standard}}_{\mathrm{Discounting}}) + \mathrm{PV}^{\mathrm{Standard}}_{\mathrm{Price}}.$$

7 Yield

The yield of a bond security is a conventional number representing the internal rate of return of *standardised* cash flows. Standardised means in this context that the exact payment dates are not taken into account but only the number of periods. The accrual factor from the standard settlement date to the next coupon is denoted w. The factor is computed using the day count convention of the bond.

The yield is written only for the bonds with unique notional payment N at the end (bullet bonds).

7.1 Simple interest in last period

Some bonds use a different calculation method in the last coupon period. The most used method is the *simple interest in last period* or *US market final period convention*.

In the final period ($n^c = 1$), the simple yield is used. The dirty price (at standard settlement date) is related to the yield by

$$N \cdot \text{Dirty} = \left(1 + w\frac{y}{m}\right)^{-1} (c_{n_c} + N).$$

7.2 Compound interest in last period

In the final period ($n^c = 1$), the compound interest is used. The dirty price (at standard settlement date) is related to the yield by

$$N \cdot \text{Dirty} = \left(1 + \frac{y}{m}\right)^{-w} (c_{n_c} + N).$$

7.3 US Street convention

The *US street* convention assumes that yield are compounded over the bond coupon period (usually semi-annually in US), including in the fractional first period.

The dirty price (at standard settlement date) is related to the yield by

$$N \cdot \text{Dirty} = \left(1 + \frac{y}{m}\right)^{-w} \left(\sum_{i=1}^{n^c} \frac{c_i}{\left(1 + \frac{y}{m}\right)^{i-1}} + \frac{N}{\left(1 + \frac{y}{m}\right)^{n^c-1}}\right)$$

In the final coupon period, the *US market final period convention* is used.

7.4 UK: DMO method

The method is similar to the US street convention except that the final period uses the same convention and not the US market final period convention. It can be different if the first period is long or short or if there is an ex-dividend period.

Let $v = (1 + y/m)^{-1}$. The coupons are all identical from the third one: $c_i = c/m$ for $i = 3, \ldots, n^c$. The formula described by DMO in Debt Management Office (2005) is

$$N\text{Dirty} = v^w \left(c_1 + c_2 v + \frac{cv^2}{m(1-v)}(1 - v^{n^c-2}) + Nv^{n^c-1}\right)$$

where c_1 is the cash-flow on the next date (may be 0 in the ex-dividend period or if the gilt has long first dividend period) and c_2 is the cash flow due on the next but one quasi-coupon date (may be greater than c/m for long first dividend periods). In the last period the formula reduces to

$$N \cdot \text{Dirty} = v^w (c_1 + N)$$

and is not replaced by the simple yield approach.

7.5 US Treasury Convention

The *US Treasury* convention assumes that yields are compounded over the bond coupon period (usually semi-annually in US), except in the fractional first period where a simple yield is used.

The dirty price (at standard settlement date) is related to the yield by

$$N \cdot \text{Dirty} = \left(1 + w\frac{y}{m}\right)^{-1} \left(\sum_{i=1}^{n^c} \frac{c_i}{\left(1+\frac{y}{m}\right)^{i-1}} + \frac{N}{\left(1+\frac{y}{m}\right)^{n^c-1}} \right)$$

In the final coupon period, the *US market final period* convention is used.

It appears that this convention is no longer commonly used.

7.6 FRANCE:COMPND METH

It is the same method as the *UK: DMO method* with compound interest in the last period.

7.7 GERMAN BONDS

It is the same method as the *US Street convention* with simple interest in the last period.

7.8 Sovereign bonds conventions

The following countries use the *US Street convention*:

- Spain
- United States

The following countries use the *UK: DMO method*:

- United Kingdom

8 Duration and convexity

8.1 Modified duration

The modified duration is a (dirty) price sensitivity measure. It is the relative derivative of the price with respect to the conventional yield, i.e.

$$\frac{\partial \text{Dirty}}{\partial y} = -\text{Duration}_{\text{Modified}}\text{Dirty}.$$

For the US street convention and UK DMO convention it is

$$\text{Duration}_{\text{Modified}} = \frac{1}{1+\frac{y}{m}} \left(\sum_{i=1}^{n^c} \frac{c_i}{\left(1+\frac{y}{m}\right)^{i-1+w}} \frac{i-1+w}{m} + \frac{N}{\left(1+\frac{y}{m}\right)^{n^c-1+w}} \frac{n-1+w}{m} \right) \frac{1}{N \cdot \text{Dirty}}. \tag{5}$$

For the simple interest in the last period it is

$$\text{Duration}_{\text{Modified}} = \left(1 + w\frac{y}{m}\right)^{-1} \frac{w}{m}. \tag{6}$$

For the compound interest in the last period it is

$$\text{Duration}_{\text{Modified}} = \left(1 + \frac{y}{m}\right)^{-1} \frac{w}{m}. \tag{7}$$

The modified duration is not scaled by the notional/quantity of bonds.

8.2 Macaulay duration

Macaulay duration, named for Frederick Macaulay, who introduced the concept, is the weighted average maturity of cash flows. The discounting and the time to maturity are in line with those of the yield convention. For US street not in the last period and DMO methods it is computed as

$$\text{Duration}_{\text{Macauley}} = \left(\sum_{i=1}^{n^c} \frac{c_i}{\left(1 + \frac{y}{m}\right)^{i+w}} \frac{(i - 1 + w)}{m} + \frac{N}{\left(1 + \frac{y}{m}\right)^{n^c - 1 + w}} \frac{(n^c + w)}{m} \right) \frac{1}{N \cdot \text{Dirty}}. \tag{8}$$

For simple interest in the last period it is

$$\text{Duration}_{\text{Macauley}} = \frac{w}{m}. \tag{9}$$

For US Street not in the last period and DMO methods, the relationship between the two durations is

$$\text{Duration}_{\text{Modified}} = \frac{1}{1 + \frac{y}{m}} \text{Duration}_{\text{Macauley}}.$$

For US Street in the last period the relationship is

$$\text{Duration}_{\text{Modified}} = \frac{1}{1 + w\frac{y}{m}} \text{Duration}_{\text{Macauley}}.$$

The Macaulay duration is not scaled by the notional/quantity of bonds.

8.3 Convexity

The convexity is the relative second order derivative of the price with respect to the conventional yield, i.e.

$$\frac{\partial \text{Dirty}}{\partial y} = \text{Convexity} \cdot \text{Dirty}.$$

For US street not in the last period and DMO methods is computed as

$$\text{Convexity} = \left(\sum_{i=1}^{n^c} \frac{c_i}{\left(1 + \frac{y}{m}\right)^{i+w+1}} \frac{(i - 1 + w)}{m} \frac{(i + w)}{m} + \frac{N}{\left(1 + \frac{y}{m}\right)^{n^c + w + 1}} \frac{(n^c - 1 + w)}{m} \frac{(n^c + w)}{m} \right) \frac{1}{N \cdot \text{Dirty}}. \tag{10}$$

For US Street in the last period the relation it is

$$\text{Convexity} = 2 \left(1 + w\frac{y}{m}\right)^{-2} \left(\frac{w}{m}\right)^2. \tag{11}$$

The convexity is not scaled by the notional/quantity of bonds.

9 Other measures

9.1 Z-spread

The z-spread is the constant spread for which the present value from a curve plus the spread matches a present value. The spread is in the convention in which the curve is stored; this convention is usually ACT/ACT, continuously compounded. The discounting curve for a spread s and an initial curve $P(t)$ is

$$P(t)\exp(-st).$$

The z-spread is not scaled by the notional/quantity of bonds.

The computation of the Z-spread depends on the way the present value is computed. The most used approach is to compute the present value from the market quoted price as described in Section6.2. This means that one needs a market price, a risk free (repo) curve and a issuer curve to compute the present value and the initial curve from which the spread is computed. The requirement is thus one price and three curves to obtain the Z-spread.

9.2 Z-spread sensitivity

This is the present value sensitivity to the z-spread. It is the parallel (in the curve convention) curve sensitivity at the curve plus spread level.

The z-spread sensitivity is scaled by the notional/quantity of bonds.

10 Implementation

In OG-Analytics, the bond security figures are computed in the class `BondSecurityDiscountingMethod` and `BondTransactionDiscountingMethod`.

In the Analytics viewer, the different settings can be changes through the following `Properties`:

1. RiskFree: the risk free/repo curve
2. Credit: the issuer specific curve
3. Curve: the base curve (in particular for Z-spread)
4. CalculationMethod: to change the calculation method. The possible values are `FromCurves`, `FromCleanPrice`, and `FromYield`.

References

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1. The Analytic Framework for Implying Yield Curves from Market Data.
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3. Option Pricing with Fourier Methods.
4. Bill Pricing.
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6. Interest Rate Futures and their Options: Some Pricing Approaches.
7. Bond Futures: Description and Pricing.
8. Forex Options Vanilla and Smile.
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13. Hull-White one factor model.
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15. Replication Pricing for Linear and TEC Format CMS.
16. Binormal With Correlation by Strike Approach to CMS Spread Pricing.
17. Libor Market Model with displaced diffusion: implementation.
18. Portfolio hedging with reference securities.
19. Compounded Swaps in Multi-Curves Framework
20. Curve Calibration in Practice: Requirements and Nice-to-Haves.

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The company's flagship product, the OpenGamma Platform, is a transparent system for front-office and risk calculations for financial services firms. It combines data management, a declarative calculation engine, and analytics in one comprehensive solution. OpenGamma also develops a modern, independently-written quantitative finance library that can be used either as part of the Platform, or separately in its own right.

Released under the open source Apache License 2.0, the OpenGamma Platform covers a range of asset classes and provides a comprehensive set of analytic measures and numerical techniques.

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