PRICING METHODOLOGY OF BLOOMBERG OVME FUNCTION FOR BOND

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In this document, we describe the models used by Bloomberg function OVME to price bond options (European and American). There are four models: Black, Black-Scholes, Normal Mean Reversion, and Lognormal. The first two models apply Black-Scholes formula to bond price (clean or dirty), while the latter two use tree construction of short rate models.

Black model

In the Black model, it is assumed that the clean price of the underlying bond follows the Black-Scholes dynamics with the dividend rate obtained from the forward price and price volatility obtained from historical yield volatility. This is equivalent to applying Black's formula to the forward price of underlying bond.

More precisely, the forward price F is computed according to the following equation:

$$P + accrued = D(T)F + \sum_i D(t_i)C_i$$

where P is the bond's clean price, $D(t)$ is the repo rate discounting for cash flow at time t, and C_i is the i-th cash flow between time 0 and T.

The dividend rate d that is to be inserted into Black-Scholes formula is computed according to the equation

$$F = P\, e^{(r-d)T}$$

where r is the continuously compounded risk free rate.

Calculation of option price with clean price P and strike K:

- **European call price:** $Call = Pe^{-dT}N(d_1) - Ke^{-rT}N(d_2) = e^{-rT}[FN(d_1) - KN(d_2)]$
- **European put price:** $Put = -Pe^{-dT}N(-d_1) + Ke^{-rT}N(-d_2) = e^{-rT}[-FN(-d_1) + KN(-d_2)]$
- $d_1 := \frac{\ln\left(\frac{P}{K}\right)+\left(r-d+\frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} = \frac{\ln\left(\frac{F}{K}\right)+\frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}$, $d_2 := \frac{\ln\left(\frac{P}{K}\right)+\left(r-d-\frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} = \frac{\ln\left(\frac{F}{K}\right)-\frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}$
- **American call/put price:** Binomial tree based on a lognormal distribution of prices for the underlying security.
- Volatility σ is Price Volatility σ_P, which is obtained from Yield Volatility σ_Y by the following formula (Y is the bond's yield and $D \coloneqq -\frac{1}{P}\frac{\partial P}{\partial Y}$ is the Macaulay duration):

$$\sigma_P = \sigma_Y Y \frac{1}{P}\frac{\partial P}{\partial Y} = -\sigma_Y Y\, D$$

The Yield Volatility σ_Y is obtained from historical data (with linear interpolation when necessary). For bonds denominated by JPY currency, implied volatility is used.

Calculation of Greeks for European options: Delta and Gamma are computed by Black-Scholes formula (with modifications, as dividend d is also dependent on underlying clean price P), and Vega is computed by increasing the volatility by 1%.

Calculation of Greeks for American options: Delta and Gamma are read off directly from binomial tree values; Vega is computed by increasing the volatility by 1%.

Black-Scholes model

In the Black-Scholes model, it is assumed that the dirty price of the underlying bond follows the Black-Scholes dynamics with zero dividend rate. The price of European option is calculated via the Black-Scholes formula, while the price of American option is calculated as the price of a Bermudan option with two dates: valuation day and the option expiry.

Calculation of option price with dirty price P and strike K:

- **European call price:** $Call = PN(d_1) - Ke^{-rT}N(d_2)$
- **European put price:** $Put = -PN(-d_1) + Ke^{-rT}N(-d_2)$
- $d_1 := \frac{\ln\left(\frac{P}{K}\right)+\left(r+\frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$, $d_2 := \frac{\ln\left(\frac{P}{K}\right)+\left(r-\frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$
- **American call/put price:** Price of a Bermudan option on two dates: valuation day and the option expiry.
- The volatility σ is Price Volatility σ_P, which is obtained from Yield Volatility σ_Y by the following formula (Y is the bond's yield and $D := -\frac{1}{P}\frac{\partial P}{\partial Y}$ is the Macaulay duration):

$$\sigma_P = \sigma_Y Y \frac{1}{P}\frac{\partial P}{\partial Y} = -\sigma_Y Y\, D$$

The Yield Volatility σ_Y is obtained from historical data (with linear interpolation when necessary). For bonds denominated by JPY currency, implied volatility is used.

Calculation of Greeks for European options: Delta and Gamma are computed by Black-Scholes formula, and Vega is computed by increasing the volatility by 1%.

Calculation of Greeks for American options: If not exercised immediately, the same as European one.

Lognormal model and Normal Mean Reversion model

These two models are based on short rate models, implemented via the Hull-White approach to tree construction of short rate models (see, for example, John Hull: *Options, Futures, & Other Derivatives*, fourth edition, section 21.13). The price of a bond option is then computed by the typical method of backward induction.

Lognormal model uses the standard lognormal model on short rate r:

$$\frac{dr}{r} = h(t)dt + \sigma_L dZ$$

The volatility σ_L is the yield volatility obtained from historical data (with linear interpolation when necessary).

Normal Mean Reversion model uses the normal, mean-reverting model on short rate r:

$$dr = (b(t) - ar)dt + \sigma_N dZ$$

where by default, the mean reversion coefficient a is set to 0.03, and the short rate volatility σ_N is given by the following formula

$$\text{if } a = 0, \sigma_N = |Y|\sigma_Y; \text{ if } a \neq 0, \sigma_N = \frac{aT|Y|\sigma_Y}{1 - e^{-aT}}$$

Here Y is the bond's yield, T is the bond's maturity, and σ_Y is the Yield Volatility. The Yield Volatility σ_Y is obtained from historical data (with linear interpolation when necessary); the above formula for short rate volatility σ_N is based on a closed form formula of zero coupon bond price under Normal Mean Reversion Model.

Calculation of Greeks: Delta and Gamma are computed by finite difference derivatives, and Vega is computed by increasing the volatility by 1%

Sensitivity Analysis

For all the four models, the sensitivities are calculated according to the following scheme. We shift the interest rate curve up by 10 bps, re-price the option, then we shift the interest rate curve down by 10 bps, re-price the option again. The sensitivity parameters are calculated from these prices together with the original price (no shift):

$$DV01 = \frac{P_- - P_+}{2\Delta r * 10000bps}$$

$$modified\ durtion = \frac{P_- - P_+}{2\Delta r * P_0}$$

$$convexity = \frac{P_+ + P_- - 2P_0}{2(\Delta r)^2 * P_0}$$

$$rho = DV01 * 100$$

where $\Delta r = 0.001\ (10bps)$, P_-, P_+, $and P_0$ are the option premium with $-\Delta r, 0, and \Delta r$ interest rate shift, respectively.

Note in the above formulas, $DV01$ is defined as the change of option premium when interest rate decreases by 1 bps, and rho is the change of option premium when interest rate changes by 1% (100bps). These conventions agree with those taken by MARS.