An Illustration of How to Price Asset Swaps Incorporating Accrued Interest

Nicholas Burgess

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Abstract

Asset swaps provide a form of asset financing, where investors borrow funds to purchase an asset, typically a bond. Asset swaps are also a good bond rich-cheap analysis tool. Such swaps can of course be used for speculative purposes. In this paper we provide a brief overview of asset swaps and derive the par-par asset swap spread incorporating bond accrued interest.

Notation

A	Annuity
A^{Fixed}	Annuity - swap fixed leg annuity
A^{Float}	Annuity - swap float leg annuity
AI	Accrued Interest
B	Bond Price
c	Bond Coupon
l_{j-1}	swap floating rate corresponding to the jth coupon, fixed in advance at time t_{j-1}
m	number of floating coupons
n	number of fixed coupons
N	swap notional in the case where it is constant
N_i	swap notional corresponding to the ith fixed coupon period
N_j	swap notional corresponding to the jth floating coupon period
p	par rate
ϕ	Indicator Function: +1 for $receiver$ swap and -1 for $payer$ swap
PV	Present Value or Price
$P(t_E, t_i)$	discount factor required to discount the ith fixed coupon to the swap effective date
$P(t_E, t_j)$	discount factor required to discount the jth floating coupon to the swap effective date
r^{Fixed}	swap fixed rate
s	floating spread over Libor
t_E	time to the swap effective or start date in years
t_i	time to the ith fixed coupon payment date in years
t_j	time to the jth floating coupon payment date in years
τ_i	accrual period or year fraction of the ith fixed coupon
τ_j	accrual period or year fraction of the jth floating coupon

Table 1: Notation

1 Asset Swaps

An Asset Swap is a swap whereby the fixed leg coupons are structured to replicate the cashflows of a bond or asset and the floating leg coupons replicate the floating leg of a standard swap plus a spread. Asset swaps are a mechanism to allow market participants to borrow (or loan) money at a rate of Libor plus a spread, s to fund a long (or short) position in a asset or bond. Asset swaps are also used for speculation purposes.

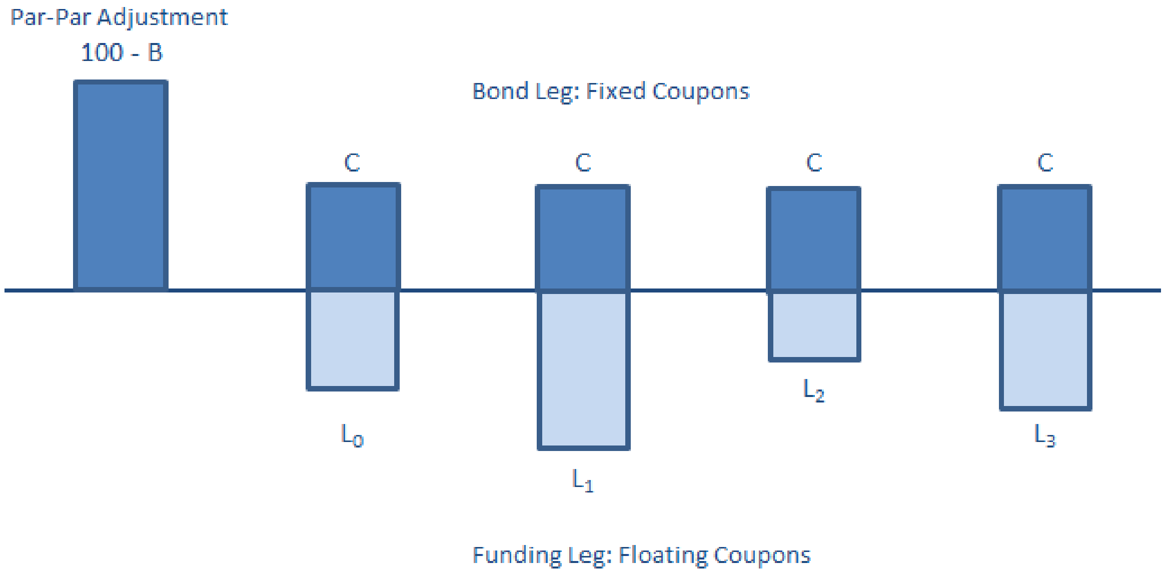

Figure 1: Asset Swap Cashflow Diagram

2 Asset Swap Spreads

The asset swap spread is a useful rich-cheap bond analysis tool.

The bonds of issuers of good credit quality, some Governments e.g. Germany and government agencies and supras and especially so for the shorter maturity bonds trade at $negative$ spreads to swaps. The opposite is true for poor or low quality credits such as Greek and Argentina Government bonds and many corporates bonds, which trade with $positive$ spreads. For good quality credits a $widening$ of the spread, which is negative, refers to a $richening$ of the bond (decline in bond yield relative to swaps) and for poor quality credits the opposite is true, namely a widening of the positive spread reflects a $cheapening$ of the bond and increase in bond yield.

It is difficult to make comparisons between different bonds based on yield alone. Bonds differ in many ways, maturity, coupon size, liquidity, futures delivery, inclusion in benchmark indices and many other idiosyncracies. Maturity is the most important of these differences so it is important to consider the term-structure of bond yields[1]. To this end we would like to fit a

[1]That is the term-structure of bond yields with the same issuer

smooth curve to the bond yield term-structure and compare yields, however all the above named differences complicate curve fitting and bond evaluation. The swap curve is free from the problems we encounter in bond curve fitting and more useful for comparison purposes. As a result swap spreads are popular for comparisons between different bonds.

3 Yield-Yield Asset Swap Spread

A long asset swap position or long swap spread position refers to owning a bond against a hedge in swaps. The yield-yield spread is defined as the yield of a bond less the swap rate of a maturity matched swap. The investor makes money as the spread widens[2], since the bond yield falls relative to the swap rate. The trade must be duration weighted so that the investor is exposed only to the spread between the swap rate and the bond yield and not to market direction. This is a first order approximation with the investor exposed to convexity risk.

The yield-yield spread is defined as the difference between the Bond yield taken at time t for a Bond maturing at time T and that of a corresponding Swap Par Rate observed at time t with a swap maturity at time T, namely

$$\textit{Yield-Yield Spread}\Big(t,T\Big) = \textit{Bond Yield}\Big(t,T\Big) - \textit{Swap Rate}\Big(t,T\Big) \tag{1}$$

4 Par-Par Asset Swap Spread

The Par-Par asset swap spread, s is defined as the floating spread such that an asset swap trades at par, which can be deduced from the following expression, where B is the Bond price[3].

$$PV^{\text{ Asset Swap}} = \underbrace{\phi r^{Fixed}\sum_{i=1}^{n} N_i\tau_i P(t_E,t_i)}_{\text{Fixed Leg}} - \underbrace{\phi\sum_{j=1}^{m} N_j(l_{j-1}+s)\tau_j P(t_E,t_j)}_{\text{Float Leg}} + \underbrace{\phi N_1\left(\frac{100-B}{100}\right)}_{\text{Par-Par Adjustment}} \tag{2}$$

setting $PV^{\text{ Asset Swap}} = 0$ in (2) and rearranging gives

$$\phi\sum_{j=1}^{m} N_j(l_{j-1}+s)\tau_j P(t_E,t_j) = \phi r^{Fixed}\sum_{i=1}^{n} N_i\tau_i P(t_E,t_i) + \phi N_1\left(\frac{100-B}{100}\right) \tag{3}$$

splitting out the spread s from the libor rates l_j gives

$$\phi\sum_{j=1}^{m} N_j s\tau_j P(t_E,t_j) = \phi r^{Fixed}\sum_{i=1}^{n} N_i\tau_i P(t_E,t_i) - \phi\sum_{j=1}^{m} N_j l_{j-1}\tau_j P(t_E,t_j) + \phi N_1\left(\frac{100-B}{100}\right) \tag{4}$$

[2]Widening here is referring to a richening of a Government Bond with a negative spread.
[3]The bond price can be clean or dirty i.e. exclude or include accrued interest respectively.

we divide the expression by ϕ and can treat the s term as a fixed rate, which we can take outside of the summation operator. In doing the later we can work with annuity expressions leading to (5) and (6) below

$$s\sum_{j=1}^{m} N_j\tau_j P(t_E,t_j) = r^{Fixed}\sum_{i=1}^{n} N_i\tau_i P(t_E,t_i) - \underbrace{\sum_{j=1}^{m} N_j l_{j-1}\tau_j P(t_E,t_j)}_{\text{Float PV}} + N_1\left(\frac{100-B}{100}\right) \tag{5}$$

noting that the floating leg PV is equivalent to the par swap fixed leg i.e. $pA^{Fixed}_{N_i}$ it follows that

$$sA^{Float}_{N_i} = r^{Fixed}A^{Fixed}_{N_i} - pA^{Fixed}_{N_i} + N_1\left(\frac{100-B}{100}\right) \tag{6}$$

simple rearrangement leads to our solution for the asset swap spread, namely the below

$$s = \left(\frac{\left(r^{Fixed}-p\right)A^{Fixed}_{N_i} + N_1\left(\frac{100-B}{100}\right)}{A^{Float}_{N_j}}\right) \tag{7}$$

for bonds with a constant notional we can express the spread formula as

$$s = \left(\frac{\left(r^{Fixed}-p\right)A^{Fixed}_{N} + N\left(\frac{100-B}{100}\right)}{A^{Float}_{N}}\right) \tag{8}$$

where B denotes the bond price, which can be clean or dirty.

5 Par-Par Asset Swap Spread with Accrued Interest

If an asset swap is traded between Bond coupons accrued interest needs to be accounted for in our calculations. Bonds coupons are always paid in full. As such bond purchases return partially accrued interest to the seller and Bond prices are adjusted accordingly. Bond prices quote as both $Dirty$ including a full next coupon and as $Clean$ after adjusting for accrued interest due to the seller.

Likewise a par-par asset swap must account for accrued interest payments. The par adjustment is based upon the bond price B, which includes an accrued interest adjustment if based upon a clean price and allows for a full next coupon if based upon a dirty bond price.

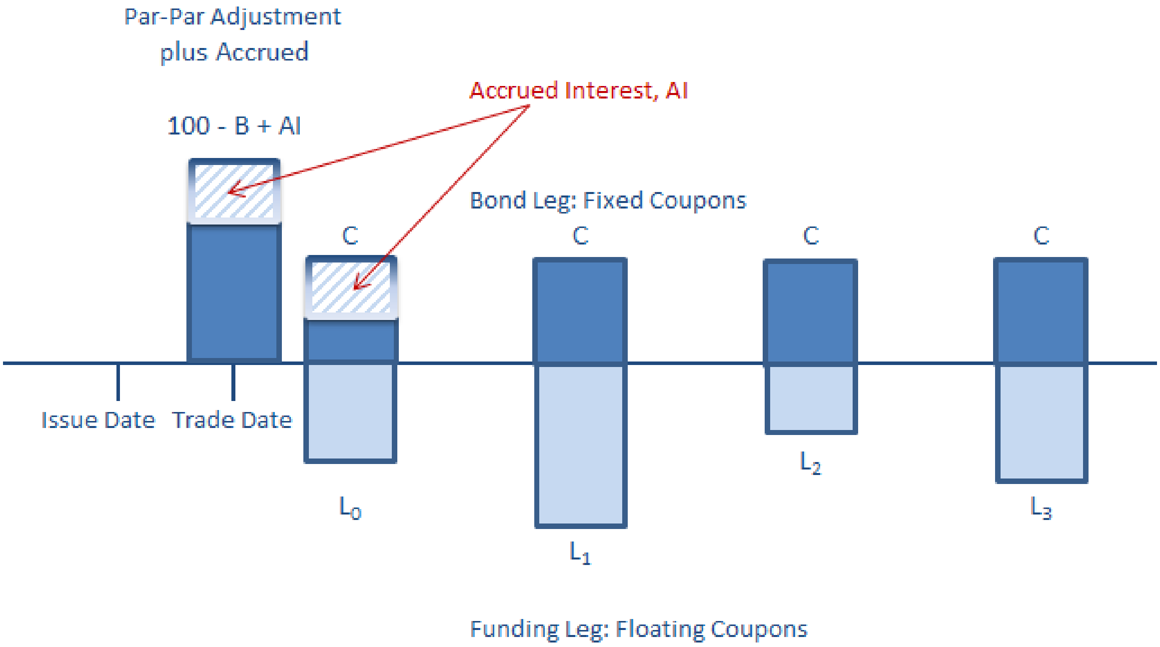

Figure 2: Asset Swap Cashflow Diagram with Accrued Interest

We note however that accrued interest, if accounted for in the par adjustment, is paid on the trade date[4], but on the fixed leg of the asset swap the accrued is paid later on the swap coupon date as shown in figure (2). As such we have a discounting difference between the accrued paid on trade date and the next coupon date, which must be accounted for.

For Bonds using the $Clean$ Bond price we modify equation (8) to account for accrued interest and the discounting difference as follows

$$s^{Clean} = \left(\frac{\left(r^{Fixed} - p\right) A_N^{Fixed} - AI + N\left(\frac{100-B}{100}\right) - DD}{A_N^{Float}} \right) \tag{9}$$

and when using a $Dirty$ Bond price we adjust only to allow for the discounting difference

$$s^{Dirty} = \left(\frac{\left(r^{Fixed} - p\right) A_N^{Fixed} + N\left(\frac{100-B}{100}\right) - DD}{A_N^{Float}} \right) \tag{10}$$

where AI denotes the accrued interest and DD the discounting difference and are defined as follows

$$\begin{aligned} AI &= \left(N \times r^{Fixed} \times \tau_i\right) \\ DD &= \left(AI \times P(t_E, t_i)\right) - AI \end{aligned} \tag{11}$$

where τ_i and t_i denote the accrued interest accrual year fraction and payment date respectively.

[4]To be more precise it is paid on the effective or settlement date

6 Par-ParAsset Swap Spread Example

Consider the 10 year benchmark German Bund, currently DBR 0.5% 2016, which has a yield of -0.18841% and is quoted with a clean price of 106.680 with 175 days of accrued interest at EUR 2,391.87. The current 10 year swap rate is 0.23359%.

Bond Definition

CleanPrice	106.680
DirtyPrice	106.919
Accrued (175 Days)	2,391.87
Ticker	DE0001102390 Govt
SecurityName	DBR 0 1/2 02/15/26
IssueDate	Fri 15-Jan-16
FirstCouponDate	Wed 15-Feb-17
MaturityDate	Sun 15-Feb-26
Coupon	0.50%
Price	106.680
isCleanPrice	TRUE
Daycount	ACT/ACT
Frequency	Annual

Figure 3: DBR 0.5% 2016 Bond Definition

The yield-yield spread for this Bond is quoted on Bloomberg is -42.2 basis points and likewise the par-par asset swap spread -41.2 basis points. Derive and explain the yield-yield and par-par asset swap spread calculations.

6.1 Yield-Yield Spread Calculation

The yield-yield spread is calculated as follows and quoted in basis points.[5]

$$\begin{aligned} \textit{Yield-Yield Spread}\Big(\textit{today}, 10Y\Big) &= \textit{Bond Yield}\Big(\textit{today}, 10Y\Big) - \textit{Swap Rate}\Big(\textit{today}, 10Y\Big) \\ &= -0.18841\% - 0.23359\% \\ &= -0.42200\% \\ &\textit{or -42.2 basis points} \end{aligned}$$

6.2 Par-Par Spread Calculation

For the par-par asset swap spread, since our bond is quoted with a clean price we use equation (9) namely

$$s^{Clean} = \left(\frac{\left(r^{Fixed} - p\right) A_N^{Fixed} - AI + N\left(\frac{100-B}{100}\right) - DD}{A_N^{Float}} \right)$$

[5]A basis points is 1/100th of a percent

for clarity we can decompose the equation (9) into the following components

$$s^{Clean} = \left(\frac{PV(Fixed) - PV(Float) - AI + ParAdj - DD}{Annuity(Float)} \right)$$

using the clean price we calculate the spread as follows

Swap Leg Breakdown

Fixed PV	51,219
Accrued Interest	2,392
Fixed PV less Accrued	48,828
Float PV	22,805

Asset Swap Calculation

Swap PV	26,022	
Par Adjustment	-66,800	Clean Px
Discounting Diff	6	
Float Annuity	9,907,204	
ASW Spread	-41.166	-41.2 Basis Points

Figure 4: Par-Par Asset Swap Spread using the Clean Bond Price

similarly we can decompose the equation (10) into the following components

$$s^{Dirty} = \left(\frac{PV(Fixed) - PV(Float) + ParAdj - DD}{Annuity(Float)} \right)$$

and using the dirty price we calculate the spread as follows

Swap Leg Breakdown

Fixed PV	51,219
Accrued Interest	0
Fixed PV less Accrued	51,219
Float PV	22,805

Asset Swap Calculation

Swap PV	28,414	
Par Adjustment	-69,192	Dirty Px
Discounting Diff	6	
Float Annuity	9,907,204	
ASW Spread	-41.166	-41.2 Basis Points

Figure 5: Par-Par Asset Swap Spread using the Dirty Bond Price

7 Conclusion

In summary we reviewed the asset swap product and secondly we discussed the asset swap pricing methodologies. Thirdly we derived the asset swap spread and finally we demonstrated how to incorporate Bond accrued interest into the asset swap spread, providing an example.

References

[1] **Brigo**, D and **Mercurio**, F (2007) Textbook: Interest Rate Models - Theory and Practice, Springer Finance

[2] **Hull**, J (2011) Textbook: Options, Futures and Other Derivatives 8ed, Pearson Education Limited