# How to Price Swaps in your Head
# An Interest Rate Swap & Asset Swap Primer

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## Abstract

Interest rate swaps are an actively traded product in the financial marketplace and are popular for hedging mortgage and corporate loan exposures against rises in interest rates. Asset swaps on the other hand provide a form of asset financing, where investors borrow funds to purchase an asset, typically a bond. Asset swaps are also a good bond rich-cheap analysis tool. Both types of swaps can of course be used for speculative purposes.

In this paper we provide an overview of both interest rate swaps and asset swaps, we explain the products and examine how they are priced & quoted in the market. Analytical and numerical risk is also considered. We conclude with a review of swap pricing formulas and examine how to price swaps quickly in one's head. We do this using simple approximations that hold extremely well in the current low interest rate environment.

## Disclaimer

The contents of this paper are based upon publicly available material for indicative swap pricing purposes only. This paper does not contain any proprietary nor copyrighted materials without explicit permission.The author would like to thank Bloomberg L.P. for their permission to demonstrate Bloomberg terminal pages within this paper.

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# Notation

The notation in table (1) will be used for pricing formulae.

| | |
|---|---|
| $A$ | Annuity |
| $A^{Fixed}$ | Annuity - swap fixed leg annuity |
| $A^{Float}$ | Annuity - swap float leg annuity |
| $A_N$ | Annuity - scaled by a constant notional $N$ |
| $A_{N_i}$ | Annuity - scaled by a variable notional $N_i$ |
| $B$ | Bond Price; can be 'Dirty' or 'Clean' i.e. with or without accrued interest respectively |
| $C$ | Cash-flow |
| $c_n$ | Bond Coupon; the nth bond coupon |
| $D_{Mac}$ | Macaulay's Duration |
| $D_{Mod}$ | Modified Duration |
| $l_{j-1}$ | swap floating rate corresponding to the $j$th coupon, fixed in advance at time $t_{j-1}$ |
| $m$ | number of floating coupons |
| $n$ | number of fixed coupons |
| $N$ | swap notional in the case where it is constant |
| $N_i$ | swap notional corresponding to the $i$th fixed coupon period |
| $N_j$ | swap notional corresponding to the $j$th floating coupon period |
| $p$ | par rate |
| $P(t_E, t_i)$ | discount factor required to discount the $i$th fixed coupon to the swap effective date |
| $P(t_E, t_j)$ | discount factor required to discount the $j$th floating coupon to the swap effective date |
| $PV$ | Present Value or Price |
| $\phi$ | Indicator Function: +1 for *receiver* swap and -1 for *payer* swap |
| $r^{Fixed}$ | swap fixed rate |
| $s$ | floating spread over Libor |
| $t_E$ | time to the trade effective or start date in years |
| $t_i$ | time to the $i$th fixed coupon payment date in years |
| $t_j$ | time to the $j$th floating coupon payment date in years |
| $\tau_i$ | accrual period or year fraction of the $i$th fixed coupon |
| $\tau_j$ | accrual period or year fraction of the $j$th floating coupon |
| $y$ | Bond Yield to Maturity |

Table 1: Notation

# 1 Interest Rate Swaps

An interest rate swap or IRS is a financial product whereby one party exchanges a series of fixed payments (the fixed leg) for a series of floating payments (the floating leg), as illustated in figure (1) below. A swap can be considered a mechanism to exchange a fixed rate loan for a variable or floating rate loan, where the floating rate of interest is typically linked to Libor[1].

The party receiving the fixed coupons would describe the swap as a *receiver* swap and likewise if paying the fixed payments a *payer* swap. Interest rate swaps are quoted in the market place as as *par rate* i.e. the fixed interest rate that makes the swap worth zero or par as at the start or $effective$ *date* of the swap. The par rate can be considered an average[2] of the floating Libor interest rates.

**Swap Terminology: Long / Short vs Payer / Receiver**
*Being long or short a swap is a reference to the floating rate, whilst the terms payer or receiver are a reference to the fixed rate. Being long a swap means we are long or receive the floating rate and pay the fixed rate i.e. we hold a payer swap. Likewise being short a swap means that we are short or pay the floating rate and receive the fixed rate i.e. we hold a receiver swap.*

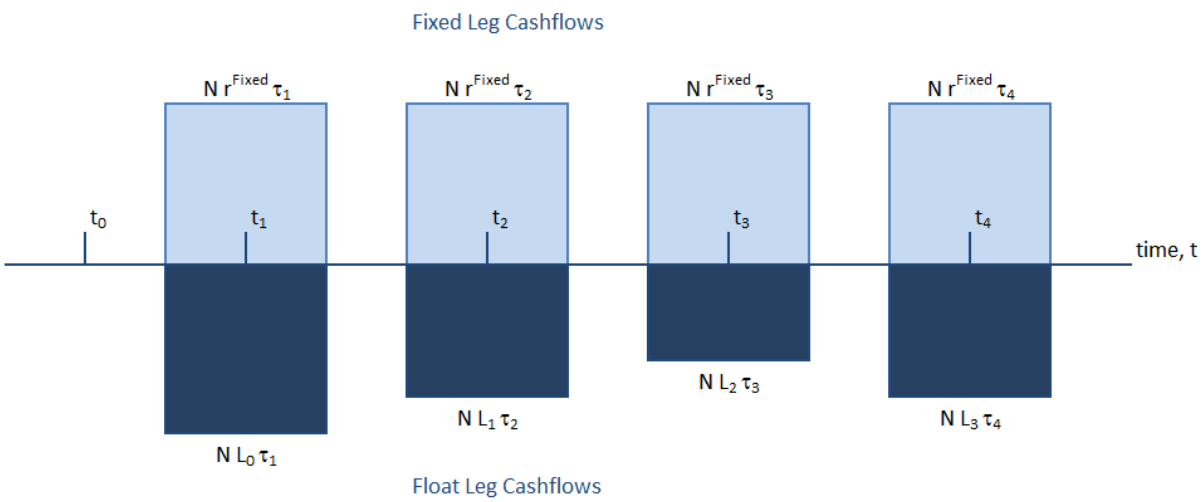



Figure 1: Interest Rate Swap Cashflow Illustration

[1]London Interbank Offer Rate. This is the rate at which a panel of London banks lend to each other.
[2]A weighted average by notional and discount factor.

**Comment: Fixing In Advance**

*As shown in figure (1) attention is drawn to the fact that fixings*[3]*, also known as resets, typically take place in advance. A floating coupon paid at time $t_1$ would have it's rate of interest $l_0$ set at time $t_0$, likewise the coupon at time $t_2$ having a rate of interest $l_1$ set at time $t_1$ and so forth.*

Counterparties enter swap transactions to hedge existing cashflows from adverse interest rate movements in the market and / or speculate on interest rate movements. For example a party paying variable / floating interest on a loan may wish to swap to a fixed rate of interest, in the belief that interest rates are going to rise. Similarly a party may simply wish to enter a swap transaction to speculate and bet on interest rates moving one way or another. Swaps are a key instrument used by mortgage lenders to hedge their interest rate exposures. Corporations also hedge their potentially large loan exposures using swaps.

Currently EUR par-swaps are being quoted in the market as shown in figure (2). The rates quoted are the market best Bid / Offer[4] par-rates respectively. The market quotes the fixed rate it is willing to exchange for the floating Libor rate i.e. the par rate. A client wishing to enter a par receiver swap would receive the market bid rate and a similarly to enter a payer swap would pay the market offer rate.

BTI

Interest Rate Swaps | 2) Tools | 3) Settings | 4) Trading Access | IRS Multi-Dealer RFQ

18:17 * Market Closed *

20) EUR | 21) EUR MAC | 22) USD | 23) USD MAC | 24) USD IMM | 25) GBP | 26) CHF | 27) AUD

Vanilla 6M | 6M Crv | 6M Bfly | Vanilla 3M | 3M Crv | 3M Bfly | Vanilla 12M | OIS | BFX | IMM 6M | IMM 3M

| 6M Euribor | | | | 6M Euribor | | |
|---|---|---|---|---|---|---|
| 30) 1 Year | 0.041 / 0.046 | -0.002 | | 48) 18 Year | 1.339 / 1.348 | -0.027 |
| 31) 18 Month | 0.052 / 0.058 | -0.002 | | 49) 19 Year | 1.360 / 1.368 | -0.030 |
| 32) 2 Year | 0.075 / 0.080 | -0.001 | | 50) 20 Year | 1.377 / 1.385 | -0.030 |
| 33) 3 Year | 0.146 / 0.151 | -0.001 | | 51) 25 Year | 1.407 / 1.416 | -0.035 |
| 34) 4 Year | 0.241 / 0.246 | -0.004 | | 52) 30 Year | 1.413 / 1.423 | -0.038 |
| 35) 5 Year | 0.351 / 0.356 | -0.004 | | 53) 40 Year | 1.429 / 1.442 | -0.036 |
| 36) 6 Year | 0.470 / 0.475 | -0.007 | | 54) 50 Year | 1.390 / 1.403 | -0.036 |
| 37) 7 Year | 0.592 / 0.597 | -0.009 | | Other Markets | | |
| 38) 8 Year | 0.711 / 0.716 | -0.011 | | 55) EURO-SCHATZ | 18:00 d 111.370 | + 0.005 |
| 39) 9 Year | 0.820 / 0.825 | -0.014 | | 56) EURO-BOBL | 18:01 d 130.790 | + 0.050 |
| 40) 10 Year | 0.917 / 0.922 | -0.015 | | 57) EURO-BUND | 18:02 d 155.860 | + 0.280 |
| 41) 11 Year | 1.002 / 1.010 | -0.015 | | 58) EURO-BUXL | 18:02 d 159.320 | + 1.360 |
| 42) 12 Year | 1.077 / 1.084 | -0.018 | | | | |
| 43) 13 Year | 1.142 / 1.150 | -0.020 | | | | |
| 44) 14 Year | 1.197 / 1.205 | -0.021 | | | | |
| 45) 15 Year | 1.243 / 1.251 | -0.024 | | | | |
| 46) 16 Year | 1.283 / 1.290 | -0.025 | | | | |
| 47) 17 Year | 1.314 / 1.322 | -0.027 | | | | |

Figure 2: Bloomberg IRS Trading Portal. Used with Permission.

At inception a swap has zero cost and has a present value or PV of zero. After entering a payer swap transaction, should Libor interest rates increase, the fixed rate payer (floating rate receiver) will benefit. Likewise should Libor rates decrease the holder of a payer swap will lose money. A positive (negative) swap PV reflects the size of the benefit (loss) to the swap holder.

[3] This is a reference to variable rates of interest on the floating leg, which periodically need fixing or resetting.

[4] The bid is the market buy rate and the offer is the market sell rate.

When the fixed rate of the swap is not equal to the par rate such swaps are quoted as PVs. In figure (3) below we show the current quote for a EUR swap with a notional of EUR 1,000,000 and fixed rate of 1% for 5 years . Expressing quotes this way reflects the cost to enter / exit such a swap. This is the quote convention for bespoke swaps and swap unwinds[5]. Swaps can also be canceled by entering a swap with equal and offsetting cashflows.

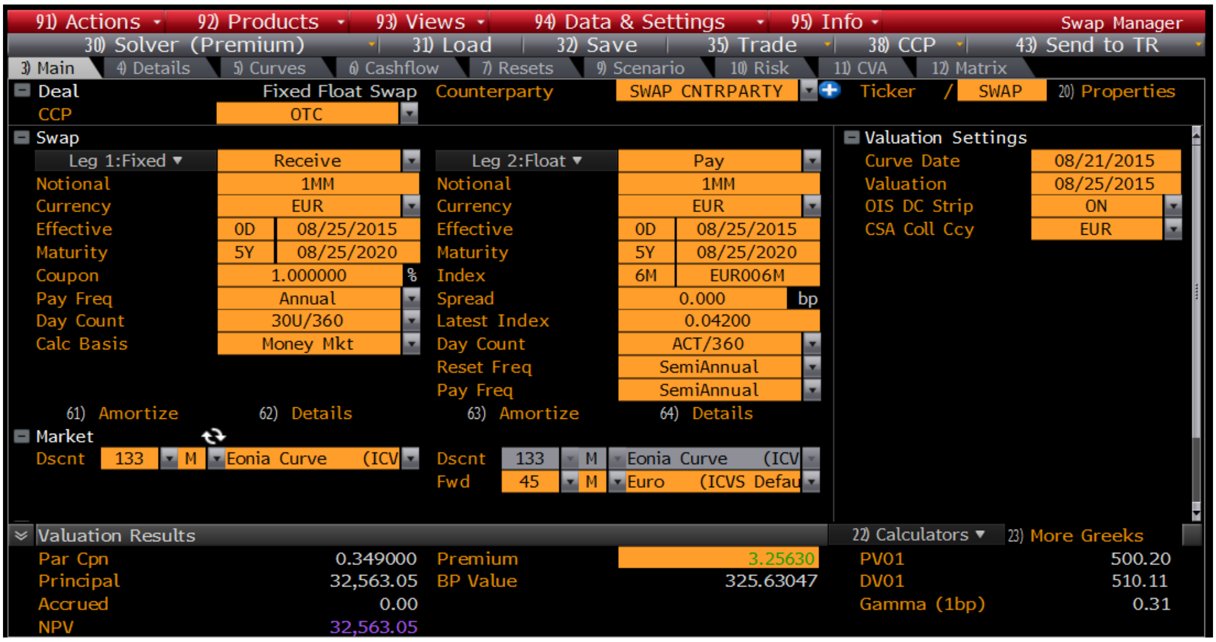



Figure 3: Bloomberg EUR 5Y Swap Quote. Used with Permission.

# 2 Interest Rate Swap Pricing Formulae

## 2.1 Annuity Definition

Let us define a stream of regular payments an *annuity*, $A$. An annuity has a unit coupon i.e. has a notional $N = 1$ as defined below.

$$A = \sum_{i=1}^{n} \tau_i P(t_E, t_i) \tag{2.1}$$

[5]An unwind is the term used to describe the cancellation of a swap

For the our purposes we extend the annuity definition as follows.

$$
\boxed{\begin{aligned}
A_N &= N\sum_{i=1}^{n} \tau_i P(t_E, t_i) && \text{when the notional is constant} \\
A_{N_i} &= \sum_{i=1}^{n} N_i \tau_i P(t_E, t_i) && \text{when the notional is variable} \\
A &= \sum_{i=1}^{n} \tau_i P(t_E, t_i) && \text{when the notional is unity}
\end{aligned}} \tag{2.2}
$$

Furthermore let us define an annuity $A^{Fixed}$ when the annuity coupon payments coincide with the fixed leg payments and likewise $A^{Float}$ for the floating leg.

**Annuity Example 1**
*Consider a EUR 5Y Swap with a constant notional of EUR 1mm. Find the value of the Annuity expressions A and $A_N$ from equation (2.2)*

*You may assume the swap has 5 yearly accrual periods with $\tau_i = 1.0$ and that interest rates are zero giving discount factors $P(t_E, t_i) = 1.0$*

$$
A = \sum_{i=1}^{5} \left( \tau_i . P(t_E, t_i) \right) = \sum_{i=1}^{5} 1 = 5
$$

$$
A_N = N \sum_{i=1}^{5} \left( \tau_i . P(t_E, t_i) \right) = 1,000,000 \sum_{i=1}^{5} 1 = \text{EUR 5mm}
$$

**Annuity Example 2**
*Consider a EUR 10Y Swap with a constant notional of USD 2.5mm. Find the value of the Annuity expressions A and $A_N$ from equation (2.2)?*

*You may again assume the swap has 10 yearly accrual periods with $\tau_i = 1.0$ and that interest rates are zero giving discount factors $P(t_E, t_i) = 1.0$*

$$
A = \sum_{i=1}^{10} \left( \tau_i . P(t_E, t_i) \right) = \sum_{i=1}^{10} 1 = 10
$$

$$
A_N = N \sum_{i=1}^{10} \left( \tau_i . P(t_E, t_i) \right) = 2,500,000 \sum_{i=1}^{10} 1 = \text{USD 25mm}
$$

**Remark 1:** ***How to use the Annuity definition***
*The fixed leg of a swap is much easier to calculate that the floating leg, where we have to forecast each and every Libor rate. The annuity definition provides a mechanism to represent a floating leg as a fixed leg. It is a key component of the swap pricing formulas in section 2 that follow, simplifying the swap pricing formulas.*

*Furthermore if we assume annual swap accrual periods and that discount factors are 1 the annuity is simple to calculate, as can be seen in the annuity examples above. These approximating assumptions work reasonably well and especially so in the current low interest rate environment, making it easy to price swaps in one's head.*

## 2.2 Swap Pricing Formulae & Definitions

In the pricing formulas and definitions that follow multiple expressions for the swap present value or PV are shown. All the expressions are equivalent and give the same answer, however the functional forms expressed in terms of the annuity and par rate, which are boxed and coloured *blue* are the most convenient to work with.

The reader should be aware that it is conventional to express swap pricing formulae in terms of a standardized market par rate $p^{Market}$, which we outline below in section (2.5). The market par rate should not be confused with the trade specific par rate $p^{Trade}$, which is less convenient to work with, see sections (2.9) and (2.9.3).

## 2.3 Fixed Leg Definition

The fixed leg of a swap refers to the fixed coupon payments of the swap. Receiver swaps receive the fixed coupons (and pay the floating coupons) and payer swaps pay the fixed coupons (and receive the floating coupons). The present value or PV of the fixed leg is defined as follows.

$$\boxed{PV^{\text{Fixed Leg}} = r^{Fixed} \sum_{i=1}^{n} N_i \tau_i P(t_E, t_i)} \tag{2.3}$$

## 2.4 Floating Leg Definition

The floating leg of a swap refers to the variable or floating Libor coupon payments of the swap. Each coupon is determined by the Libor rate at the start of the coupon period. When the Libor rate is known the rate is said to have been fixed or reset and the corresponding coupon payment is known. The present value or PV of the floating leg is defined as follows.

$$\boxed{PV^{\text{Float Leg}} = \sum_{j=1}^{m} N_j (l_{j-1} + s) \tau_j P(t_E, t_j)} \tag{2.4}$$

It is possible and sometimes convenient to bifurcate the floating leg into a floating Libor and fixed spread components as follows

$$
\begin{aligned}
PV^{\text{ Float Leg}} &= \underbrace{\sum_{j=1}^{m} N_j l_{j-1}\tau_j P(t_E,t_j)}_{\textit{Floating Libor}} + s \underbrace{\sum_{j=1}^{m} N_j \tau_j P(t_E,t_j)}_{\textit{Fixed Spread}} \\
&= \sum_{j=1}^{m} N_j l_{j-1}\tau_j P(t_E,t_j) + sA_{N_j}^{Float}
\end{aligned}
\tag{2.5}
$$

## 2.5 Par Rate Definition

In the swaps market investors want to enter swaps transactions at zero cost. On the swap effective date or start date of the swap the swap has zero value, however as time progresses this will no longer be the case and the swap will become profitable or loss making.

To this end investors want to know what fixed rate should be used to make the fixed and floating legs of a swap transaction equal, which we denote $p^{Market}$ Such a fixed rate is called the *swap* or *par rate*. Swaps that are executed with the fixed rate being set to the par rate and called *par swaps*.

The market par rate is used to convert the floating leg into a fixed leg via the following formula

$$
\begin{aligned}
PV^{\text{Float Leg}} &= PV^{\text{Fixed Leg}} \\
N\sum_{j=1}^{m} l_{j-1}\tau_j P(t_E,t_j) &= p^{Market} N \sum_{i=1}^{n} \tau_i P(t_E,t_i) \\
N\sum_{j=1}^{m} l_{j-1}\tau_j P(t_E,t_j) &= p^{Market} A_N^{Fixed}
\end{aligned}
\tag{2.6}
$$

combining (2.5) and (2.6) and noting that par swaps have a spread $s$ of zero leads to

$$
\boxed{PV^{\text{ Float Leg}} = p^{Market} A_N^{Fixed} + sA_N^{Float}}
\tag{2.7}
$$

Market participants quote a standardized[6] par rate $p^{Market}$, which is a rearrangement of (2.6) with notional $N$ terms canceling, defined as follows

$$
\boxed{p^{Market} = \frac{\sum_{j=1}^{m} l_{j-1}\tau_j P(t_E,t_j)}{\sum_{i=1}^{n} \tau_i P(t_E,t_i)}}
\tag{2.8}
$$

or alternatively as

$$
\boxed{p^{Market} = \frac{\sum_{j=1}^{m} l_{j-1}\tau_j P(t_E,t_j)}{A^{Fixed}}}
\tag{2.9}
$$

[6]As such the underlying swap has a constant notional and a Libor spread of zero. There is also a trade specific par rate, which is not usually quoted, but rather provided on request, see section (2.9) for more details.

**Remark 2:** ***Par Rate Conventions***
*The market par rate $p^{market}$ quotes published in the market place are derived from standardized par swaps; as such the underlying swaps and the corresponding par rates hold only for swaps with a fixed or constant notional and zero Libor spread. There is however a trade specific par rate, which is not published but quoted on a request, see section (2.9).*

## 2.6 Receiver Swap Definition

In a *receiver* swap the investor receives the fixed leg and pays the floating leg, see sections (2.3) and (2.4) respectively. A 'Receiver' Swap has a present value or PV of

$$\begin{aligned} PV^{Receiver} &= PV^{\text{Fixed Leg}} - PV^{\text{Float Leg}} \\ &= r^{Fixed} \underbrace{\sum_{i=1}^{n} N_i \tau_i P(t_E, t_i)}_{Annuity} - \sum_{j=1}^{m} N_j (l_{j-1} + s) \tau_j P(t_E, t_j) \\ &= \left( r^{Fixed} A_{N_i}^{Fixed} \right) - \sum_{j=1}^{m} N_j (l_{j-1} + s) \tau_j P(t_E, t_j) \end{aligned} \tag{2.10}$$

the floating leg can be expressed as a fixed leg[7] using a trade specific par rate $p^{Trade}$

$$PV^{Receiver} = \left( r^{Fixed} - p^{Trade} \right) A_{N_i}^{Fixed} \tag{2.11}$$

or alternatively using the market par rate $p^{Market}$, which holds for a constant notional $N$ only

$$\boxed{PV^{Receiver} = \left( r^{Fixed} - p^{Market} \right) A_N^{Fixed} - s A_N^{Float}} \tag{2.12}$$

## 2.7 Payer Swap Definition

In a *payer* swap the investor pays the fixed leg and receives the floating leg, see sections (2.3) and (2.4) respectively. A 'Payer' Swap has a present value a PV of

$$\begin{aligned} PV^{Payer} &= PV^{\text{Float Leg}} - PV^{\text{Fixed Leg}} \\ &= \sum_{j=1}^{m} (N_j l_{j-1} + s) \tau_j P(t_E, t_j) - r^{Fixed} \underbrace{\sum_{i=1}^{n} N_i \tau_i P(t_E, t_i)}_{Annuity} \\ &= \sum_{j=1}^{m} (N_j l_{j-1} + s) \tau_j P(t_E, t_j) - \left( r^{Fixed} A_{N_i}^{Fixed} \right) \end{aligned} \tag{2.13}$$

[7]See sections (2.5) and (2.9)

the floating leg can be expressed as a fixed leg[8] using a trade specific par rate $p^{Trade}$

$$PV^{Payer} = -\left(r^{Fixed} - p^{Trade}\right)A^{Fixed}_{N_i} \tag{2.14}$$

or alternatively using the market par rate $p^{Market}$, which holds for a constant notional $N$ only

$$\boxed{PV^{Payer} = -\left(r^{Fixed} - p^{Market}\right)A^{Fixed}_{N} + sA^{Float}_{N}} \tag{2.15}$$

## 2.8 Generic Swap Definition

In compact form we can represent the present value of a swap as follows

$$\begin{aligned} PV^{Swap} &= \phi\left(PV^{\text{ Fixed Leg}} - PV^{\text{ Float Leg}}\right) \\ &= \phi\left(r^{Fixed}\sum_{i=1}^{n} N_i\tau_i P(t_E, t_i) - \sum_{j=1}^{m} N_j(l_{j-1} + s)\tau_j P(t_E, t_j)\right) \end{aligned} \tag{2.16}$$

the floating leg can be expressed as a fixed leg[9] using a trade specific par rate $p^{Trade}$

$$PV^{Swap} = \phi\left(r^{Fixed} - p^{Trade}\right)A^{Fixed}_{N_i} \tag{2.17}$$

or alternatively using the market par rate $p^{Market}$, which holds for a constant notional $N$ only

$$\boxed{PV^{Swap} = \phi\left[\left(r^{Fixed} - p^{Market}\right)A^{Fixed}_{N} - sA^{Float}_{N}\right]} \tag{2.18}$$

The vast majority of swaps traded have a constant notional in which case equation (2.16) simplifies to

$$\begin{aligned} PV^{Swap} &= \phi N\left(r^{Fixed}\sum_{i=1}^{n} \tau_i P(t_E, t_i) - \sum_{j=1}^{m}(l_{j-1} + s)\tau_j P(t_E, t_j)\right) \\ &= \phi N\left(r^{Fixed}A^{Fixed} - \sum_{j=1}^{m}(l_{j-1} + s)\tau_j P(t_E, t_j)\right) \end{aligned} \tag{2.19}$$

or equivalently in terms of par rates[10], using a trade specific par rate $p^{Trade}$

$$\begin{aligned} PV^{Swap} &= \phi N\left(r^{Fixed} - p^{Trade}\right)A^{Fixed} \\ &= \phi\left(r^{Fixed} - p^{Trade}\right)A^{Fixed}_{N} \end{aligned} \tag{2.20}$$

or alternatively using the market par rate $p^{Market}$

$$\boxed{\begin{aligned} PV^{Swap} &= \phi N\left[\left(r^{Fixed} - p^{Market}\right)A^{Fixed} - sA^{Float}\right] \\ &= \phi\left[\left(r^{Fixed} - p^{Market}\right)A^{Fixed}_{N} - sA^{Float}_{N}\right] \end{aligned}} \tag{2.21}$$

[8] See sections (2.5) and (2.9)
[9] See sections (2.5) and (2.9)
[10] See sections (2.5) and (2.9)

## 2.9 Par Rate Overview

The par rate[11] is the fixed rate that makes the value of the fixed and floating leg of a swap identical. Suffice to say the par rate also makes the value of the swap price to par at inception i.e. $PV^{Swap} = 0$. Swap investors are usually interested in entering a swap at zero cost. The par rate was constructed and designed with this in mind i.e. to construct a swap where the fixed and floating legs are equivalent on the start or effective date, after which investors desire for the swap to trade away from par in their favor.

In terms of pricing swaps mentally in one's head the par rate provides a useful mechanism to convert a floating leg into a fixed leg and consequently simplify the pricing process.

### 2.9.1 Market Par Rates, $p^{Market}$

Interest rate swaps are quoted in the market as a par rate see figure (2); quotes span a set of benchmark maturities with all underlying swaps having a constant notional and no Libor spread. Such quotes are typically cleared on an exchange, very liquid and easy to unwind or novate[12].

Of course swaps can be tailored to suit an individual's or corporate's needs, in many different ways; one can customize the notional, vary the notional to decrease (amortize) or increase (acrete) with time, coupons and payment dates can be irregular and floating Libor spreads are often added and customized to meet client requirements. Such features are often customized, but this does not constitute an exclusive list.

Bespoke swaps are typically quoted as PVs not par rates; likewise swap unwinds are also typically quoted as a PV. In comparison to standardized swaps, bespoke and tailored swaps are not typically cleared on exchange, not so liquid, have larger bid-offer spreads and are usually more expensive to unwind or novate. After the effective or start date swaps quote as a PV not a par rate, since the par rate construct is designed for use on the start date, and not over the life of the trade. There is no reason why one could not calculate the par rate for a bespoke swap, however the par rate for customized swaps is not suitable for quotation, publication or yield curve calibration purposes.

Rather than attempt to calculate and publish every possible floating leg variation the market i.e. exchanges, brokers, investment banks and other participants publish the standard interest rate swap quotes. Such swaps have amongst other things the following characteristics:

- Constant Notional
- Standardized Coupons
- Regular Payment Dates

[11] Also known as the swap rate

[12] A type of unwind where the instead of closing out the trade or taking the opposite position, the trade is legally transferred to another counterparty

- No Libor Spread
- The price is zero on the swap effective date

Contrary to this bespoke swap prices have to be requested by market participants for individual trades via RFQ[13] over the phone or electronically. Such swaps do not price to par and are quoted in present value terms.

We can derive the par rate by simple rearrangement of (2.16) and solving for the fixed rate $r^{Fixed}$ which makes the present value of the fixed leg equal to that of the floating leg.

### 2.9.2 Market Par Rate Definition, $p^{Market}$

The market par rate is obtained by simple rearrangement of (2.16), setting $r^{Fixed} = p^{Market}$, and solving for the fixed rate that makes the present value of the swap zero, or equivalently solving for the fixed rate that makes the fixed and floating legs of a swap equal. We remind the reader that market par rate quotes are based on fixed notional swaps with no Libor spread.

Therefore we deduce

$$\begin{aligned} PV^{Floating\ Leg} &= PV^{Fixed\ Leg} \\ N\sum_{j=1}^{m} l_{j-1}\tau_j P(t_E,t_j) &= r^{Fixed} N \sum_{i=1}^{n} \tau_i P(t_E,t_i) \end{aligned} \tag{2.22}$$

recalling that the par rate $p^{Market}$ is the fixed rate that equates the fixed and floating legs gives

$$\sum_{j=1}^{m} N l_{j-1}\tau_j P(t_E,t_j) = p^{Market} \sum_{i=1}^{n} N\tau_i P(t_E,t_i) \tag{2.23}$$

rearranging for the par rate, and noting that the Notional $N$ terms cancel gives

$$p^{Market} = \frac{\sum_{j=1}^{m} N l_{j-1}\tau_j P(t_E,t_j)}{\sum_{i=1}^{n} N\tau_i P(t_E,t_i)} = \frac{\sum_{j=1}^{m} l_{j-1}\tau_j P(t_E,t_j)}{\sum_{i=1}^{n} \tau_i P(t_E,t_i)} \tag{2.24}$$

leading to several equivalent representations for the par rate quoted as a weighted average Libor rate; weighted by the discount factor and annuity or quoted in terms of the float leg present value respectively

$$\boxed{p^{Market} = \frac{\sum_{j=1}^{m} l_{j-1}\tau_j P(t_E,t_j)}{\sum_{i=1}^{n} \tau_i P(t_E,t_i)} = \frac{\sum_{j=1}^{m} l_{j-1}\tau_j P(t_E,t_j)}{A^{Fixed}} = \frac{PV^{Market\ Float\ Leg}}{A^{Fixed}}} \tag{2.25}$$

[13] Request for Quote

Finally it is important to note that the par rate provides an extremely useful mechanism to convert the floating leg of a generic swap into a fixed leg. Knowing that a generic swap may include a Libor spread we write

$$\begin{aligned} PV^{Float\ Leg} &= PV^{Market\ Float\ Leg} + s\sum_{j=1}^{m} N\tau_j P(t_E, t_j) \\ &= PV^{Market\ Float\ Leg} + sA_N^{Float} \end{aligned} \tag{2.26}$$

From (2.25)

$$PV^{\text{Market Float Leg}} = p^{Market} A_N^{Fixed} \tag{2.27}$$

Substituting (2.27) in (2.26) it can be seen that the floating leg of a swap can be expressed as follows

$$\boxed{PV^{\text{ Float Leg}} = p^{Market} A_N^{Fixed} + sA_N^{Float}} \tag{2.28}$$

Equation (2.28) provides a convenient way to price swaps, allowing us to transform the floating leg of any swap into into a fixed leg. The floating leg can in effect be treated as a spread to the fixed rate, with the spread being the market par rate $p^{market}$ with a Libor spread add-on. This simplifies the swap pricing formula, however since we are working with the market par rate $p^{market}$ we must remember to include the additional spread term as a correction to the price or an add-on.

We can rearrange equation (2.28) for a further representation for the par rate quoted in terms of the floating leg present value as follows

$$\boxed{p^{Market} = \frac{PV^{\text{ Float Leg}} - sA_N^{Float}}{A_N^{Fixed}}} \tag{2.29}$$

### 2.9.3 Trade Par Rate Definition, $p^{Trade}$

Following the same steps outlined in (2.9.2) we can derive the corresponding formula for the trade par rate $p^{Trade}$. Again simple rearrangement of (2.16) and solving for the fixed rate $r^{Fixed}$ which makes the present value of the fixed leg equal to that of the floating leg.

For any swap trade we can set $r^{Fixed} = p^{Trade}$ knowing that this par rate includes the floating leg Libor spread term $s$ and makes our swap price to par i.e. $PV^{Swap} = 0$. Consequently we have

$$\begin{gathered} PV^{\text{ Float Leg}} = PV^{\text{ Fixed Leg}} \\ \sum_{j=1}^{m} N_j(l_{j-1} + s)\tau_j P(t_E, t_j) = p^{Trade} \sum_{i=1}^{n} N_i \tau_i P(t_E, t_i) \end{gathered} \tag{2.30}$$

Rearranging (2.30) leads to the 3 expressions for the trade specific par rate $p^{trade}$, namely a par rate weighted by notional and discount factor, by annuity and a par rate expressed in terms of the float leg PV respectively

$$\begin{aligned} p^{Trade} &= \left( \frac{\sum_{j=1}^{m} N_j(l_{j-1} + s)\tau_j P(t_E, t_j)}{\sum_{i=1}^{n} N_i \tau_i P(t_E, t_i)} \right) \\ &= \left( \frac{\sum_{j=1}^{m} N_j(l_{j-1} + s)\tau_j P(t_E, t_j)}{A_{N_i}^{Fixed}} \right) \\ &= \left( \frac{PV^{\text{ Float Leg}}}{A_{N_i}^{Fixed}} \right) \end{aligned} \tag{2.31}$$

The trade par rate $p^{Trade}$ also allows us to transform the floating leg of a swap into a fixed leg without the need for the spread adjustment which was required in equation (2.28). By rearranging equation (2.31) the floating leg can be written as

$$\boxed{PV^{\text{ Float Leg}} = p^{Trade} A_{N_i}^{Fixed}} \tag{2.32}$$

When working with a trade specific par rate $p^{Trade}$ equation (2.32) provides a convenient mechanism to convert a swap floating leg into a fixed leg. The floating leg can in effect be treated as a spread to the fixed rate, where the spread is $p^{Trade}$, which simplifies the swap pricing formula.

### 2.9.4 Choice of Par Rate $p^{Trade}$ or $p^{Market}$

Should the reader for pricing and risk purposes choose to employ the market standard par rate $p^{Market}$ she must remember that the Libor spread term and other bespoke features of the swap are ignored in the quote and not included in the par rate. As such a trade specific add-on term or correction term for will be required for the Libor spread and further adjustments may be required to cater for bespoke features e.g. if the trade notional is not constant say. A trade specific par rate $p^{Trade}$ will account for all of the trade dynamics including variable notionals and Libor spreads. Trade specific par rates are not quoted in the market however, but could be calculated using (2.31) if desired.

We consider it more practical and useful to work with the market par rate and unless specified otherwise the reader should assume that all references to par rates are to the market standardized par rate $p^{Market}$. In later chapters this will enable us to convert a floating leg into a fixed leg simplifying the swap pricing process.

# 3 Interest Rate Swap Pricing Examples

## 3.1 USD Receiver Swap PV

**Example 1:**
*Consider a 5 year USD receiver swap with a constant notional of USD 1,000,000 receiving*

*fixed coupons with a rate of 1% and paying floating coupons at Libor flat i.e. Libor with no spread. Calculate the PV of such a swap.*

*Assume both fixed & floating coupons are paid annually having unit accrual periods $\tau_i$ & $\tau_j$*[14].

| $t_i$ | $\tau_i$ | $r^{Fixed}$ | $P(t_E, t_i)$ |
|---|---|---|---|
| 1 | 1.0 | 1.00 % | $P(t_0, t_1) = 0.990000$ |
| 2 | 1.0 | 1.00 % | $P(t_0, t_2) = 0.980400$ |
| 3 | 1.0 | 1.00 % | $P(t_0, t_3) = 0.970800$ |
| 4 | 1.0 | 1.00 % | $P(t_0, t_4) = 0.961200$ |
| 5 | 1.0 | 1.00 % | $P(t_0, t_5) = 0.951600$ |

Table 2: Fixed Leg Market Data

| $t_j$ | $\tau_j$ | $l_{j-1}$ % | $s$ | $P(t_E, t_i)$ |
|---|---|---|---|---|
| 1 | 1.0 | $l_0 = 0.2800\%$ | $0.0\ bps$ | $P(t_0, t_1) = 0.990000$ |
| 2 | 1.0 | $l_1 = 0.5140\%$ | $0.0\ bps$ | $P(t_0, t_2) = 0.980400$ |
| 3 | 1.0 | $l_2 = 0.7480\%$ | $0.0\ bps$ | $P(t_0, t_3) = 0.970800$ |
| 4 | 1.0 | $l_3 = 0.9820\%$ | $0.0\ bps$ | $P(t_0, t_4) = 0.961200$ |
| 5 | 1.0 | $l_4 = 1.2160\%$ | $0.0\ bps$ | $P(t_0, t_5) = 0.951600$ |

Table 3: Floating Leg Market Data

Using the receiver swap pricing equation (2.10) and decomposing the calculation into two parts comprising of the fixed and floating legs gives

$$
\begin{aligned}
PV^{Fixed} &= 1\% \sum_{i=1}^{5} 1,000,000\tau_i P(t_E, t_i) = 10,000 \sum_{i=1}^{5} \tau_i P(t_E, t_i) = 10,000[A^{Fixed}] \\
&= 10,000\Big[0.990000 + 0.980400 + 0.970800 + 0.961200 + 0.951600\Big] \\
&= 10,000\Big[4.8540\Big] \\
&= \text{ USD } 48,540.00
\end{aligned}
$$

$$
\begin{aligned}
PV^{Float} &= 1,000,000 \sum_{j=1}^{5} \Big(l_{j-1} + s\Big).\tau_j P(t_E, t_j) \\
&= 1,000,000\Big[0.2800\%(0.990000) + 0.5140\%(0.980400) + 0.7480\%(0.970800) \\
&+ 0.9820\%(0.961200) + 1.2160\%(0.951600)\Big] \\
&= 1,000,000\Big[0.27720\% + 0.50393\% + 0.72616\% + 0.94390\% + 1.15715\%\Big] \\
&= \Big[2,772.00 + 5,039.26 + 7,261.58 + 9,438.98 + 11,571.46\Big] \\
&= \text{ USD } 36,083.28
\end{aligned}
$$

[14]Note a market standard USD swap would have semi-annual fixed coupons and quarterly floating coupons.

$$
\begin{aligned}
PV^{Swap} &= \left(PV^{Fixed} - PV^{Float}\right) \\
&= 48,540.00 - 36,083.28 \\
&= \text{ USD } 12,456.72
\end{aligned}
$$

## 3.2 USD Receiver Swap Par Rate

**Example 2:**
*Consider the same swap in Example 1 above. Calculate the par rate of the swap. Note: Always assume we are working with the market par rate $p^{Market}$ unless expressly advised otherwise.*

We recall that from equation (2.29) the par rate of a swap is defined as

$$
p^{Market} = \frac{PV^{\text{ Float Leg }} - sA_N^{Float}}{A_N^{Fixed}}
$$

From example 1 we know that

$PV^{Float} = \text{ USD } 36,083.28$

We also know that the $sA_{N_j}^{Float}$ term is zero since the Libor spread for this particular swap is zero. Furthermore using the information from the fixed leg calculation from example 1 we can evaluate $A_{N_i}^{Fixed}$ as

$$
\begin{aligned}
A_{N_i}^{Fixed} &= 1,000,000\,(0.990000 + 0.980400 + 0.970800 + 0.961200 + 0.951600) \\
&= 1,000,000(4.8540) \\
&= 4,854,000
\end{aligned}
$$

Therefore we calculate the par rate, $p^{Market}$ as

$$
p^{Market} = \left(\frac{36,083}{4,854,000}\right) = 0.7434\%
$$

## 3.3 EUR Payer Swap PV

**Example 3:**
*Consider a standard 2 year payer EUR swap with a constant notional of EUR 5,000,000 paying annual fixed coupons with a rate of 0.5% and receiving semi-annual floating coupons at Libor flat i.e. Libor with no spread. Calculate the PV of such a swap.*

*Assume the fixed leg annual coupon have accrual periods $\tau_i$ equal to 1.0 and the floating leg semi-annual coupons have accrual periods $\tau_j$ equal to 0.5*

| $t_i$ | $\tau_i$ | $r^{Fixed}$ | $P(t_E, t_i)$ |
|---|---|---|---|
| 1.0 | 1.0 | 0.50 % | $P(t_0, t_1) = 1.038000$ |
| 2.0 | 1.0 | 0.50 % | $P(t_0, t_2) = 1.026000$ |

Table 4: Fixed Leg Market Data

| $t_j$ | $\tau_j$ | $l_{j-1}$ % | $s$ | $P(t_E, t_i)$ |
|---|---|---|---|---|
| 0.5 | 0.5 | $l_0 = 0.0487\%$ | $0.0\ bps$ | $P(t_0, t_1) = 1.0440$ |
| 1.0 | 0.5 | $l_1 = 0.1687\%$ | $0.0\ bps$ | $P(t_0, t_2) = 1.0380$ |
| 1.5 | 0.5 | $l_2 = 0.2887\%$ | $0.0\ bps$ | $P(t_0, t_3) = 1.0320$ |
| 2.0 | 0.5 | $l_3 = 0.4087\%$ | $0.0\ bps$ | $P(t_0, t_4) = 1.0260$ |

Table 5: Floating Leg Market Data

Using the payer swap pricing equation (2.13) and once again decomposing the calculation into two parts comprising of the fixed and floating legs gives

$$\begin{aligned}
PV^{Fixed} &= 0.5\% \sum_{i=1}^{2} 5,000,000\tau_i P(t_E, t_i) = 25,000 \sum_{i=1}^{2} \tau_i P(t_E, t_i) = 25,000[A^{Fixed}] \\
&= 25,000\Big[1.0380 + 1.0260\Big] \\
&= 25,000\Big[2.0640\Big] \\
&= \text{ EUR } 51,600.00
\end{aligned}$$

$$\begin{aligned}
PV^{Float} &= 5,000,000 \sum_{j=1}^{4} \Big(l_{j-1} + s\Big).\tau_j P(t_E, t_j) \\
&= 5,000,000\Big[0.0487\% \times 0.5 \times 1.0440 + 0.1687\% \times 0.5 \times 1.0380 + 0.2887\% \times 0.5 \times 1.0320 + 0.4087 \\
&= 5,000,000\Big[0.0254\% + 0.0876\% + 0.1490\% + 0.2097\%\Big] \\
&= \Big[1,271.07 + 4,377.77 + 7,448.46 + 10,483.16\Big] \\
&= \text{ EUR } 23,580.45
\end{aligned}$$

$$PV^{Swap} = \Big(PV^{Float} - PV^{Fixed}\Big) = 23,580.45 - 51,600.00 = -\text{ EUR } 28,019.55$$

## 3.4 EUR Payer Swap Par Rate

**Example 4:**
*Consider the same swap in Example 3 above. Calculate the par rate of the swap. Note: Here again we refer to the market par rate $p^{Market}$*

Once again we recall that from equation (2.29) the par rate of a swap is defined as

$$p^{Market} = \frac{PV^{\text{ Float Leg }} - sA_N^{Float}}{A_N^{Fixed}}$$

From example 3 we know that

$PV^{Float} = \text{ EUR } 23,580.45$
we know the term $sA_{N_j}^{Float} = 0$ since the libor spread is zero and and we also know from the fixed leg calculation that

$$\begin{aligned} A_{N_i}^{Fixed} &= 5,000,000\Big[1.0380 + 1.0260\Big] \\ &= 5,000,000\Big[2.0640\Big] \\ &= 10,320,000 \end{aligned}$$

Therefore we calculate the par rate, $p^{Market}$ as

$$p^{Market} = \left(\frac{23,580.45}{10,320,000}\right) = 0.2285\%$$

# 4 Interest Rate Swap Risk

In this section we will consider first order delta risks only i.e. the sensitivity of interest rate swaps to small changes in interest rate movements and how to immunize or hedge swaps against interest rate risk. Firstly we will look at duration matching and hedging and then outline PV01 and DV01 risk numbers, whilst considering both analytical and numerical risk.

**Remark 3:** ***PV01 & DV01 Swap Risk***
*PV01 measures the risk of a swap to changes to Libor forecast rates and DV01 measures the risk of a swap to changes in both Libor forecast rates and OIS discount factors. All the PV01 risk and the majority of the DV01 risk comes from the swap floating leg.*

## 4.1 Duration Matching & Hedging

When trading swaps it is appropriate to consider the concept of duration, which is used to measure bond / swap risk, specifically the $DV01$[15], the price sensitivity of a swap to a 1 basis point change in interest rates. The $DV01$ also known as the interest rate delta is a commonly used measure of risk for bonds and swaps. It measures the sensitivity of the instrument's price to a small change in interest rates, measured in basis points or one hundredth of a percentage point. This risk measure is used to compare, offset and hedge both bond and swap positions so that they are immune to small changes in interest rates.

Duration matched trades do not typically have the same notional quantities, but rather the hedge position has it's notional scaled by the duration hedge ratio, which is for the most part a function of the bond coupon or swap fixed rate, to give the equal and opposite $DV01$. Losses (or gains)

[15]Dollar value of one basis point, although oddly enough often not quoted in dollars.

on the original trade or offset by equal and opposite gains (or losses) on the hedge deal and are the original trade is immunized from small movements in interest rates; this is called duration matching or duration hedging.

The DV01 of a swap is derived in the following sub-sections. Firstly we consider Macaulay's Duration i.e. the weighted average time to receive all coupons, secondly and related to Macaulay's Duration we consider Modified Duration i.e. the percentage change in price for a change in yield from which we derive a useful expression for use with DV01 calculations and finally differentiate the swap PV formula using the 'useful expression' to obtain an analytical result for DV01.

### 4.1.1 Macaulay's Duration

The original duration concept was conceived by Frederick Macaulay and appropriately called $Macaulay's\ Duration$ or $D_{Mac}$. It measures the weighted average time until a bond holder would receive all the cash-flows from a bond.

Note the final bond coupon typically includes the redemption or return of the the bond notional $N$ and therefore the final coupon equals $(N + c_n)$, however since we are working with a unit notional i.e. $N = 1$ we have $(1 + c_n)$.

$$D_{Mac} = \sum_{i=1}^{n} t_i \left( \frac{\text{PV of ith Bond Coupon}}{\text{Bond Price}} \right) = \sum_{i=1}^{n} t_i \left( \frac{c_i \tau_i P(t_E, t_i)}{B} \right) = \sum_{i=1}^{n} t_i w_i \tag{4.1}$$

where

$$w_i = \left( \frac{c_i P(t_E, t_i)}{B} \right)$$

For a swap it is market practice to calculate the duration of the fixed and floating legs separately. For the fixed leg we have

$$D_{Mac}^{Fixed} = \sum_{i=1}^{n} t_i \left( \frac{\text{PV ith Fixed Coupon}}{\text{PV of Fixed Leg}} \right) = \frac{\sum_{i=1}^{n} t_i N_i r^{Fixed} \tau_i P(t_E, t_i)}{\sum_{i=1}^{n} N_i r^{Fixed} \tau_i P(t_E, t_i)} \tag{4.2}$$

the $r^{Fixed}$ terms cancel giving

$$D_{Mac}^{Fixed} = \left( \frac{\sum_{i=1}^{n} t_i N_i \tau_i P(t_E, t_i)}{A_{N_i}^{Fixed}} \right) \tag{4.3}$$

For the float leg we consider the floating leg and the floating spread separately. Firstly the floating leg itself is defined as[16]

$$D_{Mac}^{Float} = \sum_{j=1}^{m} t_j \left( \frac{\text{PV jth Float Coupon}}{\text{PV of Float Leg}} \right) = \frac{\sum_{j=1}^{m} t_j N_j l_{j-1} \tau_j P(t_E, t_j)}{\sum_{j=1}^{m} N_j l_{j-1} \tau_j P(t_E, t_j)} \tag{4.4}$$

[16]The floating spread component is not included in the calculation

Secondly for the float leg the Libor spread can be treated separately as a fixed coupon. Hence using a similar approach to (4.2) and (4.3) leads to the following expression for the duration for the floating leg Libor spread[17]

$$D_{Mac}^{Spread} = \left( \frac{\sum_{j=1}^{m} t_j N_j \tau_j P(t_E, t_j)}{A_{N_j}^{Spread}} \right) \tag{4.5}$$

Finally the Macaulay's Duration for a swap must be considered as a whole and not as the sum of the constituent fixed and floating legs. As such we calculate swap Macaulay Duration as follows

$$D_{Mac}^{Swap} = \phi \left( \frac{\sum_{i=1}^{n} t_i \times \left( PV\ ith\ Fixed\ Coupon \right) - \sum_{j=1}^{m} t_j \times \left( PV\ jth\ Float\ Coupon \right)}{PV\ Swap} \right) \tag{4.6}$$

Note that if the denominator PV Swap term is zero then Macaulay Duration $D_{Mac}^{Swap}$ is defined as zero. Substituting for the PV terms leads to the following explicit formula

$$D_{Mac}^{Swap} = \phi \left( \frac{\sum_{i=1}^{n} t_i N_i r^{Fixed} \tau_i P(t_E, t_i) - \sum_{j=1}^{m} t_j N_j \left( l_{j-1} + s \right) \tau_j P(t_E, t_j)}{\sum_{i=1}^{n} N_i r^{Fixed} \tau_i P(t_E, t_i) - \sum_{j=1}^{m} N_j \left( l_{j-1} + s \right) \tau_j P(t_E, t_j)} \right) \tag{4.7}$$

where again we note that $D_{Mac}^{Swap} = 0$ if the denominator equals zero i.e. if PV Swap =0. Should we wish to calculate swap Macaulay Duration directly from the swap fixed and float leg components we contrive an expression from equation (4.6) as follows

$$\begin{aligned} D_{Mac}^{Swap} &= \phi \left( D_{Mac}^{Fixed} - D_{Mac}^{Float} \right) \\ &= \phi \left[ \left( \frac{\sum_{i=1}^{n} t_i \times \left( PV\ ith\ Fixed\ Coupon \right)}{PV\ Swap} \right) - \left( \frac{\sum_{j=1}^{m} t_j \times \left( PV\ jth\ Float\ Coupon \right)}{PV\ Swap} \right) \right] \end{aligned} \tag{4.8}$$

**Remark 4:** ***Sign of Macaulay's Duration*** *Typically we think of Macaulay's duration as a positive number indicating the average time in years of an instrument's cashflows. Should we wish to consider the sign or direction of the duration result a positive result would reflect coupons are being received compared to a negative result indicating coupon payments. This is especially relevant in current times, since floating cashflows are negative on the short end of several yield curves due to negative interest rates.*

### 4.1.2 Modified Duration

Related to Macaulay's Duration we have on the other hand *Modified Duration* $D_{Mod}$, which measures the percent change in a bond's price for a 1% change in its yield to maturity; it is related to Macaulay duration by the below formula.

[17] Similar to the duration calculation for the fixed leg in (4.3) the Libor spread, which can be considered a fixed rate, cancels out to produce an annuity term.

$$D_{Mod}^{Bond} = \frac{D_{Mac}}{(1+y)} \tag{4.9}$$

If we were to price a bond by discounting it's cashflows using a standard yield curve the theoretical price would not typically match the market price of the bond. This is because the liquidity of the bond and credit risk of the bond issuer is not accounted for in the yield curve. Perhaps this is obvious once we realize that the standard yield curve is a swap curve i.e. constructed from swap calibration instruments.

To price a bond using this approach we should construct discount factors using the yield to maturity. The yield to maturity $y$ is the average discount rate to be applied over the life of a bond; It is the constant annualized rate such that the sum of the discounted bond coupons equals the quoted bond price in the market. The corresponding discount factor(s), when using yield to maturity, would be as per (4.10) below, where $n$ is the number of years to maturity.

$$P(t_E, t_i) = \frac{1}{(1+y)^n} = \frac{1}{(1+y)^{(t_i - t_E)}} \tag{4.10}$$

The bond markets quote both the price and yield for each and every bond as shown below.


Euro (by country) 1) Actions 3) Settings Fixed Income Trading
16:25
4) AS 5) BE 6) FI 7) FR 8) GE 9) GR 10) IR 11) IT 12) NE 13) PO 14) SP 15) SL 16) SK
21) GE Act 22) GE 0-1 23) GE 1-3 24) GE 3-7 25) GE 7-30 26) GE Regional 27) BU Bills

| Term | Security | BidPx / AskPx | BidYld / AskYld | Px Chg | Y. Close | Time |
|---|---|---|---|---|---|---|
| 31) 3M | BUBILL 0 02/10/16 | 100.081 / 100.099 | -0.347 / -0.424 | -0.001 | 100.09 | 16:24 |
| 32) 6M | BUBILL 0 05/18/16 | 100.174 / 100.208 | -0.343 / -0.411 | -0.008 | 100.197 | 16:24 |
| 33) 1Y | BUBILL 0 07/27/16 | 100.262 / 100.298 | -0.373 / -0.424 | -0.004 | 100.285 | 16:24 |
| 34) 2Y | BKO 0 09/15/17 | 100.680 / 100.695 | -0.371 / -0.379 | +0.005 | 100.685 | 16:24 |
| 35) 3Y | OBL 1 10/12/18 #167 | 103.900 / 103.905 | -0.336 / -0.338 | +0.020 | 103.882 | 16:24 |
| 36) 4Y | OBL 0 ¼ 10/11/19 #170 | 101.975 / 101.985 | -0.254 / -0.256 | +0.040 | 101.94 | 16:24 |
| 37) 5Y | OBL 0 ¼ 10/16/20 #172 | 101.830 / 101.845 | -0.121 / -0.124 | +0.075 | 101.765 | 16:24 |
| 38) 6Y | DBR 2 ¼ 09/04/21 | 113.310 / 113.325 | -0.043 / -0.046 | +0.100 | 113.217 | 16:24 |
| 39) 7Y | DBR 1 ½ 09/04/22 | 109.685 / 109.705 | 0.071 / 0.068 | +0.145 | 109.548 | 16:24 |
| 40) 8Y | DBR 2 08/15/23 | 113.685 / 113.695 | 0.215 / 0.214 | +0.195 | 113.493 | 16:25 |
| 41) 9Y | DBR 1 08/15/24 | 105.365 / 105.375 | 0.375 / 0.374 | +0.240 | 105.128 | 16:25 |
| 42) 10Y | DBR 1 08/15/25 | 104.455 / 104.465 | 0.529 / 0.528 | +0.275 | 104.183 | 16:25 |
| 43) 15Y | DBR 6 ¼ 01/04/30 | 172.160 / 172.240 | 0.820 / 0.816 | +0.535 | 171.665 | 16:25 |
| 44) 20Y | DBR 4 ¾ 07/04/34 | 160.820 / 160.925 | 1.115 / 1.111 | +0.695 | 160.18 | 16:25 |
| 45) 30Y | DBR 2 ½ 08/15/46 | 127.850 / 127.950 | 1.382 / 1.378 | +0.920 | 126.975 | 16:24 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| RXA | 157.24 | +.32 | OEA | 129.680 | +.060 | DUA | 111.595 | +.015 | ERA | 100.200 | +.000 |
| EUR | 1.0719 | -.0054 | TYA | 126-26 | + 09 | USA | 153-25 | + 13 | DAX | ↑10690.4 | -18.0 |

Figure 4: Bloomberg German Bund Quotes; Used with Permission.

**Remark 5:** ***Swap Rate Analogy to Yield to Maturity***
*The equivalent of yield to maturity for swaps is the swap or par rate $p$, which as outlined in section (2.9) is the constant annualized fixed rate which, when applied to the fixed leg, makes the swap price to par.*

The corresponding discount factor expressed in terms of the par rate $p$, where $n$ is the number of years to maturity is as follows

$$P(t_E, t_i) = \frac{1}{(1 + p^{Market})^n} = \frac{1}{(1 + p^{Market})^{(t_i - t_E)}} \tag{4.11}$$

and the equivalent of bond modified duration for swaps is

$$D_{Mod} = \frac{D_{Mac}}{(1 + p^{Market})} \tag{4.12}$$

**Remark 6: *Convexity Adjustment***
*When using the par rate to imply a discount factor a convexity adjustment is required to improve the accuracy of our calculation. This is because the par rate - discount factor relationship is non-linear. We refer the interested reader to [7] for more details.*

Substituting equation (4.3) into (4.12) leads to an expression for modified duration for the fixed leg of a swap

$$\boxed{D^{Fixed}_{Mod} = \left( \frac{\sum_{i=1}^{n} N_i t_i \tau_i P(t_E, t_i)}{A^{Fixed}_{N_i}(1 + p^{Market})} \right)} \tag{4.13}$$

From which we obtain a useful expression for use with swap DV01 calculations

$$\boxed{A^{Fixed}_{N_i} D^{Fixed}_{Mod} = \left( \frac{\sum_{i=1}^{n} N_i t_i \tau_i P(t_E, t_i)}{(1 + p^{Market})} \right)} \tag{4.14}$$

Likewise substituting equation (4.4) into (4.12) leads to an expression for modified duration for the float leg of a swap[18]

$$\boxed{D^{Float}_{Mod} = \left( \frac{\sum_{j=1}^{m} t_j N_j l_{j-1} \tau_j P(t_E, t_j)}{\sum_{j=1}^{n} N_j l_{j-1} \tau_j P(t_E, t_j)\,(1 + p^{Market})} \right)} \tag{4.15}$$

In a similar manner to (4.13), considering the floating leg's Libor spread as a fixed coupon, we arrive at an expression for the Libor spread as

$$\boxed{D^{Spread}_{Mod} = \left( \frac{\sum_{j=1}^{m} N_j t_j \tau_j P(t_E, t_j)}{A^{Spread}_{N_j}(1 + p^{Market})} \right)} \tag{4.16}$$

[18] Note that the floating leg definition here excludes the Libor spread

Finally substituting (4.7) into (4.12) gives the modified duration for the entire swap as

$$D^{Swap}_{Mod} = \phi \left( \frac{\sum_{i=1}^{n} t_i N_i r^{Fixed} \tau_i P(t_E, t_i) - \sum_{j=1}^{m} t_j N_j \left( l_{j-1} + s \right) \tau_j P(t_E, t_j)}{\underbrace{\left( \sum_{i=1}^{n} N_i r^{Fixed} \tau_i P(t_E, t_i) - \sum_{j=1}^{m} N_j \left( l_{j-1} + s \right) \tau_j P(t_E, t_j) \right)}_{PV^{Swap}} (1 + p^{Market})} \right) \tag{4.17}$$

which in shorthand can be written as

$$\boxed{D^{Swap}_{Mod} = \phi \left( \frac{\sum_{i=1}^{n} t_i N_i r^{Fixed} \tau_i P(t_E, t_i) - \sum_{j=1}^{m} t_j N_j \left( l_{j-1} + s \right) \tau_j P(t_E, t_j)}{PV^{Swap} \left( 1 + p^{Market} \right)} \right)} \tag{4.18}$$

where $D^{Swap}_{Mod} = 0$ if $PV^{Swap} = 0$ and $PV^{Swap}$ is as shown in equation (4.17).

## 4.2 Swap PV01

One useful risk measurement concept is the PV01, which measures the sensitivity of a swap to a one basis point parallel shift in interest rates. The PV01 captures the change in a swaps' present value should par rates increase by 1 basis point. Specifically the PV01 is defined as a swap's fixed leg annuity scaled by 1 basis point one basis point, namely

$$\boxed{\begin{aligned} PV01^{Swap} &= \phi A^{Fixed}_{N_i} \text{x 1 Basis Point} \\ &= \left( \phi A^{Fixed}_{N_i} \right) / 10,000 \end{aligned}} \tag{4.19}$$

Numerically we can observe the PV01 for a swap by bumping the swap yield curve calibration instruments by 1 basis point. The swap yield curve for USD would be the USD 3M Libor curve and for EUR the EUR 3M Libor (London) or 3M Euribor curve (Target).

Interestingly the PV01 can be considered as the DV01 for par swaps. The PV01 and DV01 are identical for par swaps, which can seen by setting the fixed rate $r^{Fixed}$ in the DV01 formula (4.28) to the par rate, making the $PV^{Swap}$ term equal zero.

## 4.3 Swap DV01

The DV01 of a swap however is defined as the sensitivity of a swap's present value or price to a 1 basis point **down-shift** in par rates[19]. It is common to calculate swap DV01 by decomposing the swap into a fixed and floating rate bond. This method provides good intuition into DV01,

[19]The par rate is the swap equivalent of bond yield to maturity

where DV01 for a bond is the sensitivity of a bond to a down-shift of 1 basis point in yield to maturity.

Swap DV01 measures the sensitivity of a swap to changes in both Libor forecast rates and OIS discount factors. The Libor risk is typically much larger than the discount risk, except for swaps that are trading far from par. Most of the DV01 risk comes from the floating leg, since only the floating leg is sensitive to Libor forecast rate changes, which is the dominant factor. Both legs carry discount risk, however as mentioned discount risk is typically much lower than forecast risk.

Numerically we can replicate swap DV01 by shifting the calibration instruments of both the Libor forecast and the OIS discounting yield curves by 1 basis point downwards. Heuristically this makes sense since the par rate by definition is the wighted average Libor forecast rate, with Libor rates weighted by the respective OIS discount factors.

Analytically DV01 can be calculated using a modified duration approach. Modified duration duration is useful for DV01 calculations since it indicates the percentage change in price for a percentage change in yield. Both approaches are outlined in [5] for a bond and referred to as method 1 and 2 respectively. Using the modified duration price the DV01 for a Bond is calculated as follows

$$DV01^{Bond} = -B \times D^{Bond}_{Mod}/10,000 \tag{4.20}$$

where B is the clean Bond price.

Using bond interpretation a fixed rate receiver swap is a long bond and short a bond floater and vice versa for a receiver swap. The floating rate bond has no risk to forward rate changes, since floating coupons track Libor and are naturally immunized against interest rate moves. Alternatively a floating rate bond or note can be considered as and also replicated by a strip of forward starting bond-lets. Mathematically such bond-lets cancel each other out and reduce to the single front coupon; namely the coupon where the Libor rate has fixed already.

Under the bond approach the modified duration must include the final bond redemption payment or exchange of notional and the DV01 of the floating leg collapses to the DV01 of the front payment, where the Libor rate has fixed already and no longer a floating coupon.

Immediately below we quite the DV01 for a swap. The reader should be aware that there are several similar yet competing DV01 definitions, which produce similar results. In this paper we compute the DV01 for a swap directly from the swap PV. In this computation we work with the swap equivalent of yield to maturity namely the swap par rate.

$$\begin{aligned} DV01^{Swap} &= \left(\frac{dPV^{Swap}}{dp^{Market}}\right) \textit{x - 1 Basis Point} \\ &= -\left(\frac{dPV^{Swap}}{dp^{Market}}\right) \text{ x } 0.0001 \end{aligned} \tag{4.21}$$

which is often also quoted as

$$DV01^{Swap} = -\left(\frac{dPV^{Swap}}{dp^{Market}}\right)/10,000 \tag{4.22}$$

We remind the reader that the PV of a swap can be calculated using equation (2.16)

$$PV^{Swap} = \phi\left(\left(r^{Fixed} - p^{Market}\right)A^{Fixed}_{N_i} - sA^{Float}_{N_j}\right)$$

differentiating (2.16) and substitute the result into (4.21) will lead to an analytical DV01 expression.

$$\frac{dPV^{Swap}}{dp^{Market}} = -\phi\left(A^{Fixed}_{N_i} - (r^{Fixed} - p^{Market})\frac{dA^{Fixed}_{N_i}}{dp^{Market}} - s\frac{dA^{Float}_{N_i}}{dp^{Market}}\right) \tag{4.23}$$

requoting the annuity equation (2.2) we consider differentiating the annuity term with respect to the par rate

$$A_{N_i} = \sum_{i=1}^{n} N_i \tau_i P(t_E, t_i)$$

deciding importantly to transform annuity expression using the swap equivalent of bond yield to maturity we discount with the annualized swap par rate using (4.11), namely

$$P(t_E, t_i) = \frac{1}{(1 + p^{Market})^{(t_i - t_E)}}$$

substituting (4.11) into (2.2) gives

$$A_{N_i} = \sum_{i=1}^{n} N_i \tau_i (1 + p^{Market})^{-(t_i - t_E)}$$

and now differentiating with respect to the par rate $p^{Market}$ leads to

$$\begin{aligned}
\frac{dA_{N_i}}{dp^{Market}} &= -\sum_{i=1}^{n} N_i t_i \tau_i (1 + p^{Market})^{-(t_i - t_E) - 1} \\
&= -\sum_{i=1}^{n} N_i t_i \tau_i (1 + p^{Market})^{-(t_i - t_E)} (1 + p^{Market})^{-1} \\
&= -\left(\frac{\sum_{i=1}^{n} N_i t_i \tau_i (1 + p^{Market})^{-(t_i - t_E)}}{(1 + p^{Market})}\right) \\
&= -\left(\frac{\sum_{i=1}^{n} N_i t_i \tau_i P(t_E, t_i)}{(1 + p^{Market})}\right)
\end{aligned} \tag{4.24}$$

observing that the right-hand side of (4.24) is identical to (4.14) gives the following result

$$\frac{dA_{N_i}}{dp^{Market}} = -A_{N_i} D_{Mod} \tag{4.25}$$

applying the annuity derivative from (4.25) to the swap differential in (4.23) yields

$$\frac{dPV^{Swap}}{dp^{Market}} = -\phi\left(A^{Fixed}_{N_i} + (r^{Fixed} - p^{Market})A^{Fixed}_{N_i} D^{Fixed}_{Mod} - sA^{Float}_{N_i} D^{Float}_{Mod}\right) \tag{4.26}$$

recalling the swap PV expression from equation (2.18) we recognize the above reduces to the below, note that $\phi$ is included in the $PV^{Swap}$ term

$$\frac{dPV^{Swap}}{dp^{Market}} = -\phi A_{N_i}^{Fixed} - \phi PV^{\text{Swap No Spread}} D_{Mod}^{Fixed} + \phi PV^{\text{Swap Spread}} D_{Mod}^{Float} \tag{4.27}$$

applying (4.27) to the DV01 expression (4.22) and remembering that the swap DV01 is based on a down-shift of 1 basis point leads to 4.28

$$DV01^{Swap} = \left(\phi A_{N_i}^{Fixed} + \phi PV^{\text{Swap No Spread}} D_{Mod}^{Fixed} - \phi PV^{\text{Swap Spread}} D_{Mod}^{Float}\right)/10,000 \tag{4.28}$$

Note the swap is priced without any floating spread and we consider any spread component separately. To provide further intuition and insight into the DV01 calculation let us consider (4.28) as follows

$$DV01^{Swap} = \underbrace{\left(\phi A_{N_i}^{Fixed}\right)/10,000}_{\text{PV01 or Risk from Swap Curve}} + \underbrace{\left(\phi PV^{\text{Swap No Spread}} D_{Mod}^{Fixed} - \phi PV^{\text{Swap Spread}} D_{Mod}^{Float}\right)/10,000}_{\text{DV01 Add-on or Risk from OIS Curve}} \tag{4.29}$$

which leads to the following expression for DV01

$$\boxed{DV01^{Swap} = PV01 + \phi\left(PV^{\text{Swap No Spread}} D_{Mod}^{Fixed} - PV^{\text{Swap Spread}} D_{Mod}^{Float}\right)/10,000} \tag{4.30}$$

**Remark 7:** ***DV01 Composition***
*The DV01 expression (4.30) is composed of 2 constituents namely the PV01 and a DV01 add-on. The PV01 (or annuity) component captures the effect of a 1 basis point move in the swap par rates. The second component labelled DV01 add-on in (4.29) captures OIS discounting effects for swaps that are not trading at par. Finally the DV01 for a par swap equals PV01, since* $PV^{Swap}$ *in the DV01 add-on term would be zero.*

## 4.4 Hedge Ratios & Duration Matching

DV01 measures interest rate risk and the change in a the present value of a trading position to a 1 basis point down shift in interest rates. We can hedge the interest rate risk of a swap by entering an offsetting swap with an equal and opposite DV01. This is known as duration[20] matching.

Duration or DV01 matched trades are immunized against small changes in interest rate movements, however they are not immunized against big shifts in underlying interest rates. Hedging against large movements in rates would require DV01 and convexity hedging[21].

Essentially an existing swap position is delta hedged i.e. immunized from small changes in interest rate movements, by entering a hedge swap with an offsetting DV01 value. A long (short)

[20] Modified Duration measures the percentage change in price for a percentage change in yield.
[21] Also known as Delta and Gamma hedging respectively

swap position is hedged with a short (long) offsetting position having an equal and opposite DV01 value. The hedge trade notional is weighted by the DV01 hedge ratio as demonstrated below

$$
\begin{aligned}
Qty^{Hedge} \times DV01^{Hedge} &= Qty^{\ Position} \times \ DV01^{Position} \\
Qty^{Hedge} &= Qty^{\ Position} \times \left( \frac{DV01^{Position}}{DV01^{Hedge}} \right) \\
Qty^{Hedge} &= Qty^{Position} \times Hedge\ Ratio
\end{aligned}
\tag{4.31}
$$

The notional of the hedge trade is weighted such that the DV01 hedges the delta risk of our swap position as follows, note the minus sign. As shown in (4.31) the hedge ratio is given by

$$
\boxed{Hedge\ Ratio = \left( \frac{DV01^{Position}}{DV01^{Hedge}} \right)}
\tag{4.32}
$$

and noting the minus sign the hedge quantity $Qty^{Hedge}$ is given by

$$
\boxed{
\begin{aligned}
Qty^{Hedge} &= -Qty^{Position} \times Hedge\ Ratio \\
&= -Qty^{Position} \times \left( \frac{\mathrm{DV01}^{Position}}{\mathrm{DV01}^{Hedge}} \right)
\end{aligned}
}
\tag{4.33}
$$

**Remark 8: *Duration Hedge Direction***
*The sign of the hedge quantity indicates the hedge direction. By this we mean if the hedge trade is a payer (receiver) swap then a positive hedge quantity would indicate that the hedge is also a payer (receiver) swap, but a negative sign would indicate the hedge is a receiver (payer) swap.*

**Example: Duration Matching**
*Consider yourself a swap trader with a portfolio largely hedged but with a residual interest rate risk of DV01 = USD 5,000.*

*If you were to DV01 hedge your portfolio using the following payer swap USD 1,000,000 2Y with DV01 USD 2,000. What size position in the swap hedge would be required?*

$$
\begin{aligned}
Hedge\ Quantity &= -Quantity\ Swap\ Position \times \left( \frac{\mathrm{DV01}^{Position}}{\mathrm{DV01}^{Hedge}} \right) \\
&= -1,000,000 \text{ x } \left( \frac{5,000}{2,000} \right) \\
&= -1,000,000 \text{ x } 2.5 \\
&= \text{- USD } 2,500,000
\end{aligned}
$$

The negative sign indicates that we should enter a the opposite or short position in the payer swap, in this case we need to enter a receiver swap to hedge the risk.

## 4.5 Numerical Risk: PV01 & DV01

The DV01 and PV01 risks can also be calculated numerically by shifting yield curves downwards by 1 basis point

Firstly as outlined in section (6.3) swaps are priced in a multi-curve environment. In particular we note that swaps are priced using a swap yield curve to obtain the Libor forecasting curve and an OIS discounting curve. For USD the swap forecast curve would be the USD 3M Libor curve and for EUR it would be the EUR 6M Libor curve, see section (6.4) for further details.

Numerical risk for a swap is calculated by pricing the swap in question and then bumping all the yield curve calibration instruments by 1 basis point downwards and repricing[22]

If we bump or shift the yield curve downwards by 1 basis point and re-price the swap in question the difference in the swap price or present value $PV$, will closely match the DV01 and PV01 as derived in equation (4.30).

$$DV01^{Swap} = PV01 + \phi\left(PV^{Swap\ No\ Spread}D^{Fixed}_{Mod} - PV^{Swap\ Spread}D^{Float}_{Mod}\right)/10,000$$

If we bump only the swap yield curve we will only capture the PV01 and likewise if we shift the swap curve we will observe the OIS discounting effects or DV01 add-on term for swaps that are not trading at par. Finally if we shift both the OIS and Swap yield curves we capture the full DV01 for a swap as outlined in (4.29)

$$DV01^{Swap} = \underbrace{\left(\phi A^{Fixed}_{N_i}\right)/10,000}_{PV01\ Risk\ from\ Swap\ Curve} + \underbrace{\left(PV^{Swap}D^{Fixed}_{Mod} - \phi s A^{Float}_{N_i}D^{Float}_{Mod}\right)/10,000}_{DV01\ Add\text{-}on\ Risk\ from\ OIS\ Curve}$$

# 5 Interest Rate Swap Pricing & Risk Case Study

*Next we review swap pricing and risk by means of a case study. Consider a 2 year USD receiver swap with a constant notional of USD 10,000,000 receiving fixed coupons with a rate of 1.5% and paying floating coupons at Libor flat i.e. Libor with no spread. Assume both fixed and floating coupons are paid annually[23] and have unit accrual periods i.e.* $\tau_i = \tau_j = 1$.

| $t_i$ | $\tau_i$ | $r^{Fixed}$ | $P(t_E, t_i)$ |
|---|---|---|---|
| 1 | 1.5 | 1.00 % | $P(t_0, t_1) = 0.990000$ |
| 2 | 1.5 | 1.00 % | $P(t_0, t_2) = 0.980400$ |

Table 6: Pricing & Risk Example: Fixed Leg Market Data

[22] There are several ways to bump a curve using forward, backward or central differencing for example. Furthermore we can bump all curve calibration points in a single parallel shift or we can perturb and bump each calibration point individually one by one for improved results.

[23] Note a market standard USD swap would have semi-annual fixed coupons and quarterly floating coupons.

| $t_j$ | $\tau_j$ | $l_{j-1}$ % | $s$ | $P(t_E, t_i)$ |
|---|---|---|---|---|
| 1 | 1.0 | $l_0 = 0.2800\%$ | $0.0\ bps$ | $P(t_0, t_1) = 0.990000$ |
| 2 | 1.0 | $l_1 = 0.5140\%$ | $0.0\ bps$ | $P(t_0, t_2) = 0.980400$ |

Table 7: Pricing & Risk Example: Floating Leg Market Data

## 5.1 Swap Annuity Example

*Consider the above USD 10mm 2 Year Swap and the corresponding market data show in tables (6) and (7). What is value of a) the Fixed Leg Annuity and b) Float Leg Annuity?*

**Annuity, A**

$$A_N = N \sum_{i=1}^{n} \tau_i P(t_E, t_i)$$

| **Fixed Leg** | Cashflow 1 | Cashflow 2 |
|---|---|---|
| notional, N | 10,000,000 | 10,000,000 |
| YearFrac, tau | 1.00 | 1.00 |
| DiscFact, P | 0.990000 | 0.980400 |

| | |
|---|---|
| **A( Fixed )** | 19,704,000 |

| **Float Leg** | Cashflow 1 | Cashflow 2 |
|---|---|---|
| notional, N | 10,000,000 | 10,000,000 |
| YearFrac, tau | 1.00 | 1.00 |
| DiscFact, P | 0.990000 | 0.980400 |

| | |
|---|---|
| **A( Float )** | 19,704,000 |

Figure 5: Annuity Calculation

## 5.2 Swap Par Rate Example

*For the above swap what is it's par rate?*

| **Fixed Leg** | Cashflow 1 | Cashflow 2 |
|---|---|---|
| notional, N | 10,000,000 | 10,000,000 |
| fixed rate, r | 1.50% | 1.50% |
| YearFrac, tau | 1.00 | 1.00 |
| DiscFact, P | 0.990000 | 0.980400 |

A( Fixed ) 19,704,000

| | |
|---|---|
| **Par Rate, p** | 0.39643% |

| **Float Leg** | Cashflow 1 | Cashflow 2 |
|---|---|---|
| notional, N | 10,000,000 | 10,000,000 |
| float rate, l | 0.28% | 0.51% |
| YearFrac, tau | 1.00 | 1.00 |
| DiscFact, P | 0.990000 | 0.980400 |

PV( Float ) 78,113

Figure 6: Swap Par Rate Calculation

## 5.3 Swap PV Example

*What is the Present Value or PV of a) the fixed leg, b) the floating leg and c) the swap?*

$$PV^{\text{Fixed Leg}} = r^{Fixed} \sum_{i=1}^{n} N_i \tau_i P(t_E, t_i)$$

$$PV^{\text{Float Leg}} = \sum_{j=1}^{m} N_j (l_{j-1} + s) \tau_j P(t_E, t_j)$$

| **Fixed Leg** | Cashflow 1 | Cashflow 2 |
|---|---|---|
| notional, N | 10,000,000 | 10,000,000 |
| fixed rate, r | 1.50% | 1.50% |
| YearFrac, tau | 1.00 | 1.00 |
| DiscFact, P | 0.990000 | 0.980400 |

PV( Fixed ) 295,560

| **Float Leg** | Cashflow 1 | Cashflow 2 |
|---|---|---|
| notional, N | 10,000,000 | 10,000,000 |
| float rate, l | 0.28% | 0.51% |
| YearFrac, tau | 1.00 | 1.00 |
| DiscFact, P | 0.990000 | 0.980400 |

PV( Float ) 78,113

$$PV^{Swap} = \phi \left( PV^{\text{Fixed Leg}} - PV^{\text{Float Leg}} \right)$$

| **Swap** | |
|---|---|
| Swap Type | Receiver |
| Phi | 1 |
| PV( Swap ) | 217,447 |

Figure 7: Swap PV Calculation

## 5.4 Macaulay's Duration Example

*What is the Macaulay's Duration for a) the fixed leg and b) the float leg?*

**Macaulay's Duration, D**

$$D_{Mac}^{Fixed} = \left( \frac{\sum_{i=1}^{n} t_i N_i \tau_i P(t_E, t_i)}{A_{N_i}^{Fixed}} \right)$$

$$D_{Mac}^{Float} = \frac{\sum_{j=1}^{m} t_j N_j l_{j-1} \tau_j P(t_E, t_j)}{\sum_{j=1}^{m} N_j l_{j-1} \tau_j P(t_E, t_j)}$$

| Fixed Leg | Cashflow 1 | Cashflow 2 |
|---|---|---|
| time, t | 1.00 | 2.00 |
| notional, N | 10,000,000 | 10,000,000 |
| fixed rate, r | 1.50% | 1.50% |
| YearFrac, tau | 1.00 | 1.00 |
| DiscFact, P | 0.990000 | 0.980400 |

| | |
|---|---|
| Numerator | 442,620 |
| Denominator | 295,560 |

| | |
|---|---|
| D( Fixed ) | 1.4976 |

| Float Leg | Cashflow 1 | Cashflow 2 |
|---|---|---|
| time, t | 1.00 | 2.00 |
| notional, N | 10,000,000 | 10,000,000 |
| float rate, l | 0.28% | 0.51% |
| YearFrac, tau | 1.00 | 1.00 |
| DiscFact, P | 0.990000 | 0.980400 |

| | |
|---|---|
| Numerator | 128,505 |
| Denominator | 78,113 |

| | |
|---|---|
| D ( Float ) | 1.6451 |

Figure 8: Macaulay's Duration Calculation

## 5.5 Modified Duration Example

*What is the Modified Duration for a) the fixed leg and b) the float leg?*

**Modified Duration, MD**

$$D_{Mod} = \frac{D_{Mac}}{(1 + p^{Market})}$$

**Fixed Leg**

| | |
|---|---|
| D( Fixed ) | 1.4976 |
| Par Rate, p | 0.39643% |
| MD( Fixed ) | 1.4917 |

**Float Leg**

| | |
|---|---|
| D( Float ) | 1.6451 |
| Par Rate, p | 0.39643% |
| MD( Float ) | 1.6386 |

Figure 9: Modified Duration Calculation

## 5.6 Swap PV01 Example

*Calculate the PV01 or present value of a basis point for the swap.*

$$\boxed{\begin{aligned} PV01^{Swap} &= \phi A_{N_i}^{Fixed} \text{x 1 Basis Point} \\ &= \left(\phi A_{N_i}^{Fixed}\right)/10,000 \end{aligned}}$$

**Swap PV01**

| | |
|---|---|
| Swap Type | Receiver |
| Phi | 1 |
| A( Fixed ) | 19,704,000 |
| PV01( Swap ) | 1,970 |

Figure 10: PV01 Calculation

## 5.7 Swap DV01 Example

*Calculate the DV01 for the swap. What does the DV01 tell us about the swap?*

$$\boxed{DV01^{Swap} = PV01 + \phi\left(PV^{Swap\ No\ Spread} D_{Mod}^{Fixed} - PV^{Swap\ Spread} D_{Mod}^{Float}\right)/10,000}$$

**Swap DV01**

| | |
|---|---|
| Swap Type | Receiver |
| Phi | 1 |

**Libor Forecast Rate Risk**

| | |
|---|---|
| PV01( Swap ) | 1,970 |

**OIS Discounting Risk ( Swap )**

| | |
|---|---|
| PV( Swap ) | 217,447 |
| MD( Fixed ) | 1.4917 |
| OIS Risk | 32 |

**OIS Discounting Risk ( Spread )**

| | |
|---|---|
| PV( Spread ) | 0 |
| MD( Float ) | 1.6386 |
| OIS Risk | 0 |

**Risk Breakdown**

| | | |
|---|---|---|
| Libor Risk | 1,970 | Forecast Risk |
| OIS Risk | 32 | Discounting Risk |
| DV01( Swap ) | 2,003 | Total Risk |

Figure 11: DV01 Calculation

**What does the DV01 tell us about the swap?**
The DV01 calculates the profit or loss our swap will make from a down-shift in interest rates. Specifically it calculates the P&L from a 1 basis point down-shift in both OIS rates, used to calculate discount factors, and Libor forecast rates. The DV01 calculation is made up of a PV01 term, which indicates the risk to Libor forecast rates, and a DV01 add-on which indicates the risk to OIS discounting rates.

## 5.8 Hedge Ratio Example

*Suppose we want to hedge the DV01 risk of the 2 year USD 10mm Payer Swap using a 5 year USD 1mm Hedge Receiver Swap with DV01 5,000. What notional size for the hedge Swap would be required for the Hedge Swap*

$$\boxed{\text{Hedge Ratio} = \left(\frac{DV01^{Position}}{DV01^{Hedge}}\right)}$$

$$\boxed{\begin{aligned} Qty^{Hedge} &= -Qty^{Position} \times \text{Hedge Ratio} \\ &= -Qty^{Position} \times \left(\frac{DV01^{Position}}{DV01^{Hedge}}\right) \end{aligned}}$$

| | |
|---|---|
| DV01( Swap ) | 2,003 |
| DV01( Hedge ) | 5,000 |
| Hedge Ratio | 0.4006 |

| | |
|---|---|
| Swap Notional | 10,000,000 |
| Hedge Qty | 4,005,671 |

Figure 12: Hedge Ratio Calculation

# 6 Asset Swaps

An Asset Swap is a swap whereby the fixed leg coupons are structured to replicate the cashflows of a bond or asset and the floating leg coupons replicate the floating leg of a standard swap plus a spread. Asset swaps are a mechanism to allow market participants to borrow (or loan) money at a rate of Libor plus a spread, $s$ to fund a long (or short) position in a asset or bond. Asset swaps are also used for speculation purposes.

**Long & Short Trading Positions**

*When trading swaps a long or short position refers to the swap float leg. In a long (short) swap one receives (pays) the floating leg. Therefore payer swap is a long swap position and a receiver swap is a short swap position.*

*For bond positions the long or short position refers the yield; a long bond position refers to lending cash in exchange for receiving bond coupon or interest payments i.e. earning the yield and vice versa for short bond positions.*

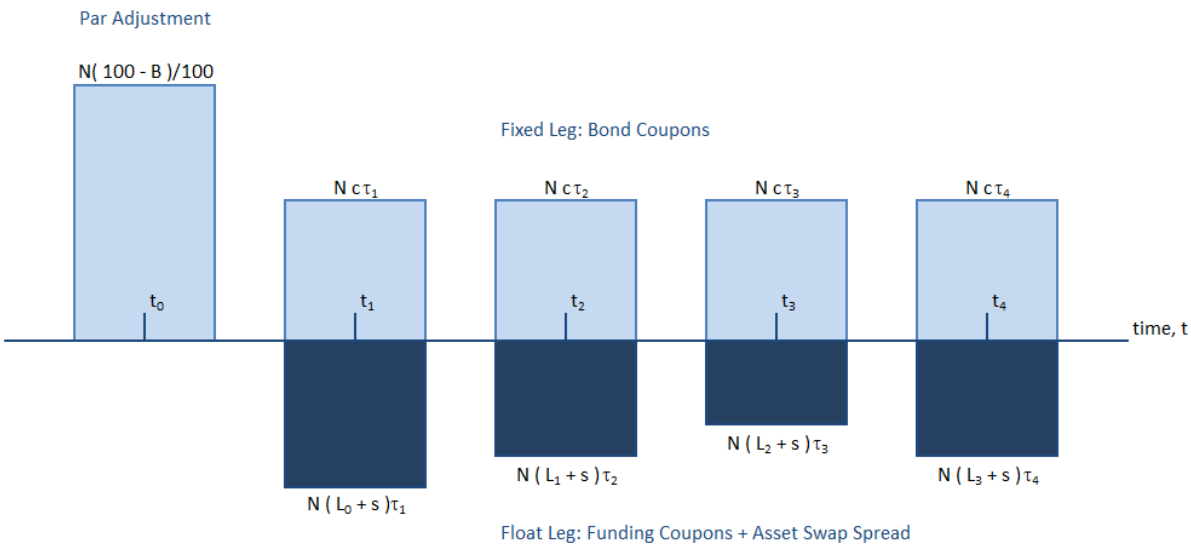



Figure 13: Par-Par Asset Swap Cashflow Illustration

## 6.1 Credit Risk

The underlying bond in the asset swap package is credit risky and can default. The bond purchaser's claim is on the recovery value of the bond, however she must continue to honor the bond coupons on the swap. Of course she may choose to close the swap position at market value. The asset swap seller is therefore taking on this credit risk, which could be hedged with Credit Default Swaps[24]. Consequently asset swap investors receive credit protection should the underlying asset default.

## 6.2 Asset Swap Spreads

Asset swaps are quoted in the market as a spread, which incorporates both funding costs and a premium for credit protection. The spread attempts to make the swap in some way equivalent to a bond. The asset swap spread is also a useful rich-cheap bond analysis tool.

The bonds of issuers of good credit quality, of some governments such as Germany and good quality government agency bonds trade at *negative* spreads to swaps, and especially so for the shorter maturity bonds. The opposite is true for poor or low quality credits such as Greek and Argentina sovereigns and many corporate bonds, which trade with *positive* spreads. For good quality credits a *widening* of the spread, which is negative, refers to a *richening* of the bond (decline in bond yield relative to swaps) and for poor quality credits the opposite is true, namely a widening of the positive spread reflects a *cheapening* of the bond and increase in bond yield.

[24] A Credit Default Swap is an insurance contract whereby credit protection can be taken out in return for a premium or fee. The premium is usually quoted as a rate in basis points.

It is difficult to make comparisons between different bonds based on yield alone. Bonds differ in many ways, maturity, coupon size, liquidity, futures delivery, inclusion in benchmark indices and many other idiosyncracies. Maturity is the most important of these differences so it is important to consider the term-structure of bond yields[25]. To this end we would like to fit a smooth curve to the bond yield term-structure and compare yields, however all the above named differences complicate curve fitting and bond evaluation. The swap curve is free from the problems we encounter in bond curve fitting and more useful for comparison purposes. As a result swap spreads are popular for comparisons between different bonds.

## 6.3 Multiple Swap Curves & Multiple Yield Curve Bootstrapping

As mentioned above asset swaps are quoted as a spread to the swap curve. Recent market developments concerning yield curve construction, also known as bootstrapping, has lead to the introduction of multiple swap curves. This situation naturally led to the obvious question; which curve should asset swap spread benchmarked against? In this section we provide some background as to why multiple swap curves were introduced and in the subsequent section we give an overview of the benchmark swap curves.

After the credit crisis and the collapse of Lehman Brothers that followed in 2008 Libor rates, the rates at which banks lend to one another, were no longer considered risk-free. Interbank lending begain to incorporate credit risk and Libor rates began to price in, different levels of credit risk dependent on the loan term. The market standard 1M, 3M, 6M and 12M Libor rates started to reflect, for the first time, increasing levels of credit risk respectively as longer-term interbank lending became associated with greater credit risk[26].

Furthermore, post credit crisis, tenor basis swaps[27] began to quote with non-negligible spreads and as a result discounting and forecasting Libor rates from single yield curve was no longer viable. Yield curves were subsequently required to be built from tenor-homogenous instruments; that is instruments with the same coupon frequency and similar credit risk for consistency purposes.

The market consequently adopted the overnight OIS[28] rate as the risk-free discounting benchmark. This was because OIS swaps have a coupon frequency of 1 day, the shortest quoted frequency in the market, and market participants therefore adopted this rate to be the closest proxy to risk-free lending. In contrast however Libor forecasting needs to be derived from an appropriate Libor curve, namely the Libor curve calibrated with tenor homogeneous instruments i.e. instruments whose coupon frequency match the Libor rate to be forecasted. Typical market swap curves in this multi-curve regime are quoted with coupon frequencies of 1 day, 1 month, 3 month, 6 month and 12 month and often referred to simply as an OIS, 1ML, 3ML, 6ML and 12L curve respectively, where the L denotes Libor.

[25]That is the term-structure of bond yields with the same issuer

[26]Of course if there is short term distress in the market this might not be the case.

[27]A tenor basis swap is a swap is similar to a regular fixed-float swap, except that we have 2 floating legs with different floating rate frequencies e.g. 3M Libor vs 6M Libor.

[28]Overnight Index Swap

## 6.4 Benchmark Swap Curves

Current EUR yield curves are charted below for reference in figure (14). The reader's attention is drawn to the fact that EUR 1M and 3M Libor forward rates are negative on the short end of the curve. OIS rates are also negative in this region and the corresponding discount factors are greater than 1.0, which might have been considered unusual in previous years.

A EUR swap is quoted as an annual fixed rate versus a semi-annual 6M Euribor rate as standard. Consequently the EUR benchmark swap curve has a floating frequency of 6 months. EUR asset swap spreads should be considered relative to the 6M Euribor curve. USD swaps are quoted semi-annual fixed versus 3M USD Libor, so the USD benchmark is the 3M Libor curve. This indicates that some basic familiarity with swap conventions is required to know which swap curve an asset swap spread is being quoted and benchmarked against.

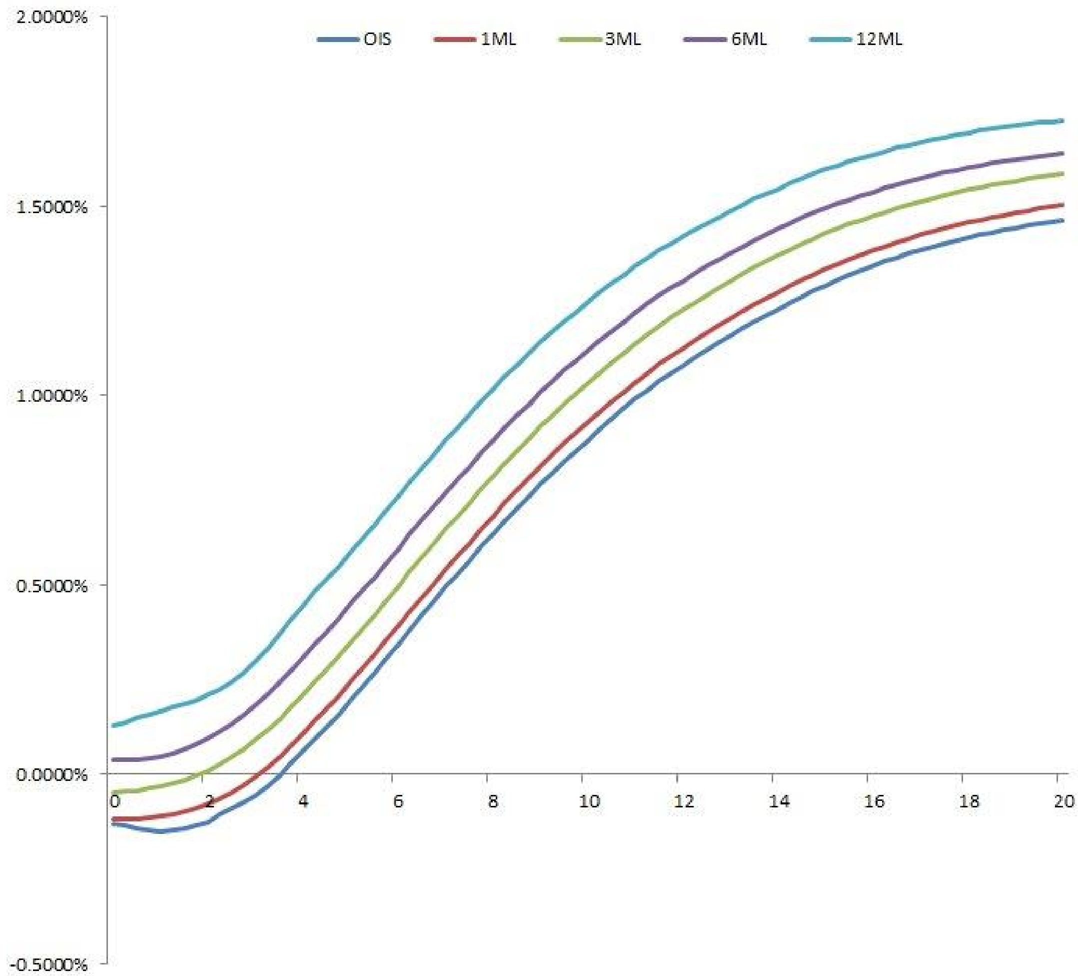



Figure 14: Current EUR Forward Rate Curves

**Swap Conventions**
To help identify the swap benchmark curves we outline swap coupon frequency and daycount

conventions below for major currencies. We remind the reader that the benchmark curves of interest correspond to the floating leg of a swap, so we are particularly interested in the float leg frequency, which determines the benchmark swap frequency.

| Currency | Fixed Leg | Float Leg | Swap Benchmark Curve |
|---|---|---|---|
| $USD$ | Semi-Annual 30/360 | Quarterly ACT/360 | 3M USD LIBOR |
| $EUR$ | Annual 30E/360 | Semi-Annual ACT/360 | 6M EUR LIBOR / EURIBOR |
| $GBP$ | Semi-Annual ACT/365 | Semi-Annual ACT/365 | 6M GBP LIBOR |
| $JPY$ | Semi-Annual ACT/365F | Semi-Annual ACT 360 | 6M JPY LIBOR / TIBOR |

Table 8: Swap Conventions

## 6.5 Curve Risk

In an upward sloping yield curve environment, a high coupon bond normally has a lower modified duration than a low coupon bond. For example consider and compare the 1 5/8% 5/2026 and 6% 6/2026 US Treasuries. The low coupon bond has a modified duration of 9.122 and the high coupon bond 7.697.

If the yield curve steepens we expect the yield to rise further on the low coupon bond relative to the high coupon bond. Hence selling the low coupon bond and buying the high coupon bond in duration-neutral matched amounts will leave us with a steepening exposure in much the same way as if we were to buy an 8-year bond and sell a 10-year bond. This steepening exposure we call curve risk.

Curve risk often manifests when trading asset swaps on a Yield-Yield basis since the duration of the swap is typically not identical to that of the offsetting bond.

## 6.6 Convexity Risk

The term convexity refers to non-linear changes or second order effects on trading positions, which traders call *Gamma*. As Bond prices increase their corresponding yield decreases and vice versa, Bond and Swap prices however are not a linear function of yield. Plotting bond prices versus yield would result in a convex or curved looking function as can be seen below. For small changes in yield we can approximate the Price-Yield function as being linear, but for larger changes in yield this is clearly inadequate.

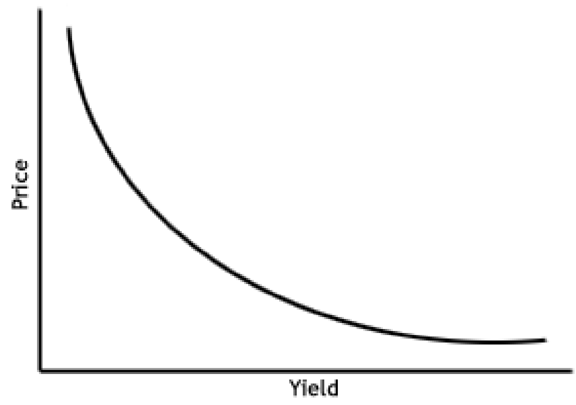



Figure 15: Price Yield Relationship

**Remark 9:** ***Convexity Increases with Coupon Size***
*Bonds and swaps with larger coupons, ceterus paribus*[29]*, are subject to higher convexity risk or Gamma.*

**Remark 10:** ***Convexity Decreases with Bond / Swap Tenor***
*Bonds and swaps with longer tenors, ceterus paribus*[30]*, are subject to lower convexity risk or Gamma.*

If an asset swap position[31] is not re-hedged regularly shifts in the yield curve may trigger a loss on the bond position that is not offset by the profit in the swap position or vice versa. The risk per basis point shift in the yield for the bond does not typically match that of the swap hedge due to a convexity mismatch between the bond and swap. For small changes in yield the mismatch is small and often negligible, but this is not the case for larger shifts. Asset swap positions require regular rehedging to manage this convexity risk.

## 6.7 Par Adjustments, Funding & Collateral

There are several asset swapping methods, which centre upon making a *par adjustment*. As such they quote the swap spread that makes an asset swap price to par and have zero present value. However since the Bond Price B is not typically trading at 100 a par adjustment payment of $(100 - B)$ is required to compensate the asset swap investor, which can become extremely large as bonds trade away from par.

As a further caveat, when executing asset swaps, one has to question what is the correct funding charge to apply, because the swap and bond are not independent transactions. The correct

[29] or all things being equal
[30] or all things being equal
[31] Long Swap (receive float) and Short Bond (pay fixed) or vice versa.

funding rate depends on the collateral agreement in place with the counterparty. The interest rate implicit on the $(100 - B)$ adjustment loan is repaid via a swap at Libor. This rate may not be the appropriate rate to charge given the collateral provided and the CSA[32]. The funding distortion can be substantial and can vary depending on the underlying yield curve term-structure.

The interest rate implicit on the par adjustment or upfront fee is Libor since it is repaid via a swap for which we currently use OIS discounting with a standard or automatic CSA[33]. This may not be the appropriate funding charge to apply.

The distortion due to inappropriate funding assumption can be substantial when a bond trades far from par, which is when upfront payments can be extremely large. This issue is problematic to resolve, so much so that several other methods of structuring and pricing assets swaps were introduced to circumvent the problem, which we outline in section (7). These alternative asset swapping methodologies differ mainly in the timing of the par adjustment payment and provide flexibility in how potential funding distortions are managed. Adjusting the timing of the par adjustment payment can potentially reduce, transfer and/or mitigate any potential counterparty risk should the counterparty default.

# 7 Asset Swap Pricing Methodologies

There are several ways to structure asset swaps and quote the asset swap spread, namely the below. In this paper we give particular attention to the Yield-Yield and Par-Par methods, since these approaches are popular in the market due to their simplicity.

- Yield-Yield
- Par-Par
- MVA ( Market Value Adjusted )
- Yield Accrete
- Z-Spread
- CDS Spread

## 7.1 Yield-Yield Asset Swap Spread

A long asset swap position or long swap spread position refers to owning a bond against a hedge in swaps. The yield-yield spread is defined as the yield of a bond[34] less the swap rate of a maturity matched swap. The investor makes money as the spread widens[35], since the bond

[32]Credit Support Annex

[33]This means that the currency of the CSA collateral is assumed to be the same as that of the trade in question. So all EUR Swaps would be assumed to have EUR collateral and USD Swaps would be assumed to have USD collateral and likewise JPY Swaps JPY collateral, even when all these swap types are with the same counterparty. Hence the specifics of the counterparty CSA are being ignored.

[34]Note that the bond yield here is derived from the bond's dirty price, which includes accrued interest.

[35]Widening here is referring to a richening of a Government Bond with a negative spread.

yield falls relative to the swap rate. The trade must be duration weighted so that the investor is exposed only to the spread between the swap rate and the bond yield and not to market direction. This is a first order approximation with the investor exposed to convexity risk.

As can be seen below Bond Futures on German Sovereign Bonds (Schatz, Bobl, Bund & Buxl) trade at a discount to the 6m Euribor rate see figures (16) and (17) indicating German Sovereign bonds are of good credit quality and less credit risky compared to interbank EUR lending with 6 monthly coupons.

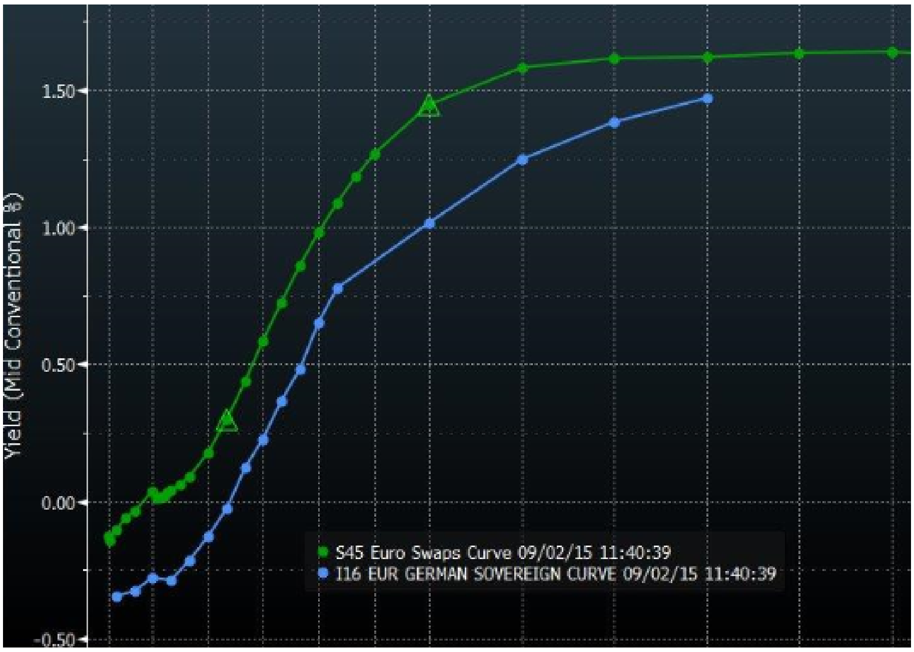



Figure 16: German Sovereign Yields vs EUR 6M Swap Rates

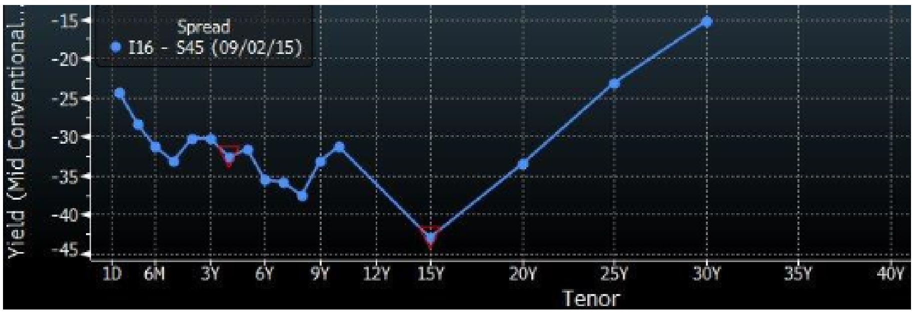



Figure 17: German Sovereign Asset Swap Spreads

Likewise, and not surprisingly, Greek Government bonds, which are in distress, trade at a large premium to 6m Euribor. This indicates poor credit quality and high credit risk relative to interbank lending see figures (18) and (19).

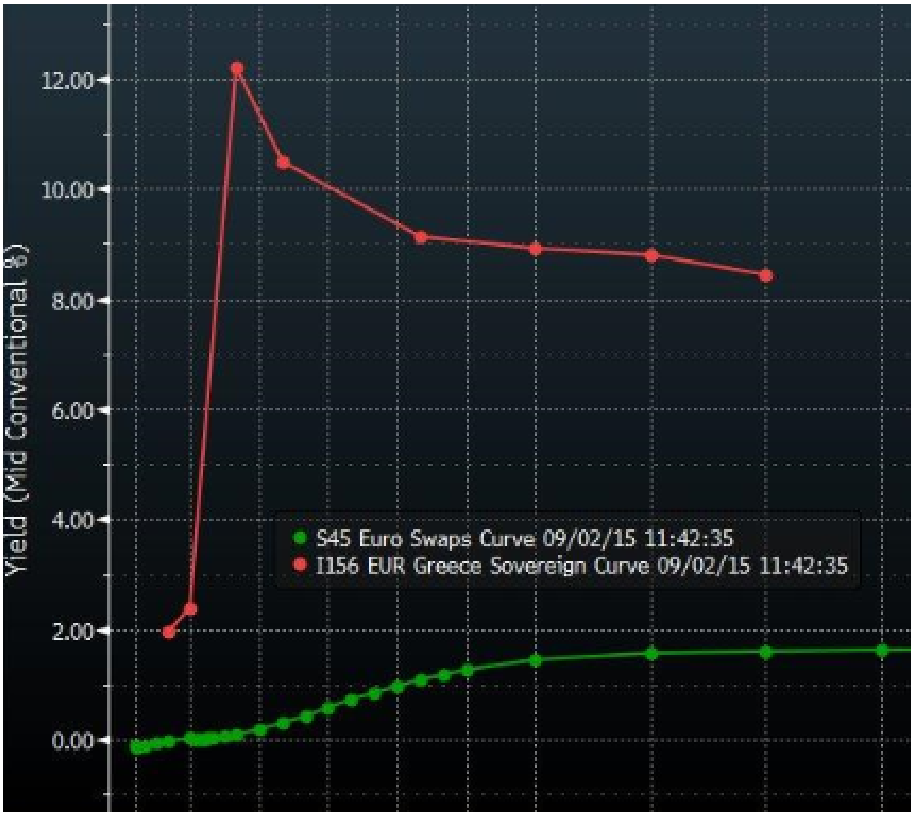



Figure 18: Greek Sovereign Yield vs EUR 6M Swap Rates


Figure 19: Greek Sovereign Asset Swap Spreads

## 7.2 Par-Par Asset Swaps

In the case where the asset is a Bond a trading desk structuring such an Asset Swap for a client might take the following steps to create the asset swap.

1. Borrow 100 (Par) from the Treasury desk to fund the Bond purchase and pay the treasury desk *Libor* plus a funding spread

2. Purchase a Bond at it's clean price, $B$
3. The trader passes the cash difference i.e. ( 100 - B ) on to the client.
4. The trader then receives the bond coupons and passes them on to the client in exchange for Libor plus a spread. The spread is typically higher than the funding spread to allow the trader to make a profit and account for credit risk, see below.

## 7.3 Par-Par Asset Swap Structuring & Cashflows

To better understand how par-par asset swaps are structured and traded consider the cashflow diagrams that follow. Structurers and traders usually consider the following distinct groups of cashflows, notional exchanges and events

- Upfront Payments
- Interim Coupons
- Maturity Payments

The cashflow diagrams & tables for a par-par swap are show below. The net sum of present values from each distinct group of cashflows forms the trade price, with some commission applied of course. All cashflows are considered with respect to the Asset Swap trader, shown in grey in the diagrams that follow, an out-flow would carry a negative value and likewise an in-flow a positive value.

### 7.3.1 Upfront Payments

Any cashflows on or before the start or effective date of the swap are considered here as upfront payments. This includes any exchanges of notional or principal. The upfront payments in a par-par asset swap are shown below in figure (20), which reflects the following events

1. The trader borrows the face value of 100 from her treasury desk
2. She uses these funds to purchase the bond for a value of $B$
3. Any left-over or shortfall of funds from the treasury borrowing and the bond purchase $(100 - B)$ is passed on to the client

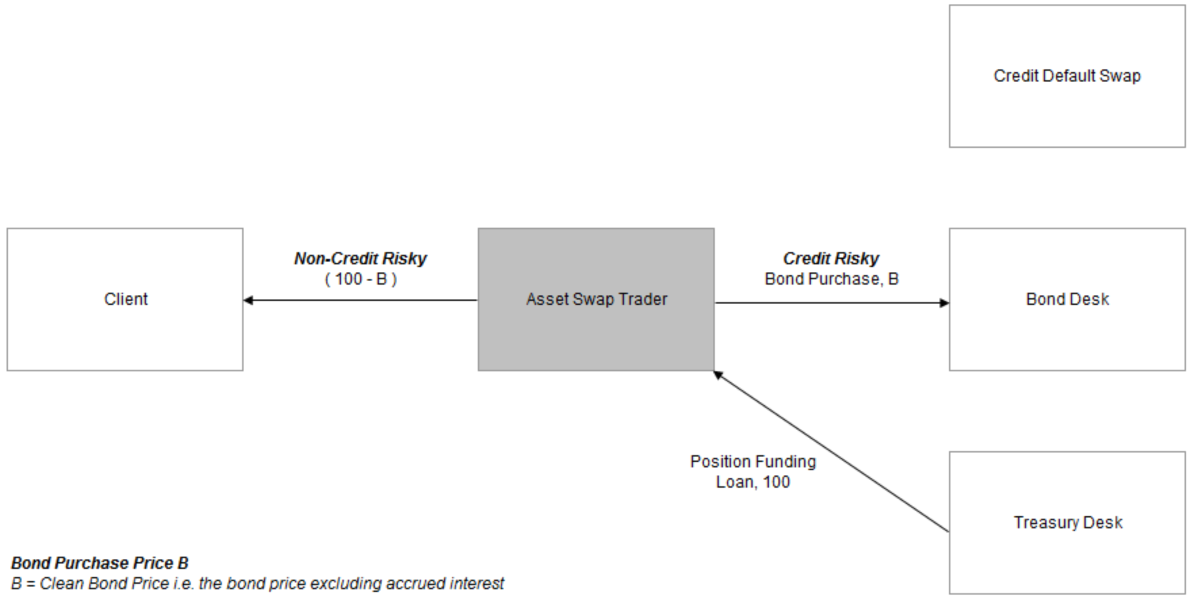



Figure 20: Asset Swap Cashflow Diagram for Cashflows on the Effective or Start Date

As shown in the following cashflow table in figure (21) the trader has a flat upfront cash position and is long the bond. Being long the bond the trader is exposed to the credit risk[36] that that the bond issuer may default. In this scenario she must continue to honour any asset swap payments that are due. The client has a cash position of $(100 - B)$, which would be negative if the underlying bond is trading above par.

**Trader Position**

*Long Bond + Long Credit Risk*

***Trader Cashflows***

| *Counterparty* | *Cashflow* | *Description* |
|---|---|---|
| *Treasury Desk* | *+100* | *Loan Funds to purchase Bond* |
| *Bond Desk* | *- B* | *Bond Purchase ( B = Clean Price )* |
| *Client* | *- ( 100 - B )* | *Difference* |
| *Net Cashflow* | *0* | |

Figure 21: Trader Upfront Cashflows

**Client Position**

*Upfront Cash Payment ( 100 - B )*

[36]The trader may opt to cover the credit risk by purchasing a CDS insurance contract. However many bonds do not have a liquid CDS market and often only quote with 5 year maturities, giving mismatched credit duration. The trader usually has to carry the risk in full or at least some credit basis.

**Client Cashflows**

| Counterparty | Cashflow | Description |
|---|---|---|
| Swap Trader | ( 100 - B ) | Credit / Debit difference arising from Position Funding and the Bond Purchase. |
| Net Cashflow | ( 100 - B ) | |

Figure 22: Client Upfront Cashflows

### 7.3.2 Interim Coupons

Next we consider interim coupons and again any exchanges of notional. The later occurring only the swap notional is amortising or accreting over time i.e. non-constant.

In the cashflow diagram (24) shown below the reader's attention is drawn to the fact that the funding spreads $S_1$ and $S_2$ are not identical. The asset swap trader would usually charge his commission as a spread , rather than as a fee, and so she would adjust the spread from the client to incorporate commission. In the case shown $S_1 > S_2$ with commission added to the client spread $S_1$.

The interim events here reflect the following

1. The trader may purchase credit protection / insurance from her credit desk to protect against losses on the bond, should the issuer default.
2. The trader receives the bond coupons $c$ and passes these on to the client
3. The client pays the asset swap trader loan coupons $L + S_1$ to cover the trader's financing costs to purchase the bond, plus a fee which incorporates commission and credit protection / insurance costs.
4. The trader passes loan proceeds of $L + S_2$ onto her treasury desk, retaining part of the loan proceeds from the client as commission and to cover costs

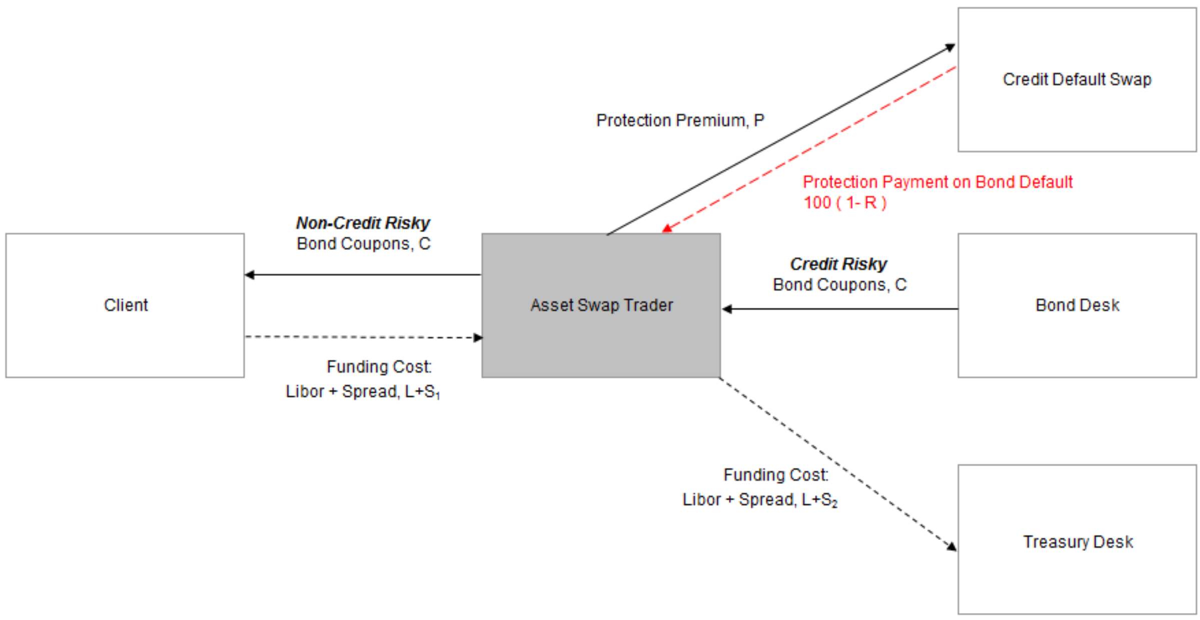



Figure 23: Asset Swap Cashflow Diagram for the Interim Coupons

As shown in the following cashflow table in figure (24) the trader has a cash position of $(S_1 - S_2 - P)$, where $P$ is the CDS spread or premium for credit protection[37]. This amount is the trader's revenue, she should structure the asset swap package so that this amount is greater than zero. This cash return can be seen as the trader's reward for taking the credit risk of the bond, where she is likely to be exposed to the full credit risk of the bond or to be carrying some credit basis risk resulting from an imperfect credit hedge, which is often the case.

**Trader Position with CDS Hedge**
*Interim Coupons ( $S_1$ - $S_2$ - $P$ ) + Long Bond + Long Residual Credit Risk or Basis*

**Trader Position without CDS Hedge**
*Interim Coupons ( $S_1$ - $S_2$ ) + Long Bond + Long Credit Risk*

[37]we have assumed that the trader has put on a credit hedge having purchased credit protection in the CDS market

***Trader Cashflows***

| *Counterparty* | *Cashflow* | *Description* |
|---|---|---|
| *Bond Desk* | $+ C$ | *Bond Coupons* |
| *Treasury Desk* | $- ( L+S_2 )$ | *Funding Cost: Libor + Spread* $S_2$ |
| *CDS Desk* | $- P$ | *Credit Protection Premium* |
| *Client* | $- C$ | *Bond Coupons* |
| *Client* | $+ ( L+S_1 )$ | *Funding Cost: Libor + Spread* |
| *Net Cashflow* | $S_1 - S_2 - P$ | *Difference between spreads*† |
| | $= F$ | *This equals trader commission / fee ( F )* |

**† *Spreads*** $S_1 = S + P + F$

$S_2 = S$

***Trader Cashflows in the Event the Bond Defaults***

| *Counterparty* | *Cashflow* | *Description* |
|---|---|---|
| *CDS Desk* | $+ 100 ( 1 - R )$ | *Credit Protection in Event of Bond Default* |
| | | *Where* |
| | | *R = Recovery Rate % from Bond Default* |

Figure 24: Trader Interim Coupons

## Client Position

*Interim Coupons* $( C - ( L + S_1 ) )$

***Client Cashflows***

| *Counterparty* | *Cashflow* | *Description* |
|---|---|---|
| *Swap Trader* | $+ C$ | *Bond Coupons* |
| *Swap Trader* | $- ( L+S_1 )$ | *Funding Cost: Libor + Spread* $S_1$ |
| *Net Cashflow* | $C - ( L + S_1 )$ | *Bond Coupons less Funding* |

***Libor Spreads $S_1$ and $S_2$***

$S_1 > S_2$

*Spread $S_1$ includes funding spread ( S ), credit risk premium ( P ) and* trader commission / fee ( F )

$S_1 = S + P + F$

*Spread $S_2$ includes funding spread ( S ) only*

$S_2 = S$

Figure 25: Client Interim Coupons

**Remark 11:** ***Hedging Credit Risk***
*A trader may want to or be able to hedge the credit risk. Many bonds do not have a liquid CDS market and even when they do the trader often cannot match the hedge to the maturity of the asset swap, since often only the 5Y CDS contracts are liquid. Often credit risk is hedged using a proxy bond issuer or CDS reference entity, whereby we receive a protection or insurance payout if a highly correlated credit defaults, but we don't receive a credit protection on the exact same bond issuer referenced in our asset swap.*

*Finally we should also note that in the event of a default we receive a payout to cover losses on our bond notional, but we are not compensated for losses on the bond coupons themselves. Suffice to say if and when traders purchase credit protection using CDS contracts this does not typically provide a perfect hedge and we carry some residual credit risk or credit basis risk.*

### 7.3.3 Maturity Payments

Finally we consider and examine the cashflows and principal exchanges at maturity, which is trivial. At maturity if the bond has not defaulted the bond will redeem at par. The trader will receive 100 from the bond issuer, which she will use to replay the bond financing loan with her treasury desk.

Strictly speaking there are bond and libor coupons at maturity, but for brevity and simplicity we can treat these as interim payments without effecting the pricing of the product in any way.

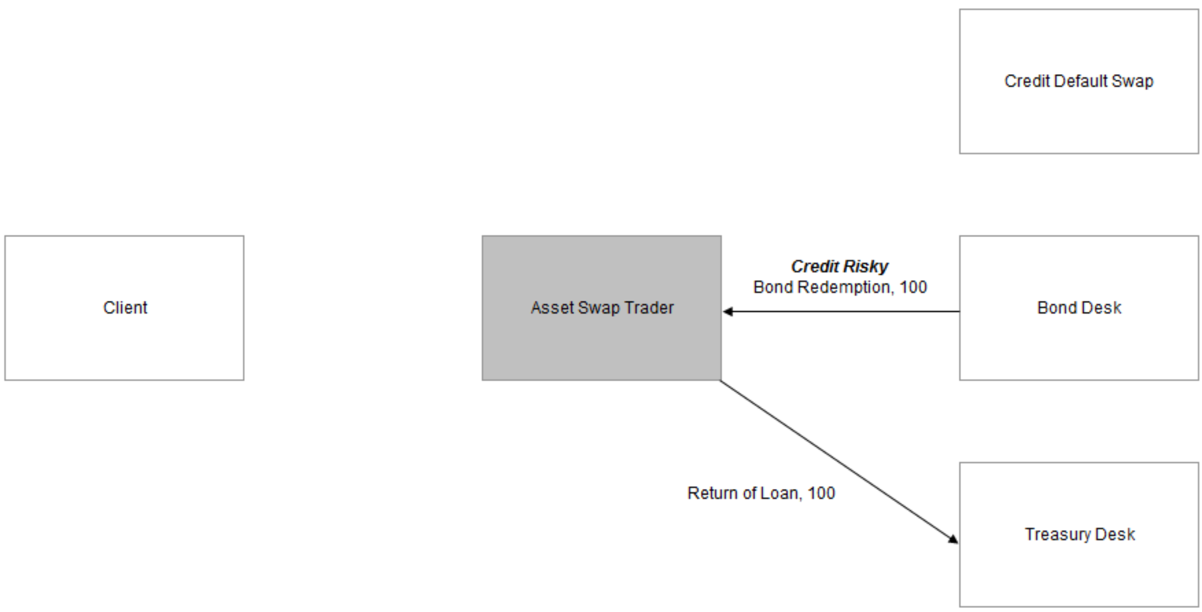



Figure 26: Asset Swap Cashflow Diagram for Cashflows at Maturity

At maturity the client and trader positions are flat. Both parties have no further risk exposures not cash payments that require settlement.

**Trader Position**
*Flat*

| ***Trader Cashflows*** | | |
|---|---|---|
| Counterparty | Cashflow | Description |
| Bond Desk | +100 | Bond Redemption |
| Treasury Desk | -100 | Return of Loan |
| Net Cashflow | 0 | |

Figure 27: Trader Cashflows at Maturity

**Client Position**
*Flat*

| ***Client Cashflows*** | | |
|---|---|---|
| Counterparty | Cashflow | Description |
| Net Cashflow | 0 | |

Figure 28: Client Cashflows at Maturity

### 7.3.4 Overall Structure

In this section we consider the risk reward profile that the trader and the client encounter when trading par-par asset swaps. We would like to draw to the reader's attention that asset swap pricing formulas are based upon the client's perspective, whereby all commissions and risk charges et al are built into the asset swap spread.

**Trader Risk Reward Profile with CDS Hedge**
*Long Residual Credit Risk or Basis*
*Trader Profit: Interim Coupons of* ( $S_1$ - $S_2$ - $P$ )

**Trader Risk Reward Profile without CDS Hedge**
*Long Credit Risk*
*Trader Profit: Interim Coupons of* ( $S_1$ - $S_2$ )

A trader or structurer would consider the sum of upfront, interim and maturity cashflows to determine profitability and evaluate the risk reward profile for the asset swap. The trader has to manage or carry the credit risk and consider if the reward if sufficient to compensate for the risk. The trader may wish to hedge the credit risk with CDS contracts, which often provides only a partial hedge leaving her with some residual credit risk, as mentioned in the commentary in section (7.3.2).

**Client Risk Reward Profile**
*Long Interest Rate Risk*
*Client Cash Position: ( 100 - B ) + C - ( L +* $S_1$ *)*

Likewise the client should evaluate their cash position and determine if the bond coupons less the respective financing charges adequately compensate for taking on the interest rate exposure, which comes from the fact that the client are receiving typically $fixed$ bond cashflows and must pay $floating$ financing charges to the asset swap trader. If interest rates were to rise they would incur the extra finance charges and if interest rates fall receive the benefit.

To determine the fair value for the asset swap on trade date the market quotes the spread that makes the asset swap price to zero i.e. $PV = 0$. In section (8.2) we review how to price asset swaps and calculate the spread using equations (8.4) and (8.10) respectively.

## 7.4 Summary of Asset Swap Pricing Methodologies

In this section we summarize the yield-yield, par-par and alternative asset swap pricing methodologies. The later differ primarily in their treatment of the par adjustment and how best to manage the corresponding funding, collateral issues and counterparty risk, which we outlined in section (6.7). We remind the reader that the dominant factor in pricing asset swaps is the par adjustment factor, which can be significantly large for bonds that are trading far away from par.

A summary of other asset swap pricing methodologies follows below, we remind the reader that asset swaps are priced in terms of the asset swap spread, which is a convenient way to evaluate performance and an excellent rich-cheap indicator.

### 7.4.1 Yield-Yield Asset Swap Spread

**Trade:**
*Bonds against a swap to the same maturity, duration weighted*

**Spread:**
*Difference between bond yield and swap par rate*

**Curve Risk:**
*Spreads widen as curve steepens for bonds trading at a premium to par*

**Directionality:**
*Trade is duration neutral, but not convexity hedged. Rehedging is required for medium to large yield curve moves.*

**Use:**
*This is the most popular method for asset swap spreading, due to it's simplicity. This method works well when the yield curve is relatively flat, which is the current situation, but performs poorly when comparing bonds with with very different coupons and the yield curve is steep, when duration and convexity profile differences would be more pronounced.*

### 7.4.2 Par-Par Asset Swap Spread

**Trade:**
*Bond bought for par plus a payer swap with the fixed coupons matching the bond coupons. The trade notional is set to par and the floating leg is traded at Libor + Par-Par Spread. The par adjustment (100-B) is paid to the asset swap buyer as an upfront payment.*

**Spread:**
*The floating leg spread that makes the Swap Present Value, PV = (100 - B)*

**Curve Risk:**
*The spread decreases as swaps curve steepens*

**Directionality:**
*Trade is NOT duration neutral. Trade can lose money in a sell off.*

**Use:**
*As a popular method the par-par spread is used for rich-cheap bond analysis, relative value and bond comparison. However the spread is highly influenced by the bond price, B in the par adjustment factor, which is undesirable and introduces idiosyncracies.*

### 7.4.3 Market Value Adjusted (MVA)

**Trade:**
*Bond bought for par plus a payer swap with the fixed coupons matching the bond coupons. The trade notional is set to par and the floating leg is traded at Libor + Par-Par Spread. The par adjustment (100-B) is paid to the asset swap buyer at maturity.*

**Spread:**
*The floating leg spread that makes the Swap Present Value, PV = (100 - B)*

$$\text{MVA Spread} = \left(\frac{\text{Par-Par Spread}}{B}\right) \times 100 \tag{7.1}$$

**Curve Risk:**
*The spread decreases as swaps curve steepens*

**Directionality:**
*Trade is duration neutral. MVA spread rises as yields rise*

**Use:**
*Like the par-par method the MVA spread is used for rich-cheap bond analysis, relative value and bond comparison. The MVA spread though is not as highly influenced by the bond price B as the par-par method making it less biased and a more objective tool for relative value analysis. This is not a popular method however.*

#### 7.4.4 Yield Accrete

**Trade:**
*Bond bought for par plus a payer swap with the fixed coupons matching the bond coupons. The trade notional is set to par and the floating leg is traded at Libor + Par-Par Spread. The par adjustment (100-B) is paid to the asset swap buyer in installments. The floating leg notional accretes*[38] *from the bond price, B to Par according to a pre-agreed schedule. The accretion in swap notional is paid to the asset swap holder with the total sum of these payments totalling (100-B).*

**Spread:**
*The floating leg spread that makes the Swap Present Value, PV =0. The spread should lie between the Par-Par spread and the MVA spread.*

**Curve Risk:**
*Limited curve risk in line with the accretion schedule. Quite small compared to Par-Par and MVA methods*

**Directionality:**
*Limited directionality in line with the accretion schedule. Quite small compared to Par-Par and MVA methods*

**Use:**
*This method should be better for relative value calculations, however the spread is hard to calculate. Making the method unpopular. The Z-Spread method is often preferred to this method.*

#### 7.4.5 Z-Spread

**Description:**
*The spread applied to the swap zero curve such that the PV of bond cashflows priced by this shifted swap zero curve gives a bond price equal to the bond price quoted in the market.*

**Use:**
*A simple, straightforward and popular relative value tool.*

#### 7.4.6 CDS Spread

**Trade:**
*A CDS trade is a credit insurance contract. The protection buyer pays a spread usually quarterly and in the event of a credit event*[39]*, such as a default, the protection seller must pay the contingency payment so that the buyer receives no losses from the default*[40].

[38] The swap floating leg notional starts from the bond face value scaled by the bond price and accretes to a pre-arranged schedule until the notional equals the bond face value scaled by par i.e. the actual bond face value.

[39] A credit event is typically a default, but also includes amongst other things restructuring, issuer downgrades, moratorium, repudiation and other events specified in the CDS contract.

[40] The contingency payment made on default protects the credit protection buyer from losses resulting from the default of the underlying bond or reference entity. A payment of $N(1-R)$ is typically made, where $N$ is the notional

**Spread:**
*The annual premium payable to the CDS writer in exchange for credit protection on a par amount of bonds. CDS are quoted instruments, no calculation is required.*

**Use:**
*Trading CDS is similar to trading asset swap spreads. The CDS spread is quoted on the market*[41]*, tracking the perceived credit worthiness of the underlying bond or reference entity. A credit event does not need to be triggered to make a profit. One can simply unwind their position. For example if after 1 year the credit spread widens from 10bps to 20bps say, the buyer of a 5 year CDS contract could unwind their position by selling a 4 year CDS contract for a running profit of 10bps.*

# 8 Asset Swap Pricing Formulae

In what follows we outline how yield-yield and par-par asset swaps are priced respectively.

## 8.1 Yield-Yield Asset Swap Spread

As discussed in section (7.4.1) the yield-yield asset swap comprises of a long bond yield and short swap rate position or vice versa. Such a position is formed by taking a long position in the bond and paying fixed in a duration matched swap trade or vice versa. Duration matching was discussed in detail in sections (4.1) and (4.3).

The Yield-Yield asset swap spread varies with both the pricing or valuation time $t$ and Bond / Swap maturity $T$. It is calculated as follows

$$\text{Yield-Yield Spread}\big(t,T\big) = \text{Bond Yield}\big(t,T\big) - \text{Swap Spread}\big(t,T\big) \tag{8.1}$$

The bond yield in (8.1) is based upon the Bond's dirty price, which includes accrued interest to date. The dirty price of a bond can be calculated as follows. Note that the final coupon $c_n$ usually includes an exchange of bond principal, where the the bond notional is redeemed and the face value of the bond is returned to bond investors.

$$\textit{Dirty Bond Price} = \frac{c_1}{(1+y/m)^{1\times m}} + \frac{c_2}{(1+y/m)^{2\times m}} + \frac{c_3}{(1+y/m)^{3\times m}} + \ldots + \frac{c_n}{(1+y/m)^{n\times m}} \tag{8.2}$$

where $m$ is a daycount fraction representing the number of bond coupon payments per year. For completeness the clean price of a bond is calculated as

$$\textit{Clean Bond Price} = \textit{Dirty Bond Price} - \textit{Accrued Interest} \tag{8.3}$$

of the CDS contract and $R$ is the recovery rate (in percent) of the bond, making $(1-R)$ the percentage loss on the bond.

[41]Note that CDS contracts are liquid for the benchmark 5 year maturity and less liquid for other maturities.

## 8.2 Par-Par Asset Swap Spread

The Par-Par asset swap spread, $s$ is defined as the floating spread such that an asset swap trades at par, which can be deduced from the following expression, where $B$ is defined as the Clean Bond Price[42].

$$PV^{\text{Asset Swap}} = \underbrace{\phi r^{Fixed} \sum_{i=1}^{n} N_i \tau_i P(t_E, t_i)}_{\text{Fixed Leg}} - \underbrace{\phi \sum_{j=1}^{m} N_j (l_{j-1} + s) \tau_j P(t_E, t_j)}_{\text{Float Leg}} + \underbrace{\phi N_1 \left( \frac{100 - B}{100} \right)}_{\text{Par Ajdustment}} \tag{8.4}$$

where $N_1$ is the upfront face value of the bond i.e. the initial bond notional[43]

Setting $PV^{\text{Asset Swap}} = 0$ in (8.4) and rearranging gives

$$\phi \sum_{j=1}^{m} N_j (l_{j-1} + s) \tau_j P(t_E, t_j) = \phi r^{Fixed} \sum_{i=1}^{n} N_i \tau_i P(t_E, t_i) + \phi N_1 \left( \frac{100 - B}{100} \right) \tag{8.5}$$

Splitting out the spread $s$ from the libor rates $l_j$ gives

$$\phi \sum_{j=1}^{m} N_j s \tau_j P(t_E, t_j) = \phi r^{Fixed} \sum_{i=1}^{n} N_i \tau_i P(t_E, t_i) - \phi \sum_{j=1}^{m} N_j l_{j-1} \tau_j P(t_E, t_j) + \phi N_1 \left( \frac{100 - B}{100} \right) \tag{8.6}$$

We divide the expression by $\phi$ and can treat the $s$ term as a fixed rate, which we can take outside of the summation operator. In doing the later we can work with annuity expressions leading to (8.7) and (8.8) below

$$s \sum_{j=1}^{m} N_j \tau_j P(t_E, t_j) = r^{Fixed} \sum_{i=1}^{n} N_i \tau_i P(t_E, t_i) - \underbrace{\sum_{j=1}^{m} N_j l_{j-1} \tau_j P(t_E, t_j)}_{\text{Float PV}} + N_1 \left( \frac{100 - B}{100} \right) \tag{8.7}$$

Noting that the floating leg PV is equivalent to the par swap fixed leg i.e. $p^{Market} A_{N_i}^{Fixed}$ it follows that

$$\begin{aligned} s A_{N_j}^{Float} &= r^{Fixed} A_{N_i}^{Fixed} - p^{Market} A_{N_i}^{Fixed} + N_1 \left( \frac{100 - B}{100} \right) \\ &= \left( r^{Fixed} - p^{Market} \right) A_{N_i}^{Fixed} + N_1 \left( \frac{100 - B}{100} \right) \end{aligned} \tag{8.8}$$

simple rearrangement leads to our solution for the asset swap spread namely

$$\boxed{s = \left( \frac{\left( r^{Fixed} - p^{Market} \right) A_{N_i}^{Fixed} + N_1 \left( \frac{100 - B}{100} \right)}{A_{N_j}^{Float}} \right)} \tag{8.9}$$

[42] This is the bond price adjusted to remove accrued interest.

[43] In most cases asset swaps are non-amortizing, that is to say they have a constant notional $N$. In such cases the variable notional terms $N_1$, $N_i$ and $N_j$ would simplify to $N$ to signify a constant notional

For the vast majority of asset swaps the underlying bond has a constant notional whereby all $N$ terms cancel and equation (8.9) becomes

$$\boxed{s = \left(\frac{\left(r^{Fixed} - p^{Market}\right)A^{Fixed} + \left(\frac{100-B}{100}\right)}{A^{Float}}\right)} \tag{8.10}$$

where $B$ denotes the clean bond price.

# 9 Asset Swap Pricing Examples

## 9.1 Yield-Yield Asset Swap Spread Examples

**Example 9.1.** Consider the 10 year German Bund DBR 0.5% 2026 which is currently trading at a clean price of 99.020 and having yield of 0.640%. Given that the 10 year EUR swap rate is 0.920% what is the yield-yield asset swap spread?

As outlined in section 8.1 and equation (8.1)

$$\text{Yield-Yield Spread}\left(t,T\right) = \text{Bond Yield}\left(t,T\right) - \text{Swap Spread}\left(t,T\right)$$

Therefore the asset swap (yield-yield) spread, $s$ is simply

$$\begin{aligned} s &= 0.640\% - 0.920\% \\ &= -0.280\% \\ &\textit{or -28.00 basis points} \end{aligned}$$

**Example 9.2.** Consider the 10 year Greek Sovereign Bond GGB 3.0% 2026 which is currently trading at a clean price of 65.398 and having yield of 9.188%. Given that the 10 year EUR swap rate is 0.920% what is the yield-yield asset swap spread?

As outlined in section 8.1 and equation (8.1)

$$\text{Yield-Yield Spread}\left(t,T\right) = \text{Bond Yield}\left(t,T\right) - \text{Swap Spread}\left(t,T\right)$$

Therefore the asset swap (yield-yield) spread, $s$ is simply

$$\begin{aligned} s &= 9.188\% - 0.920\% \\ &= 8.268\% \\ &\textit{or 826.80 basis points} \end{aligned}$$

## 9.2 Par-Par Asset Swap Spread Examples

**Example 9.3.** Consider the 10 year German Bund DBR 0.5% 2026 which is currently trading at a clean price of 104.58. Given that the 10 year EUR swap rate is 0.44% what is the par-par asset swap spread for this bond? For this exercise assume the all Annuity Factors have a value of 10.0 for simplicity.

Recall the Asset Swap Spread $s$ can be calculated using equation (8.10), namely

$$s = \left( \frac{\left(r^{Fixed} - p^{Market}\right) A^{Fixed} + \left(\frac{100-B}{100}\right)}{A^{Float}} \right)$$

Firstly we calculate the swap components giving

$$\left(r^{Fixed} - p^{Market}\right) A^{Fixed} = (0.5\% - 0.44\%) \times 10.0 = 0.600\%$$

The Par-Par adjustment is the dominating term and evaluates to

$$\textit{Par-Par Adjustment} = \left(\frac{100 - 104.58}{100}\right) = -4.580\%$$

We proceed to calculate the asset swap spread $s$ as

$$\begin{aligned} s &= \frac{0.600\% - 4.580\%}{10.0} \\ &= -0.3980\% \\ &\textit{or -39.80 basis points} \end{aligned}$$

**Example 9.4.** Consider the 10 year Greek Government Bond GGB 3.0% 2026 which is currently trading at a clean price of 75.280. Given that the 10 year EUR swap rate is 0.440% what is the par-par asset swap spread for this bond?

Using the Asset Swap Spread $s$ can be calculated using equation (8.10), namely

$$s = \left( \frac{\left(r^{Fixed} - p^{Market}\right) A^{Fixed} + \left(\frac{100-B}{100}\right)}{A^{Float}} \right)$$

Again we calculate the swap components giving

$$\left(r^{Fixed} - p^{Market}\right) A^{Fixed} = (3.0\% - 0.44\%) \times 10.0 = 25.60\%$$

The Par-Par adjustment evaluates to

$$\textit{Par-Par Adjustment} = \left(\frac{100 - 75.280}{100}\right) = 24.72\%$$

We proceed to calculate the asset swap spread $s$ as

$$\begin{aligned} s &= \frac{25.60\% + 24.72\%}{10.0} \\ &= 5.0320\% \\ &\textit{or 503.20 basis points} \end{aligned}$$

# 10 Doing it all in your Head

In this section we introduce the concepts and metal mathematics of how to price swaps and asset swaps in one's head in a way that is both quick and accurate, albeit using an approximation. The approximation centres upon approximating the annuity factor, which significantly simplifies calculations, making it possible to price swaps in one's head[44]. This works extremely well for in low interest rate environments, which we are experiencing today, and holds particularly well for short dated swaps.

Notice all of the swap pricing formulas introduced in section (2) rely upon the the annuity factor term defined in (2.1). Furthermore the asset swap spread using the par-par method see (8.2) also relies upon this same factor. For yield-yield asset swap spreads however the calculation process is already sufficiently simple, which is why the method is popular in the marketplace.

To be able to price Swaps and Asset Swaps in one's head the familiarity with swap quote conventions, market benchmark swap / par rates[45] and the annuity approximation is required, all of which we outline below.

A swap comprises of fixed and floating cashflows; the floating coupons however can be converted into a strip of fixed cashflows, thus simplifying the swap pricing process. The Libor floating rate can be converted into a fixed rate and the Libor spread can be considered a fixed rate. In particular the reader is reminded that the Libor rate can be transformed into fixed rate using the par rate introduced in section (2.9).

In what follows we outline swap conventions that allow us to form a base case for pricing swaps. Secondly we review market par rates and demonstrate how to convert floating legs into fixed legs. This allows us to combine the floating and fixed legs of a swap into a single fixed leg and price the swap accordingly. Thirdly we look at pricing swaps in our head using the single fixed leg representation. Finally we look at several examples to demonstrate the concepts presented.

## 10.1 Swap Quote Conventions

Understanding Swap quote conventions allows us to price and consider all swaps as a scalar multiple of a base case, making it simple work to price swaps in one's head. The base case is a 1mm Notional 1 year Swap with a coupon of 1%, we can consider all swaps as scalar multiples of this base case swap, upon which we elaborate further below.

With the above aim in mind, of pricing swaps in multiples of a base case swap, familiarity with interest rate swap quote conventions is required. In particular one should be aware that swaps are typically executed and quoted with notional units usually quoted in millions[46]. Additionally one should also not that floating rate spreads are quoted in basis points or 1/100 th of a percent, that is to say 1 Basis Point = 0.01% = 0.0001. Understanding these conventions helps to prepare to price swaps mentally as will be shown below.

[44] Subject to the reader having some basic familiarity of the market benchmark swap rates outlined in (10.2)
[45] This is a reference to the highly liquid par swaps used in yield curve calibration
[46] Swaps trading with a notional value of a billion or more are also not uncommon

### 10.1.1 Interest Swaps are Quoted with Notionals in Millions

Swaps are traded with notionals typically quoted in millions. This is not to say that the basic denomination of a swap is a 1,000,000, but just an indication and measure of trade size. When pricing swaps mentally we should be looking to price swap coupons on annual[47] per million notional basis and scale accordingly by the number of millions and number of years to maturity.

**Remark 12:** ***Units of Millions per Year***
*For mental calculations we recommend working in units of millions per year. One should be comfortable working in units of a million and consider coupon calculations consider annualized or yearly cashflows. This helps us to calculate the fixed leg of a swap quickly.*

### 10.1.2 Libor Spreads are Quoted in Basis Points, 0.01%

The floating leg of a swap may have a floating Libor spread. All Libor spreads are quoted in basis points i.e. 0.01% or 0.0001. Par swaps DO NOT have a spread. Other bespoke interest rate swaps may or may not have a Libor spread. Either way we need to understand how to price swaps with basis point spreads.

The best practice, mentally at least, for pricing swaps is to incorporate the spread into the fixed leg and to consider the floating leg to have no spread. This allows us, as outlined in section (2.5), to treat the floating leg as a fixed leg with the fixed rate being the par rate.

Par swaps have no Libor spread[48] and are quoted as an interest rate in %. The rate quoted is called the par or swap rate; it is the fixed rate such that if applied to the fixed leg would result in the swap pricing to par i.e. $PV = 0$ as outlined in section (2.9).

### 10.1.3 Bid-Offer Spread

The bid (buy) and offer(sell) or bid-ask prices in the marketplace indicate the market buy and sell prices respectively. The difference indicates the commission or how much the market player will make in profit from a simultaneous buy and sell transaction.
The current typical bid-offer spreads in the market place, see figure (2), is around 1/10 th of a basis point, that is 0.001 or 0.00001. That does not leave the trader with much profit when working in units of millions per year. Such a tight bid-offer spread results in a trader making a handsome profit of USD 10 on a 1 year USD 1,000,000 swap[49]

From the speculators perspective this explains why the market trades in such large sizes, with

[47] We consider annual coupons in line with approximation 1 in section (10.3)
[48] Par rates can be calculated for swaps with a libor spread, but market standard quotes typically exclude the Libor spread for simplicity and liquidity purposes
[49] The reader can confirm this using the annuity approximation from section (10.3).

swap notionals in the order of hundreds of millions being common place and even billions being regularly traded. A further reason is of course supply and demand with there being ample need for corporates and other parties to hedge their interest rate exposures and in large volumes, as metioned in section (1).

## 10.2 Market Benchmark Swap Rates

In the swaps marketplace par swaps are highly liquid instruments. These instruments form the basis for yield curve construction for both Libor[50] and OIS discounting and are fundamental calibration instruments used for curve construction both as outright instruments and when as components when forming a tenor basis.

Par swaps are quoted as in interest in percent with the underlying zero Libor spread $s$ on the floating leg[51]. The rate quoted is called the swap rate or par rate. The par rate is defined in section (2.9) and is the constant rate such that if applied to the fixed leg of the swap would result in the swap pricing to par i.e. $PV^{Swap} = 0$. The par rate allows us to express the floating leg as a fixed leg equivalent.

As shown in section (2.9.2) and in particular equation (2.28) the floating leg can expressed as a fixed leg of equivalent value using the par rate, namely

$$PV^{\text{Float Leg}} = p^{Market} A_N^{Fixed} + s A_N^{Float} \tag{10.1}$$

Note that we wish to work with the market par rate $p^{Market}$ since such par rates are readily available in the market and do not require calculation. Furthermore the market par rate will be a known, highly visible and liquid number, whereas a trade specific par rate will not be observable. The reader is reminded that the par rate for a standardized market swap has zero Libor spread $s$ and as such we must account for the the spread term separately.

Equation (2.28) allows us to convert all floating legs into a fixed leg, which is extremely convenient and helpful for mental swap pricing. This allows us to price a swap using equations (2.18) and (2.21) which we combine and repeat below for convenience.

$$PV^{Swap} = \phi\Big(\Big(r^{Fixed} - p^{Market}\Big) A_N^{Fixed} - s A_N^{Float}\Big) \tag{10.2}$$

With knowledge of the market par rates $p^{Market}$ the floating leg of of any swap trade can be converted into a fixed leg equivalent using equation (10.1) and likewise using equation (10.2) we can price any swap without the need to evaluate the floating leg directly. Consequently the reader is not required to have knowledge of Libor fixings or projected rates $L_j$ associated with the floating leg. This greatly simplifies the swap pricing process.

[50]Discounting with a single Libor curve is largely obsolete in today's markets

[51]Par rates can indeed be calculated for swaps with a non-zero Libor spread on the floating leg, but standard quotes quoted in the market typically exclude the spread for simplicity and uniformity

We recommend the reader when wanting to price swaps familiarize themselves with such quotes for the currencies and benchmark maturities of interest and commit these to memory. Specifically we recommend the reader be familiar with the 1Y, 2Y, 5Y and 10Y maturities for USD and other currencies of interest. The current interest rate swap market par rate quotes for USD, EUR, GBP and JPY benchmark swap maturities are shown in appendix A.

**Example 1: Floating Leg Present Value**
*Consider a 1 year par swap with a constant notional of USD 1,000,000, fixed rate $r^{Fixed}$ of 1% and a floating Libor spread of 10 basis points. Using swap par rate data from figure (37) in Appendix A and given that $A_N^{Fixed} = A_N^{Float} = 1{,}000{,}000$ i.e. $= NT$, what is the present value of the floating leg?*

Firstly from figure (37) we observe $P^{Market}(Bid) = 0.750\%$ and $P^{Market}(Offer) = 0.754\%$, which are the best market buy and sell par rates respectively.

Secondly we recall equation (10.1)

$$PV^{\text{Float Leg}} = \left(p^{Market} A_N^{Fixed} + sA_N^{Float}\right)$$

Thirdly we apply this formula using the bid par rate, which gives

$$\begin{aligned} PV^{\text{Float Leg}} &= \left(0.750\% \times 1,000,000 + 0.10\% \times 1,000,000\right) \\ &= 1,000,000 \times 0.850\% \\ &= \text{USD } 8,500 \end{aligned}$$

and likewise when using the offer par rate

$$\begin{aligned} PV^{\text{Float Leg}} &= \left(0.754\% \times 1,000,000 + 0.10\% \times 1,000,000\right) \\ &= 1,000,000 \times 0.854\% \\ &= \text{USD } 8,540 \end{aligned}$$

Therefore we can sell the swap ( i.e. pay float receive fixed ) at the bid of 0.750 for a PV of USD 8,500 and we can buy the swap ( i.e. pay fixed receive float ) at the offer of 0.754 for a PV of USD 8,540.

**Example 2: Swap Present Value**
*Consider the swap specified in example 1. What is the present value of a payer swap with this specification?*

Firstly from figure (37) we observe $P^{Market}(Bid) = 0.750\%$ and $P^{Market}(Offer) = 0.754\%$, which are the best market buy and sell par rates respectively.

Secondly we recall equation (10.2)

$$PV^{Swap} = \phi\left(\left(r^{Fixed} - p^{Market}\right) A_N^{Fixed} - sA_N^{Float}\right)$$

Thirdly we apply the formula and the insert the bid par rate which gives

$$\begin{aligned} PV^{Swap} &= -1 \times \big((1.0\% - 0.750\%) \times 1,000,000 - (0.10\% \times 1,000,000)\big) \\ &= -1,000,000 \times \big(0.25\% - 0.10\%\big) \\ &= -1,000,000 \times 0.15\% \\ &= \text{ USD } -1,500 \end{aligned}$$

and likewise when using the offer par rate

$$\begin{aligned} PV^{Swap} &= -1 \times \big((1.0\% - 0.754\%) \times 1,000,000 - (0.10\% \times 1,000,000)\big) \\ &= -1,000,000 \times \big(0.21\% - 0.10\%\big) \\ &= -1,000,000 \times 0.11\% \\ &= \text{ USD } -1,100 \end{aligned}$$

**Example 3: Floating Leg Present Value**
*Consider a 5 year swap with constant notional of EUR 10,000,000, fixed rate* $r^{fixed}$ *of 0.5% and a floating Libor spread of 25 basis points. Using swap par rate data from figure (38) in Appendix A and given that* $A_N^{Fixed} = A_N^{Float} = 50{,}000{,}000$ *i.e.* $= NT$*, what is the present value of the floating leg?*

Firstly from figure (38) we observe $P^{Market}(Bid) = 0.256\%$ and $P^{Market}(Offer) = 0.260\%$, which are the best market buy and sell par rates respectively.

Secondly we recall equation (10.1)

$$PV^{\text{ Float Leg}} = \left(p^{Market} A_N^{Fixed} + s A_N^{Float}\right)$$

Thirdly we apply the formula using the bid par rate, which gives

$$\begin{aligned} PV^{\text{ Float Leg}} &= \big(0.256\% \times 50,000,000 + 0.25\% \times 50,000,000\big) \\ &= 50,000,000 \times 0.506\% \\ &= \text{EUR } 253,000 \end{aligned}$$

and likewise for the offer

$$\begin{aligned} PV^{\text{ Float Leg}} &= \big(0.260\% \times 50,000,000 + 0.25\% \times 50,000,000\big) \\ &= 50,000,000 \times 0.510\% \\ &= \text{EUR } 255,000 \end{aligned}$$

Therefore we can sell the swap ( i.e. pay float receive fixed ) at the bid of 0.256 with a float leg PV of EUR 253,000 and we can buy the swap ( i.e. pay fixed receive float ) at the offer of 0.260 with a float leg PV of EUR 255,000.

**Example 4: Swap Present Value**
*Consider the swap specified in example 3. What is the present value of a receiver swap with this specification?*

Firstly from figure (38) we observe $P^{Market}(Bid) = 0.256\%$ and $P^{Market}(Offer) = 0.260\%$, which are the best market buy and sell par rates respectively.

Secondly we recall equation (10.2)

$$PV^{Swap} = \phi\Big(\Big(r^{Fixed} - p^{Market}\Big)A_N^{Fixed} - sA_N^{Float}\Big)$$

Thirdly we apply the formula and the insert the bid par rate which gives

$$\begin{aligned}
PV^{Swap} &= +1 \times \Big((1.0\% - 0.256\%) \times 50,000,000 - (0.250\% \times 50,000,000)\Big) \\
&= \Big((0.744\% \times 50,000,000) - (0.250\% \times 50,000,000)\Big) \\
&= 50,000,000 \times \Big(0.744\% - 0.250\%\Big) \\
&= 50,000,000 \times 0.494\% \\
&= \text{ EUR } 247,000
\end{aligned}$$

and likewise for the offer par rate

$$\begin{aligned}
PV^{Swap} &= +1 \times \Big((1.0\% - 0.260\%) \times 50,000,000 - (0.250\% \times 50,000,000)\Big) \\
&= \Big(0.740\% \times 50,000,000 - 0.250\% \times 50,000,000\Big) \\
&= 50,000,000 \times \Big(0.740\% - 0.250\%\Big) \\
&= 50,000,000 \times 0.490\% \\
&= \text{ EUR } 245,000
\end{aligned}$$

## 10.3 The Annuity Approximation

The annuity approximation we are about to introduce is the amalgamation of the two approximations that follow.

The first approximation is that all coupons and cashflows are annualized paying yearly coupons with a unit accrual basis i.e. accruing interest over one year precisely. So for example as can be seen in table (8) USD, GBP and JPY swaps pay their fixed coupons semi-annually we should consider such swaps to pay fixed annual coupons and accrue interest over exactly one year. This is a mild approximation making that does not have a large price impact and makes pricing swaps mentally a quick and easy task.

The second is that interest rates are zero, which in the current interest rate environment for many currencies is close to reality, in particular for the EUR and JPY markets, but less so for the USD interest rate market. Zero interest rates imply that discount factors are one. Together these approximations lead to the annuity approximation as explained below.

### 10.3.1 Approximation 1: Annualize All Cashflows ($\tau_i = 1$)

*Assume and consider all cashflows to be paid annually. This simplifies the pricing of swap cashflows. In the swaps pricing formulas highlighted in section (2) and in particular equations (2.16) and (2.19) this makes the all year fraction or accrual period terms $\tau$ equal to 1.*

$$\boxed{\tau_i = 1} \tag{10.3}$$

### 10.3.2 Approximation 2: Assume Zero Interest Rates ($P(t_E, t_i) = 1$)

*Assume all interest rates are zero. This is equivalent to assuming that all discount factors are one. This comes from the discount factor kernel $P(t,T) = e^{-\int_t^T r(u)\,du}$, whereby $P(t,T) = e^0 = 1$.*

$$\boxed{P(t_E, t_i) = 1} \tag{10.4}$$

### 10.3.3 Approximation 3: The Annuity Approximation ($A = T$)

*The above approximations 1 and 2 lead to the annuity approximation, substituting $\tau = 1$ and $P(t,T) = 1$ into equations (2.1) and (2.2) quoted in section (2.1) lead to the following approximations for the annuity factors.*

Applying approximations 1 and 2 to equation (2.1) leads to

$$A = \sum_{i=1}^{n} \tau_i P(t_E, t_i) = \sum_{i=1}^{n} 1 = n \tag{10.5}$$

The reader's attention is drawn to the fact that $n$ equals number of coupons, which according to approximation 1 are now annualized giving $n = T$ i.e. n equals the swap tenor $T$ or swap time to maturity in years leading to

$$\boxed{A = T} \tag{10.6}$$

For swaps pricing in one's head we intend to work primarily with equation (10.6), but for completeness and reference applying the same approximations to equation (2.2) gives

$$\boxed{\begin{array}{ll} A_N = N \displaystyle\sum_{i=1}^{n} \tau_i P(t_E, t_i) = NT & \text{when the notional is constant} \\ A_{N_i} = \displaystyle\sum_{i=1}^{n} N_i \tau_i P(t_E, t_i) = \sum_{i=1}^{n} N_i & \text{when the notional is variable} \\ A = \displaystyle\sum_{i=1}^{n} \tau_i P(t_E, t_i) = T & \text{when the notional is unity} \end{array}} \tag{10.7}$$

### 10.3.4 Annuity Approximation Base Case

Next we present some examples of how to apply the annuity approximation. When applying the above annuity approximations it is natural to do so via a base case(s), which we demonstrate below. Notice that example 1 is a base case and that all the other examples evolve naturally as multiples of the base case.

**Example 1: (*Base Case*)**
*Consider a 1 year par receiver swap with a constant notional of EUR 1,000,000 and a fixed rate of 1%. What is the approximate present value of the fixed leg? You may assume that interest rates are at 0% and annual accrual periods of exactly 1.0.*

Using equation (2.10) and annuity approximations from (10.7) leads to

$$\begin{aligned} PV^{\text{Fixed Leg}} &= r^{Fixed} A_{N_i}^{Fixed} \\ &= r^{Fixed} NT \\ &= 1\% \text{ x } 1,000,000 \text{ x } 1.0 \\ &= \text{EUR } 10,000 \end{aligned}$$

**Example 2: (*Base Case x5*)**
*Consider a 5 year par receiver swap with a constant notional of USD 1,000,000 and a fixed rate of 1%. What is the approximate present value of the fixed leg? You may assume that interest rates are at 0% and annual accrual periods of exactly 1.0.*

$$\begin{aligned} PV^{\text{Fixed Leg}} &= r^{Fixed} A_{N_i}^{Fixed} \\ &= r^{Fixed} NT \\ &= 1\% \text{ x } 1,000,000 \text{ x } 5.0 \\ &= \text{USD } 50,000 \end{aligned}$$

**Example 3: (*Base Case x5*)**
*Consider a 2 year par payer swap with a constant notional of EUR 5,000,000 and a fixed rate of 0.5%. What is the approximate present value of the fixed leg? You may assume that interest rates are at 0% and annual accrual periods of exactly 1.0.*

Using equation (2.13) and the annuity approximations from (10.7) leads to

$$\begin{aligned} PV^{\text{Fixed Leg}} &= -r^{Fixed} A_{N_i}^{Fixed} \\ &= -r^{Fixed} NT \\ &= -0.5\% \text{ x } 5,000,000 \text{ x } 5.0 \\ &= \text{- EUR } 50,000 \end{aligned}$$

## 10.4 Pricing Quickly in Your Head

In the above examples notice that the swap in example 1 forms a base case, which can be used for all swaps pricing. In particular note that the base swap has a tenor of 1 year swap with a notional of EUR 1,000,000 and a fixed coupon of %1. All other swaps can be priced as a multiple of the base case units.

To elaborate further on the base case, we consider the base case swap to be quoted with the following features and units

- Notional of 1,000,000
- Coupon of 1%
- Tenor of 1 year
- $PV^{\text{Fixed Leg}} = 10,000$

All other prices can be deduced as multiples of this base case. Consider the following swaps and their corresponding fixed leg pvs ceteras paribas or with all other all things remaining equal.

- 2 Year Swap (x2)
- 5% Coupon (x5)
- 20mm Notional[52] (x20)

Such swaps would have fixed leg PVs being multiples of the base case PV of 10,000, namely 20,000 (x2), 50,000 (x5) and 200,000 (x20) respectively.

## 10.5 Par Rate, p

An expression for the market par rate, $p^{Market}$ expressed in annuity terms was defined in (2.25). Substituting for the annuity term using (10.7) leads to

$$p^{Market} = \frac{\sum_{j=1}^{m} l_{j-1}\tau_j P(t_E, t_j)}{A^{Fixed}} = \frac{\sum_{j=1}^{m} l_{j-1}\tau_j P(t_E, t_j)}{T} \tag{10.8}$$

Recalling the annuity approximations $\tau = 1$ and $P(t_E, t_j) = 1$ from (10.3) and (10.4) respectively gives

$$\boxed{p^{Market} = \frac{\sum_{j=1}^{m} l_{j-1}}{T}} \tag{10.9}$$

Alternatively working with (2.29) and substituting the annuity approximations from (10.7) leads to an alternative approximation for the par rate represented in terms of the floating leg present value as follows

$$\boxed{p^{Market} = \frac{PV^{\text{Float Leg}} - sNT}{NT} = \frac{PV^{\text{Float Leg}}}{NT} - s} \tag{10.10}$$

[52]That is a notional of 20,000,000

## 10.6 Fixed Leg PV

An expression for the present value of the fixed leg of a swap, $PV^{\text{Fixed}}$ was defined in (2.3) substituting the the annuity approximations from (10.7) into this expression gives

$$\boxed{PV^{\text{ Fixed Leg }} = r^{Fixed} NT} \tag{10.11}$$

## 10.7 Floating Leg PV

An expression for the present value of the floating leg of a swap, $PV^{\text{Float Leg}}$ was defined in (2.4) substituting the the annuity approximations from ( 10.7 ) into this expression gives

$$\boxed{PV^{\text{ Float Leg }} = \left(p^{Market} + s\right) NT} \tag{10.12}$$

## 10.8 Swap PV

An expression for the present value of a swap, $PV^{\text{Swap}}$ was defined in (10.2) substituting the the annuity approximations from (10.7) into this expression gives

$$\boxed{PV^{Swap} = \phi\left(r^{Fixed} - p^{Market} - s\right) NT} \tag{10.13}$$

## 10.9 Swap PV01

Recalling (4.19) we have an expression for $PV01$ as

$$\begin{aligned} PV01^{Swap} &= \phi A_{N_i}^{Fixed} \text{x 1 Basis Point} \\ &= \left(\phi A_{N_i}^{Fixed}\right)/10,000 \end{aligned}$$

applying the annuity approximations from (10.7) gives

$$\boxed{PV01^{Swap} = \phi NT/10{,}000} \tag{10.14}$$

## 10.10 Swap Duration

Swap duration is often considered separately for the fixed and floating legs. First we consider the swap fixed leg then the floating leg and finally the swap as a whole.

If we consider the simple compounded discount factor is 1.0 as shown in (10.15), this implies that the par rate $p$ in the modified duration expression is zero making the approximation of macaulay and modified durations equal i.e. we ignore the discounting effect in the modified duration expression. Perhaps this is a valid approximation in a low interest rate environment.

$$\begin{aligned} P(t_E, t_i) &\approx \frac{1}{(1+p)} \\ P(t_E, t_i) = 1.0 &\implies p = 0 \end{aligned} \tag{10.15}$$

Now from (4.13) we know that swap duration is defined as follows for the fixed leg

$$D^{Fixed}_{Mod} = \left( \frac{\sum_{i=1}^{n} N_i t_i \tau_i P(t_E, t_i)}{A^{Fixed}_{N_i}(1+p)} \right)$$

applying the annuity approximations from section (10.7) gives

$$D^{Fixed}_{Mod} = \left( \frac{\sum_{i=1}^{n} N t_i}{NT(1+p)} \right) = \left( \frac{\sum_{i=1}^{n} t_i}{T} \right) \left( \frac{1}{(1+p)} \right) \tag{10.16}$$

leading to

$$D^{Fixed}_{Mod} = \frac{\bar{t}}{(1+p)} \tag{10.17}$$

where $\bar{t}$ denotes the average value of $t$. This represents the average value of $t$ simple discounted by the par rate. If we assume and further approximate that the simple compounded discount factor using the par rate is 1.0, or equivalently that the par rate used for the discount factor calculation is zero, then this expression reduces to

$$\boxed{D^{Fixed}_{Mod} = \bar{t}} \tag{10.18}$$

The spread on the floating leg can be consider as and treated as a fixed rate and hence we reuse equation ( 10.19) and deduce that

$$\boxed{D^{Spread}_{Mod} = \bar{t}} \tag{10.19}$$

Now for the floating leg, with no spread, we must consider each Libor rate independently and not use the par rate representation for the aggregated libor rates, since the later does not consider the term-structure of Libor forward rates. Applying the section (10.7) approximations to (4.15)

$$D^{Float}_{Mod} = \left( \frac{N\tau \sum_{j=1}^{m} t_j \left( l_{j-1} + s \right)}{PV^{Swap} \left(1 + p^{Market}\right)} \right) \tag{10.20}$$

where we also assume that we have a constant accrual period $\tau$[53]. We assume the simple discount factor is 1.0, or equivalently that the par rate used for the discount factor calculation is zero, as above, to further reduce this to

$$\boxed{D^{Float}_{Mod} = \left( \frac{N\tau \sum_{j=1}^{m} t_j \left( l_{j-1} + s \right)}{PV^{Swap}} \right)} \tag{10.21}$$

[53] Where the accrual year fraction $\tau$ is assumed to be 1.0 for annual coupons, 0.5 for semi-annual coupons, 0.25 for quarterly coupons etc.

Once again applying the annuity approximations from section (10.7) to the swap modified duration expression in (4.18) gives

$$\boxed{D^{Swap}_{Mod} = \phi\left(\frac{NT\bar{t}r^{Fixed} - N\tau\sum_{j=1}^{m} t_j\left(l_{j-1} + s\right)}{PV^{Swap}\left(1 + p^{Market}\right)}\right)} \tag{10.22}$$

where again we assume as in (10.20) that we have a constant accrual period.

## 10.11 Swap DV01

From (4.30) we know that

$$DV01^{Swap} = PV01 + \phi\left(PV^{Swap\ No\ Spread} D^{Fixed}_{Mod} - PV^{Swap\ Spread} D^{Float}_{Mod}\right)/10,000$$

which can be written approximately using (10.7) and (10.13) as

$$DV01^{Swap} = \phi\left(NT + (r^{Fixed} - p - s)\bar{t}NT\right)/10,000 \tag{10.23}$$

giving

$$\boxed{DV01^{Swap} = \phi NT\left(1 + (r^{Fixed} - p - s)\bar{t}\right)/10,000} \tag{10.24}$$

## 10.12 Asset Swap Spread, s

The following approximation for the asset swap spread, $s$ is derived from (8.10) with all annuity terms set using the annuity approximations from (10.7) above. Note the par-par adjustment term is usually the dominating factor in this expression.

$$\boxed{s = r^{Fixed} - p^{Market} + Par\text{-}Par\ Adjustment} \tag{10.25}$$

where

$$\boxed{Par\text{-}Par\ Adjustment = \left(\frac{100 - B}{100}\right)/T}$$

# 11 Pricing Examples

In this section we review the material presented so far and provide pricing examples based upon the annuity and other approximations presented in this section (10)

## 11.1 Annuity Examples

### 11.1.1 Unit Notional

**Example 11.1.** Consider a 2 year USD swap with unit notional. What is the value of the annuity factor $A$?

$$A = T = 2$$

**Example 11.2.** Consider a 8 year EUR swap with unit notional. What is the value of the annuity factor $A$?

$$A = T = 8$$

### 11.1.2 Constant Notional

**Example 11.3.** Consider a 1 year EUR swap with a constant notional of EUR 1,000,000. What is the value of the annuity factor $A_N$?

$$\begin{aligned} A_N &= NT \\ &= 1,000,000 \times 1 \\ &= \text{EUR } 1,000,000 \end{aligned}$$

**Example 11.4.** Consider a 5 year EUR swap with a constant notional of EUR 4,000,000. What is the value of the annuity factor $A_N$?

$$\begin{aligned} A_N &= NT \\ &= 4,000,000 \times 5 \\ &= \text{EUR } 20,000,000 \end{aligned}$$

**Example 11.5.** Consider a 5 year USD swap with a constant notional of USD 6,000,000. What is the value of the annuity factor $A_N$?

$$\begin{aligned} A_N &= NT \\ &= 6,000,000 \times 5 \\ &= \text{USD } 30,000,000 \end{aligned}$$

**Example 11.6.** Consider a 10 year GBP swap with a constant notional of GBP 50,000,000. What is the value of the annuity factor $A_N$?

$$\begin{aligned} A_N &= NT \\ &= 50,000,000 \times 10 \\ &= \text{GBP } 500,000,000 \end{aligned}$$

**Example 11.7.** Consider a 10 year JPY swap with a constant notional of JPY 250,000,000. What is the value of the annuity factor $A_N$?

$$\begin{aligned} A_N &= NT \\ &= 250,000,000 \times 10 \\ &= \text{JPY } 2,500,000,000 \end{aligned}$$

### 11.1.3 Variable Notional

**Example 11.8.** Consider a 2 year EUR swap with a notional of EUR 10,000,000 for the first year and EUR 6,000,000 for the second year. What is the value of the annuity factor $A_{N_i}$?

$$\begin{aligned} A_{N_i} &= \sum_{i=1}^{2} N_i \\ &= 10,000,000 + 6,000,000 \\ &= \text{EUR } 16,000,000 \end{aligned}$$

**Example 11.9.** Consider a 5 year USD swap with an initial notional of USD 10,000,000. The notional steps down by USD 2,000,000 at the end of each year of the swap. What is the value of the annuity factor $A_{N_i}$?

$$\begin{aligned} A_{N_i} &= \sum_{i=1}^{5} N_i \\ &= 10,000,000 + 8,000,000 + 6,000,000 + 4,000,000 + 2,000,000 \\ &= \text{USD } 30,000,000 \end{aligned}$$

**Example 11.10.** Consider a 10 year JPY swap with an initial notional of JPY 500,000,000. The notional steps down by JPY 50,000,000 at the end of each year of the swap. What is the value of the annuity factor $A_{N_i}$?

$$\begin{aligned} A_{N_i} &= \sum_{i=1}^{10} N_i \\ &= 500,000,000 + 450,000,000 + 400,000,000 + 350,000,000 + 300,000,000 \\ &+ 250,000,000 + 200,000,000 + 150,000,000 + 100,000,000 + 50,000,000 \\ &= \text{JPY } 2,750,000,000 \end{aligned}$$

## 11.2 Par Rate Examples

**Example 11.11.** The EUR 6m Libor spot rate is -0.10% and inspecting the EUR 6m swap curve we observe the rate is 0.40% in 1y and 1.20% in 2y. What are the 2 year and 3 year EUR 6m Libor swap rates?

$$p^{Market}(2y) = \frac{\left(\sum_{j=1}^{2} l_{j-1}\right)}{T} = \frac{l_0 + l_1}{T} = \left(\frac{-0.10\% + 0.40\%}{2}\right) = 0.15\%$$

$$p^{Market}(3y) = \frac{\left(\sum_{j=1}^{3} l_{j-1}\right)}{T} = \frac{l_0 + l_1 + l_2}{T} = \left(\frac{-0.10\% + 0.40\% + 1.20\%}{3}\right) = 0.50\%$$

**Example 11.12.** The USD 3m Libor spot rate is 1.00% and inspecting the USD 3m swap curve we observe the rate is 2.25% in 1y and 3.75% in 2y. What are the 2 year and 3 year USD 3m Libor swap rates?

$$p^{Market}(2y) = \frac{\left(\sum_{j=1}^{2} l_{j-1}\right)}{T} = \frac{l_0 + l_1}{T} = \left(\frac{1.00\% + 2.25\%}{2}\right) = 1.625\%$$

$$p^{Market}(3y) = \frac{\left(\sum_{j=1}^{3} l_{j-1}\right)}{T} = \frac{l_0 + l_1 + l_2}{T} = \left(\frac{1.00\% + 2.25\% + 3.75\%}{3}\right) = 2.33\%$$

**Example 11.13.** The JPY 6m Libor spot rate is 0.10% and inspecting the JPY 6m swap curve we observe the rates are 0.11%, 0.10%, 0.10%, and 0.12% for the 1y, 2y, 3y and 4y swap points respectively. What is the 4 year JPY 6m Libor swap rate?

$$\begin{aligned} p^{Market}(5y) &= \frac{\left(\sum_{j=1}^{5} l_{j-1}\right)}{T} = \frac{l_0 + l_1 + l_2 + l_3 + l_4}{T} \\ &= \left(\frac{0.10\% + 0.11\% + 0.10\% + 0.10\% + 0.12\%}{5}\right) \\ &= 0.106\% \end{aligned}$$

## 11.3 Fixed Leg Examples

**Example 11.14.** Consider a 5 year GBP swap with a constant notional of GBP 10mm with a fixed rate of 1.30%. What is the present value of the fixed leg?

$$PV^{Fixed}(5y) = r^{Fixed}NT = 1.30\% \times 10mm \times 5 = \text{GBP } 65,000$$

**Example 11.15.** Consider a 10 year JPY swap with a constant notional of JPY 100mm with a fixed rate of 0.42%. What is the present value of the fixed leg?

$$PV^{Fixed}(10y) = r^{Fixed}NT = 0.42\% \times 100mm \times 10 = \text{JPY } 4.2mm$$

**Example 11.16.** Consider a 3 year EUR swap with a constant notional of EUR 50mm with a fixed rate of 0.02%. What is the present value of the fixed leg?

$$PV^{Fixed}(10y) = r^{Fixed}NT = 0.02\% \times 50mm \times 3 = \text{EUR } 30,000$$

**Example 11.17.** Consider a 5 year USD swap with a constant notional of USD 150mm with a fixed rate of 1.60%. What is the present value of the fixed leg?

$$PV^{Fixed}(5y) = r^{Fixed}NT = 1.60\% \times 150mm \times 5 = \text{USD } 12mm$$

## 11.4 Float Leg Examples

### 11.4.1 When the Par Rate is Known

**Example 11.18.** Consider a 2 year JPY swap with a constant notional of JPY 800mm a fixed rate of 0.20% and no Libor spread. The market 2 year par-rate is 0.10% What is the present value of the floating leg?

$$PV^{Float} = p^{2Y}NT = 0.10\% \times 800mm \times 2 = \text{JPY } 1.6mm$$

### 11.4.2 When the Par Rate is Unknown

**Example 11.19.** Consider a 6 year EUR swap with a constant notional of EUR 250mm a fixed rate of 0.50% and no Libor spread. Suppose there is no 6 year par-rate being quoted in the market, but the market is publishing the 5 year par rate as 0.25% and the 7 year par rate as 0.50%. What is the present value of the floating leg?

Firstly we interpolate for the 6 year par rate

$$p^{6Y} = \frac{p^{5Y} + p^{7Y}}{2} = \frac{0.25\% + 0.50\%}{2} = 0.375\%$$

then we evaluate the float leg

$$PV^{Float} = p^{6Y}NT = 0.375\% \times 250mm \times 6 = \text{EUR } 5.625mm$$

### 11.4.3 When there is a Libor Spread

**Example 11.20.** Consider a 5 year GBP swap with a constant notional of GBP 150mm a fixed rate of 1.00% and a Libor spread of 30 basis points on the floating leg. The market 5 year par-rate is 1.3% What is the present value of the floating leg?

$$PV^{Float} = \left(p^{5Y} + s\right)NT = (1.30\% + 0.30\%) \times 150mm \times 5 = \text{GBP } 12mm$$

## 11.5 Interest Rate Swap Examples

### 11.5.1 When the Par Rate is Known

**Example 11.21.** Consider a 10 year USD payer swap with a constant notional of USD 250mm a fixed rate of 3.00% and a Libor spread of 50 basis points on the floating leg. The market 10 year par-rate is 2.1% What is the present value of the swap?

$$\begin{aligned}
PV^{Float} &= \left(p^{5Y} + s\right) NT \\
&= \phi\left(r^{Fixed} - p^{10Y} - s\right) NT \\
&= -1 \times \left(3.00\% - 2.10\% - 0.50\%\right) \times 250mm \times 10 \\
&= -0.40\% \times 250mm \times 10 \\
&= \text{USD } -10mm
\end{aligned}$$

**Example 11.22.** Consider a 2 year EUR receiver swap with a constant notional of EUR 125mm a fixed rate of 0.70% and a Libor spread of 25 basis points on the floating leg. The market 2 year par-rate is -0.05% What is the present value of the swap?

$$\begin{aligned}
PV^{Swap} &= \phi\left(r^{Fixed} - p^{2Y} - s\right) NT \\
&= +1 \times \left(0.70\% + 0.05\% - 0.25\%\right) \times 125mm \times 2 \\
&= 0.50\% \times 125mm \times 2 \\
&= \text{EUR } 1.25mm
\end{aligned}$$

**Example 11.23.** Consider a 5 year GBP payer swap with a constant notional of GBP 200mm a fixed rate of 1.50% and a Libor spread of 10 basis points on the floating leg. The market 5 year par-rate is 1.30% What is the present value of the swap?

$$\begin{aligned}
PV^{Swap} &= \phi\left(r^{Fixed} - p^{5Y} - s\right) NT \\
&= -1 \times \left(1.50\% - 1.30\% - 0.10\%\right) \times 200mm \times 5 \\
&= -0.10\% \times 200mm \times 5 \\
&= \text{GBP } -1mm
\end{aligned}$$

**Example 11.24.** Consider a 10 year JPY receiver swap with a constant notional of JPY 1,000mm a fixed rate of 0.80% and a Libor spread of 20 basis points on the floating leg. The market 5 year par-rate is 0.40% What is the present value of the swap?

$$\begin{aligned}
PV^{Swap} &= \phi\left(r^{Fixed} - p^{10Y} - s\right) NT \\
&= +1 \times \left(0.80\% - 0.40\% - 0.20\%\right) \times 1,000mm \times 10 \\
&= +0.20\% \times 1,000mm \times 10 \\
&= \text{JPY } 20mm
\end{aligned}$$

### 11.5.2 When the Par Rate is Unknown

**Example 11.25.** Consider a 6 year USD payer swap with a constant notional of USD 200mm a fixed rate of 2.00% and a Libor spread of 50 basis points on the floating leg. Suppose there is no 6 year par rate quote available in the market, but the market is quoting 5 year and 7 year par rates as 1.60% and 1.80% respectively. What is the present value of the swap?

Firstly we calculate the interpolated 6 year par rate

$$p^{6Y} = \frac{p^{5Y} + p^{5Y}}{2} = \frac{1.60\% + 1.80\%}{2} = 1.70\%$$

then we evaluate the swap

$$\begin{aligned} PV^{Swap} &= \phi \left( r^{Fixed} - p^{10Y} - s \right) NT \\ &= -1 \times \left( 2.00\% - 1.70\% - 0.50\% \right) \times 250mm \times 6 \\ &= -0.20\% \times 200mm \times 6 \\ &= \text{USD } 2.4mm \end{aligned}$$

**Example 11.26.** Consider a 3 year EUR receiver swap with a constant notional of EUR 100mm a fixed rate of 0.02% and no Libor spread on the floating leg. Suppose there is no 3 year par rate quote available in the market, but the market is quoting 2 year and 4 year par rates as -0.05% and 0.15% respectively. What is the present value of the swap?

Firstly we calculate the interpolated 3 year par rate

$$p^{3Y} = \frac{p^{2Y} + p^{4Y}}{2} = \frac{-0.05\% + 0.15\%}{2} = 0.05\%$$

then we evaluate the swap

$$\begin{aligned} PV^{Swap} &= \phi \left( r^{Fixed} - p^{3Y} - s \right) NT \\ &= +1 \times \left( 0.02\% - 0.05\% \right) \times 100mm \times 3 \\ &= -0.03\% \times 100mm \times 3 \\ &= \text{EUR } -90K \end{aligned}$$

## 11.6 PV01 Examples

**Example 11.27.** Consider a 5 year USD payer swap with a constant notional of USD 150mm a fixed rate of 1.75% and a Libor spread of 10 basis points on the floating leg. The market 5 year par-rate is 1.60% What is the present value of the swap and it's PV01?

$$\begin{aligned} PV^{Swap} &= \phi \left( r^{Fixed} - p^{5Y} - s \right) NT \\ &= -\left( 1.75\% - 1.60\% - 0.10\% \right) \times 150mm \times 5 \\ &= -0.05\% \times 150mm \times 5 \\ &= \text{USD } -375K \end{aligned}$$

$$\begin{aligned} PV01^{Swap} &= \phi NT \times \text{ 1 Basis Point} \\ &= \phi NT \times 0.01\% \\ &= -150mm \times 5.0 \times 0.01\% \\ &= \text{USD } -75,000K \end{aligned}$$

**Example 11.28.** Consider a 2 year JPY receiver swap with a constant notional of JPY 500mm a fixed rate of 0.25% and a Libor spread of 5 basis points on the floating leg. The market 2 year par-rate is 0.10% What is the present value of the swap and it's PV01?

$$
\begin{aligned}
PV^{Swap} &= \phi\left(r^{Fixed} - p^{2Y} - s\right) NT \\
&= +\left(0.25\% - 0.10\% - 0.05\%\right) \times 500mm \times 2 \\
&= 0.10\% \times 500mm \times 2 \\
&= \text{JPY } 1mm
\end{aligned}
$$

$$
\begin{aligned}
PV01^{Swap} &= \phi NT \times \text{ 1 Basis Point} \\
&= \phi NT \times 0.01\% \\
&= 500mm \times 2.0 \times 0.01\% \\
&= \text{JPY } 100K
\end{aligned}
$$

## 11.7 Swap Duration Examples

### 11.7.1 Fixed Leg Duration

**Example 11.29.** Consider a 10 year EUR payer swap with a constant notional of EUR 25mm a fixed rate of 0.75% and a Libor spread of 25 basis points on the floating leg. The market 10 year par-rate is 0.90% What is the macaulay and modified duration of the fixed Leg to 4 decimal places? You may assume coupons are paid annually on both legs.

$$D^{Fixed}_{Mac} = \bar{t}_{Fixed} = \frac{1 + 2 + 3 + ... + 10}{10} = 5.5$$

$$D^{Fixed}_{Mod} = \frac{\bar{t}_{Fixed}}{1 + p} = \frac{5.5}{1 + 0.90\%} = \frac{5.5}{1.0090} = 5.4509$$

**Example 11.30.** Consider a 2 year USD payer swap with a constant notional of USD 10mm a fixed rate of 0.90% and a Libor spread of 10 basis points on the floating leg. The market 2 year par-rate is 1.00% What is the macaulay and modified duration of the fixed leg to 4 decimal places? You may assume coupons are paid annually on both legs.

$$D^{Fixed}_{Mac} = \bar{t}_{Fixed} = \frac{1 + 2}{2} = 1.5$$

$$D^{Fixed}_{Mod} = \frac{\bar{t}_{Fixed}}{1 + p} = \frac{1.5}{1 + 1.00\%} = \frac{1.5}{1.0100} = 1.4851$$

### 11.7.2 Float Leg Duration

**Example 11.31.** Consider a 5 year GBP payer swap with a constant notional of GBP 50mm a fixed rate of 1.25% and no Libor spread on the floating leg. The market 5 year par-rate is 1.30% The front coupon has set at 0.83%. What is the macaulay and modified duration of the floating leg to 4 decimal places? You may assume coupons are paid annually on both the fixed and floating leg.

*Note: Forecast rates set in advance, the 12M GBP Libor forecast rates for the swap 1Y, 2Y, 3Y, and 4Y floating coupons are 1.07%, 1.32%, 1.45% and 1.50% respectively.*

$$\begin{aligned} PV^{Float} &= (p^{Market} + s)NT = -(1.30\% + 0) \times 50mm \times 5 \\ &= 1.30\% \times 250mm \\ &= \text{GBP } 3.25mm \end{aligned}$$

$$\begin{aligned} D^{Float}_{Mac} &= \frac{N \sum_{i=1}^{m} t_j l_{j-1}}{PV^{Float}} \\ &= 50mm \times \left( \frac{1 \times 0.83\% + 2 \times 1.07\% + 3 \times 1.32\% + 4 \times 1.45\% + 5 \times 1.50\%}{3.25mm} \right) \\ &= 50mm \times \left( \frac{0.83\% + 2.14\% + 3.96\% + 5.80\% + 7.50\%}{3.25mm} \right) \\ &= 50mm \times \left( \frac{20.23\%}{3.25mm} \right) \\ &= 3.1123 \end{aligned}$$

$$D^{Float}_{Mod} = \frac{D^{Float}_{Mac}}{1 + p} = \frac{3.1123}{1 + 1.30\%} = \frac{3.1123}{1.0130} = 3.0724$$

### 11.7.3 Swap Duration

**Example 11.32.** Consider a 3 year JPY payer swap with a constant notional of JPY 250mm a fixed rate of 0.05% and no Libor spread on the floating leg. The market 3 year par-rate is 0.10% The front coupon has set at 0.07%. What is the modified duration of the swap to 4 decimal places? You may assume coupons are paid annually on both the fixed and floating leg.

*Note: Forecast rates set in advance, the 12M GBP Libor forecast rates for the swap 1Y and 2Y floating coupons are 0.11% and 0.14% respectively.*

$$\begin{aligned} PV^{Swap} &= \phi(r_{Fixed} - p^{Market} + s)NT = -(0.05\% - 0.10\% + 0) \times 250mm \times 3 \\ &= 0.05\% \times 750mm \\ &= \text{JPY } 0.375mm \end{aligned}$$

$$
\begin{aligned}
D^{Swap}_{Mac} &= \phi\left(\frac{NTr^{Fixed} - N\tau\sum_{j=1}^{m} l_{j-1}}{PV^{Swap}}\right) \\
&= -\left(\frac{(250mm \times 3 \times 0.05\%) - 250mm \times 1.0 \times (0.07\% + 0.11\% + 0.14\%)}{0.375mm}\right) \\
&= -\left(\frac{375K - 800K}{0.375mm}\right) \\
&= 1.1333
\end{aligned}
$$

$$
D^{Swap}_{Mod} = \frac{D^{Swap}_{Mac}}{1+p} = \frac{1.1333}{1+0.10\%} = \frac{1.1333}{1.0010} = 1.1322
$$

## 11.8 DV01 Examples

### 11.8.1 DV01 for Par Swaps

**Example 11.33.** Consider a 2 year USD receiver swap trading at par with a constant notional of USD 150mm a fixed rate of 1.60% and no Libor spread on the floating leg. The market 2 year par-rate is 1.60% What is the present value of the swap and it's PV01 and DV01? Note the USD floating leg coupon frequency is 3m, which can be assumed to be 0.25.

$$
\begin{aligned}
PV^{Swap} &= \phi\left(r^{Fixed} - p^{2Y} - s\right)NT \\
&= +\left(1.60\% - 1.60\%\right) \times 150mm \times 2 \\
&= 0.0\% \times 150mm \times 2 \\
&= \text{USD } 0
\end{aligned}
$$

A PV of zero is expected since the swap as mentioned is trading at par.

$$
D^{Fixed}_{Mac} = \bar{t}_{Fixed} = \frac{1+2}{2} = 1.5
$$

$$
D^{Fixed}_{Mod} = \left(\frac{\bar{t}_{Fixed}}{1+p}\right) = \left(\frac{1.5}{1+1.60\%}\right) = 1.4764
$$

$$
PV01^{Swap} = \frac{\phi NT}{10,000} = \frac{+150mm \times 2}{10,000} = \frac{300mm}{10,000} = \text{USD } 30,000
$$

$$
\begin{aligned}
DV01^{Swap} &= \frac{\phi\left(1+\left(r^{Fixed}-p^{Market}+s\right)D^{Fixed}_{Mod}\right)NT}{10,000}\\
&= \frac{+(1+(1.60\%-1.60\%+0.0\%)\times 1.4764)\times 150mm\times 2}{10,000}\\
&= \frac{(1+0.0\%)\times 300mm}{10,000}\\
&= \frac{300mm}{10,000}\\
&= \text{USD } 30,000
\end{aligned}
$$

As can be seen the PV01 and DV01 for par swaps is the same. PV01 captures Libor forecast risk and DV01 captures both Libor forecast risk and discount risk. Par swaps, by definition, have zero present value with the fixed and float legs having equal and opposite present values. Therefore since there is no net cashflow payments being made by the swap, there is no net discount risk. Hence the total PV01 and DV01 for par swaps are the same.

### 11.8.2 DV01 for Non-Par Swaps

**Example 11.34.** Consider a 5 year GBP payer swap with a constant notional of GBP 50mm a fixed rate of 1.50% and a Libor spread of 40 basis points on the floating leg. The market 5 year par-rate is 1.30% What is the present value of the swap and it's DV01? Note the GBP floating leg coupon frequency is 6m, which can be assumed to be 0.5.

$$
\begin{aligned}
PV^{Swap} &= \phi\left(r^{Fixed}-p^{5Y}-s\right)NT\\
&= -1\times\left(1.50\%-1.30\%-0.40\%\right)\times 50mm\times 5\\
&= -0.20\%\times 50mm\times 5\\
&= \text{EUR } -500K
\end{aligned}
$$

$$
\bar{t}_{Fixed} = \frac{1+2+3+4+5}{5} = 3.0
$$

$$
\begin{aligned}
D^{Fixed}_{Mod} &= \left(\frac{\bar{t}_{Fixed}}{1+p}\right)\\
&= \left(\frac{3.0}{1+1.30\%}\right)\\
&= 2.9615
\end{aligned}
$$

$$
\begin{aligned}
DV01^{Swap} &= \frac{\phi\left(1+\left(r^{Fixed}-p^{Market}+s\right)D_{Mod}^{Fixed}\right)NT}{10,000}\\
&= \frac{-(1+(1.50\%-1.30\%+0.40\%)\times 2.9615)\times 50mm\times 5}{10,000}\\
&= \frac{-(1+(0.60\%\times 2.9615)\times 250mm)}{10,000}\\
&= \frac{-(1+1.7769\%)\times 250mm}{10,000}\\
&= \frac{-254.44mm}{10,000}\\
&= \text{EUR } -25,444
\end{aligned}
$$

## 11.9 DV01 Duration Matching Examples

**Example 11.35.** A 3 year GBP receiver swap with a constant notional of GBP 250mm has DV01 GBP 75,000 and a 2 year GBP payer swap with a constant notional of GBP 250mm has DV01 GBP 50,000. What size notional in the 2 year payer swap would fully hedge the 3 year receiver swap from small changes in interest rates?

Firstly we calculate the hedge ratio

$$
\text{Hedge Ratio} = \left(\frac{\text{DV01}^{Trade}}{\text{DV01}^{Hedge}}\right) = \left(\frac{75,000}{50,000}\right) = 1.5
$$

from which we can deduce the hedge position

$$
\begin{aligned}
\text{Hedge Position} &= \text{Hedge Notional } \times \text{Hedge Ratio}\\
&= 250mm \times 1.5\\
&= \text{GBP } 375mm
\end{aligned}
$$

Therefore the GBP 250mm position in the 3 year receiver GBP swap could be fully hedged with a GBP 375mm position in the 2 year GBP payer swap.

**Example 11.36.** A 5 year USD payer swap with a constant notional of USD 100mm has DV01 USD 50,000 and a 2 year USD receiver swap with a constant notional of USD 1mm has DV01 USD 200. What size notional in the 2 year USD receiver swap would fully hedge the 5 year payer swap from small changes in interest rates?

Firstly we calculate the hedge ratio

$$
\text{Hedge Ratio} = \left(\frac{\text{DV01}^{Trade}}{\text{DV01}^{Hedge}}\right) = \left(\frac{50,000}{200}\right) = 250
$$

from which we can deduce the hedge position

$$
\begin{aligned}
\text{Hedge Position} &= \text{Hedge Notional } \times \text{Hedge Ratio} \\
&= 1mm \times 250 \\
&= \text{USD } 250mm
\end{aligned}
$$

Therefore the USD 100mm swap position in the 5 year USD payer swap could be fully hedged with a USD 250mm position in the 2 year USD receiver swap.

## 11.10 Asset Swap Spread (Par-Par) Examples

**Example 11.37.** Consider the 10 year German Bund DBR 0.5% 2026 which is currently trading at a clean price of 99.020. Given that the 10 year EUR swap rate is 0.920% what is the par-par asset swap spread for this bond?

Firstly we recall the par-par asset swap spread approximation given in equation (10.12)

$$s = r^{Fixed} - p^{Market} + \textit{Par-Par Adjustment}$$

where

$$\textit{Par-Par Adjustment} = \left( \frac{100 - \; B}{100} \right) / T$$

The Par-Par adjustment is the dominating term and evaluates to

$$\textit{Par-Par Adjustment} = \left( \frac{100 - 99.020}{100} \right) / 10 = 0.098\%$$

knowing this we can calculate the asset swap spread $s$ as

$$
\begin{aligned}
s &= 0.500\% - 0.920\% + 0.098\% \\
&= -0.322\% \\
&\textit{or -32.20 basis points}
\end{aligned}
$$

**Example 11.38.** Consider the 10 year Greek Government Bond GGB 3.0% 2026 which is currently trading at a clean price of 65.398. Given that the 10 year EUR swap rate is 0.920% what is the par-par asset swap spread for this bond?

Again we recall the par-par asset swap spread approximation given in equation (10.12)

$$s = r^{Fixed} - p^{Market} + \textit{Par-Par Adjustment}$$

where

$$\textit{Par-Par Adjustment} = \left( \frac{100 - \; B}{100} \right) / T$$

The Par-Par adjustment is the dominating term and evaluates to

$$\textit{Par-Par Adjustment} = \left( \frac{100 - 65.398}{100} \right) / 10 = 3.4602\%$$

knowing this we can calculate the asset swap spread $s$ as

$$\begin{aligned} s &= 3.000\% - 0.920\% + 3.4602\% \\ &= 5.5402\% \\ &\textit{or 554.02 basis points} \end{aligned}$$

# 12 Pricing Approximation Accuracy

In this section we perform some brief analysis on the accuracy of our 'do-it-in-your-head' approximations versus actual market prices by applying our approximations to the case study we originally considered above in section (5).These approximations we introduced in section (10.3) have their foundations based on the current low interest rate environment.

The approximations assume unit discount factors and assume that swaps have annualized coupon accrual periods having year fractions also of unity or 1.0. The later assumption holds well when interest rate curves are flat with low curve convexity.

The accuracy of approximation results are presented below, which we find are quite satisfactory. Importantly PV, Par Rate, PV01 and DV01 calculations are shown to be reasonable approximations to expected market quotes. The accuracy of these approximations implies that we can indeed price swaps in our head successfully using the formulae outlined in section (10.4) and summarized in appendix D to get a good indication of the true expected market prices and risk in the current low interest rate environment.

## 12.1 Annuity Approximation

Expected Annuity USD 19,704,000 vs. approximation USD 20,000,000. The difference of USD 296,000 or 1.5%, but this is not unreasonable considering we are approximating two cashflows of USD 10,000,000 each.

**Annuity Approximation**

| **Fixed Leg** | Cashflow 1 | Cashflow 2 |
|---|---|---|
| notional, N | 10,000,000 | 10,000,000 |
| YearFrac, tau | 1.00 | 1.00 |
| DiscFact, P | 1.000000 | 1.000000 |

| **Float Leg** | Cashflow 1 | Cashflow 2 |
|---|---|---|
| notional, N | 10,000,000 | 10,000,000 |
| YearFrac, tau | 1.00 | 1.00 |
| DiscFact, P | 1.000000 | 1.000000 |

| | |
|---|---|
| **A( Fixed )** | 20,000,000 |
| +/- | 296,000 |
| +/- % | 1.5% |

| | |
|---|---|
| **A( Float )** | 20,000,000 |
| +/- | 296,000 |
| +/- % | 1.5% |

Figure 29: Annuity Approximation Calculation

## 12.2 Par Rate Approximation

Expected Par Rate 0.39643% vs. approximation 0.39700%. The difference of 0.00057% being ca. 1/20th of a basis point, which is very good indeed.

**Par Rate Approximation**

| **Fixed Leg** | Cashflow 1 | Cashflow 2 |
|---|---|---|
| notional, N | 10,000,000 | 10,000,000 |
| fixed rate, r | 1.50% | 1.50% |
| YearFrac, tau | 1.00 | 1.00 |
| DiscFact, P | 1.000000 | 1.000000 |

| **Float Leg** | Cashflow 1 | Cashflow 2 |
|---|---|---|
| notional, N | 10,000,000 | 10,000,000 |
| float rate, l | 0.28% | 0.51% |
| YearFrac, tau | 1.00 | 1.00 |
| DiscFact, P | 1.000000 | 1.000000 |

| | |
|---|---|
| A( Fixed ) | 20,000,000 |
| PV( Float ) | 79,400 |

| | |
|---|---|
| **Par Rate, p** | 0.39700% |
| +/- % | 0.00057% |

Figure 30: Par Rate Approximation Calculation

## 12.3 Swap PV Approximation

Expected Swap PV USD 217,447 vs approximation USD 220,600. The difference of USD 3,153 or 1.4% is reasonable.

**Swap PV Approximation**

| **Fixed Leg** | Cashflow 1 | Cashflow 2 |
|---|---|---|
| notional, N | 10,000,000 | 10,000,000 |
| fixed rate, r | 1.50% | 1.50% |
| YearFrac, tau | 1.00 | 1.00 |
| DiscFact, P | 1.000000 | 1.000000 |

| **Float Leg** | Cashflow 1 | Cashflow 2 |
|---|---|---|
| notional, N | 10,000,000 | 10,000,000 |
| float rate, l | 0.28% | 0.51% |
| YearFrac, tau | 1.00 | 1.00 |
| DiscFact, P | 1.000000 | 1.000000 |

| | |
|---|---|
| PV( Fixed ) | 300,000 |
| PV( Float ) | 79,400 |

$$PV^{Swap} = \phi\left(PV^{\text{Fixed Leg}} - PV^{\text{Float Leg}}\right)$$

| | |
|---|---|
| Swap Type | Receiver |
| Phi | 1 |
| PV( Swap ) | 220,600 |
| +/- | 3,153 |
| +/- % | 1.4% |

Figure 31: Swap PV Approximation Calculation

## 12.4 Macaulay's Duration Approximation

The expected Macaulay's Duration for the fixed and floating legs of the swap were 1.4976 and 1.6451 respectively vs. 1.5000 and 1.6474 for the approximate durations respectively. The resulting differences being 0.0024 for the fixed leg Macaulay's duration and 0.0022 for the floating leg.

**Macaulay's Duration Approximation**

| Fixed Leg | Cashflow 1 | Cashflow 2 |
|---|---|---|
| time, t | 1.00 | 2.00 |
| notional, N | 10,000,000 | 10,000,000 |
| fixed rate, r | 1.50% | 1.50% |
| YearFrac, tau | 1.00 | 1.00 |
| DiscFact, P | 1.000000 | 1.000000 |
| | | |
| Numerator | 450,000 | |
| Denominator | 300,000 | |
| | | |
| D( Fixed ) | 1.5000 | |
| +/- | 0.0024 | |

| Float Leg | Cashflow 1 | Cashflow 2 |
|---|---|---|
| time, t | 1.00 | 2.00 |
| notional, N | 10,000,000 | 10,000,000 |
| float rate, l | 0.28% | 0.51% |
| YearFrac, tau | 1.00 | 1.00 |
| DiscFact, P | 1.000000 | 1.000000 |
| | | |
| Numerator | 130,800 | |
| Denominator | 79,400 | |
| | | |
| D ( Float ) | 1.6474 | |
| +/- | 0.0022 | |

Figure 32: Macaulay's Duration Approximation Calculation

## 12.5 Modified Duration Approximation

The expected Modified Durations for the fixed and floating legs of the swap were 1.4917 and 1.6386 respectively vs. 1.4941 and 1.6408 for the approximate durations respectively. The resulting differences being 0.0024 for the fixed leg Modified duration and 0.0022 for the floating leg.

The modified duration is a key component of the DV01 risk figures. For risk purposes we scale this number by 1 basis point or the reciprocal of 10,000, which implies that the order of approximation accuracy is sufficiently good for reasonable risk calculations.

**Modified Duration Approximation**

| Fixed Leg | |
|---|---|
| D( Fixed ) | 1.5000 |
| Par Rate, p | 0.39700% |
| MD( Fixed ) | 1.4941 |
| +/- | 0.0024 |

| Float Leg | |
|---|---|
| D( Float ) | 1.6474 |
| Par Rate, p | 0.39700% |
| MD( Float ) | 1.6408 |
| +/- | 0.0022 |

Figure 33: Modified Duration Approximation Calculation

## 12.6 PV01 Approximation

Expected PV01 is USD 1,970 vs approximation USD 2,000. The difference is USD 30 or 1.5%. The approximation proves to be a good estimator for PV01.

**Swap PV01 Approximation**

| | |
|---|---|
| Swap Type | Receiver |
| Phi | 1 |
| A( Fixed ) | 20,000,000 |
| PV01( Swap ) | 2,000 |
| +/- | 30 |
| +/- % | 1.5% |

Figure 34: PV01 Approximation Calculation

## 12.7 DV01 Approximation

Expected DV01 is USD 2,003 vs approximation USD 2,033. The difference is USD 30 or 1.5%, which is perfectly acceptable as an approximation estimate.

**Swap DV01 Approximation**

| | |
|---|---|
| Swap Type | Receiver |
| Phi | 1 |

**Libor Forecast Rate Risk**

| | |
|---|---|
| PV01( Swap ) | 2,000 |

**OIS Discounting Risk ( Swap )**

| | |
|---|---|
| PV( Swap ) | 220,600 |
| MD( Fixed ) | 1.4941 |
| OIS Risk | 33 |

**OIS Discounting Risk ( Spread )**

| | |
|---|---|
| PV( Spread ) | 0 |
| MD( Float ) | 1.6408 |
| OIS Risk | 0 |

**Risk Breakdown**

| | | |
|---|---|---|
| Libor Risk | 2,000 | Forecast Risk |
| OIS Risk | 33 | Discounting Risk |
| DV01( Swap ) | 2,033 | Total Risk |
| +/- | 30 | |
| +/- % | 1.5% | |

Figure 35: DV01 Approximation Calculation

To elaborate further on the approximation accuracy the expected DV01 comprises of Libor forecast risk or PV01 component with a value of USD 1,970 and an OIS discounting risk component with a value of USD 32. The error in the Libor forecast risk term is USD 29 and for the OIS discounting risk the error is only USD 1, giving a DV01 risk total of USD 30. The majority of the approximation error comes from the Libor risk or PV01 component. OIS risk is usually much lower than Libor risk, however this is less so and not always the case for swaps trading far from par.

### 12.8 Hedge Ratio Approximation

The expected hedge ratio is 0.4006 giving a hedge quantity of 4,005,671 vs the approximation 0.4066 and 4,065,918. The hedge quantity error us 60,247 or 1.5%, which is also an acceptable estimate for a swap with USD 10,000,000 notional.

**Hedge Ratio Approximation**

| | |
|---|---|
| DV01( Swap ) | 2,033 |
| DV01( Hedge ) | 5,000 |
| Hedge Ratio | 0.4066 |

| | |
|---|---|
| Swap Notional | 10,000,000 |
| Hedge Qty | 4,065,918 |
| +/- | 60,247 |
| +/- % | 1.5% |

Figure 36: Hedge Ratio Approximation Calculation

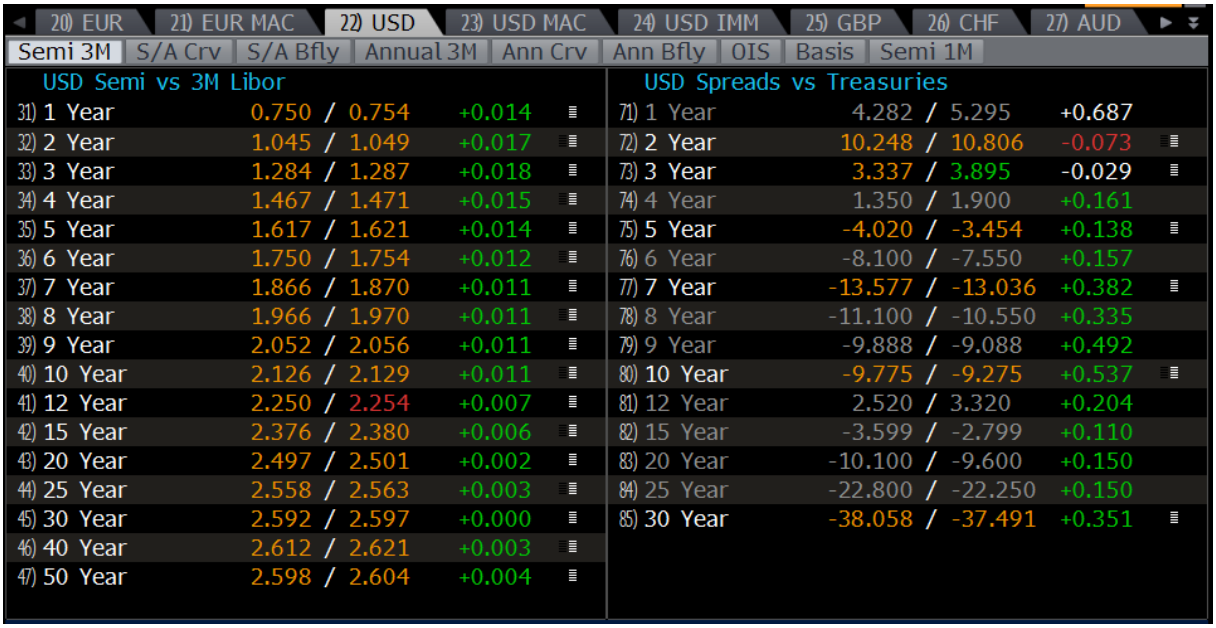
## 13 Conclusion

In summary we have reviewed the interest rate and asset swap products, examining how they are priced and quoted in the interest rate market place. We reviewed the pricing and risk formulae for both interest rate and asset swap products, providing many examples of how to price and risk swap instruments. We also proposed approximation formulae that can be used to price and risk swaps in our head quickly and accurately.

Finally by means of a case study we presented the accuracy of such pricing and risk approximations to demonstrate that we can indeed use such approximations to price swaps quickly in one's head and to further show that the approximations hold well in the current low interest rate environment.

# Appendix A: Market Par Rates

In this appendix we display current market par rate quotes for USD, EUR, GBP and JPY currencies as displayed in the Bloomberg Interest Rate Swap trading portal, which can be found on the Bloomberg terminal on the BBTI pages.


20) EUR | 21) EUR MAC | 22) USD | 23) USD MAC | 24) USD IMM | 25) GBP | 26) CHF | 27) AUD

Semi 3M | S/A Crv | S/A Bfly | Annual 3M | Ann Crv | Ann Bfly | OIS | Basis | Semi 1M

| USD Semi vs 3M Libor | | | USD Spreads vs Treasuries | | |
|---|---|---|---|---|---|
| 31) 1 Year | 0.750 / 0.754 | +0.014 | 71) 1 Year | 4.282 / 5.295 | +0.687 |
| 32) 2 Year | 1.045 / 1.049 | +0.017 | 72) 2 Year | 10.248 / 10.806 | -0.073 |
| 33) 3 Year | 1.284 / 1.287 | +0.018 | 73) 3 Year | 3.337 / 3.895 | -0.029 |
| 34) 4 Year | 1.467 / 1.471 | +0.015 | 74) 4 Year | 1.350 / 1.900 | +0.161 |
| 35) 5 Year | 1.617 / 1.621 | +0.014 | 75) 5 Year | -4.020 / -3.454 | +0.138 |
| 36) 6 Year | 1.750 / 1.754 | +0.012 | 76) 6 Year | -8.100 / -7.550 | +0.157 |
| 37) 7 Year | 1.866 / 1.870 | +0.011 | 77) 7 Year | -13.577 / -13.036 | +0.382 |
| 38) 8 Year | 1.966 / 1.970 | +0.011 | 78) 8 Year | -11.100 / -10.550 | +0.335 |
| 39) 9 Year | 2.052 / 2.056 | +0.011 | 79) 9 Year | -9.888 / -9.088 | +0.492 |
| 40) 10 Year | 2.126 / 2.129 | +0.011 | 80) 10 Year | -9.775 / -9.275 | +0.537 |
| 41) 12 Year | 2.250 / 2.254 | +0.007 | 81) 12 Year | 2.520 / 3.320 | +0.204 |
| 42) 15 Year | 2.376 / 2.380 | +0.006 | 82) 15 Year | -3.599 / -2.799 | +0.110 |
| 43) 20 Year | 2.497 / 2.501 | +0.002 | 83) 20 Year | -10.100 / -9.600 | +0.150 |
| 44) 25 Year | 2.558 / 2.563 | +0.003 | 84) 25 Year | -22.800 / -22.250 | +0.150 |
| 45) 30 Year | 2.592 / 2.597 | +0.000 | 85) 30 Year | -38.058 / -37.491 | +0.351 |
| 46) 40 Year | 2.612 / 2.621 | +0.003 | | | |
| 47) 50 Year | 2.598 / 2.604 | +0.004 | | | |

Figure 37: USD IRS Quotes from Bloomberg IRS Trading Portal - Used with Permission.

Interest Rate Swaps | 2) Tools | 3) Settings | 4) Trading Access | IRS Multi-Dealer RFQ

10:56 BGL

20) EUR | 21) EUR MAC | 22) USD | 23) USD MAC | 24) USD IMM | 25) GBP | 26) CHF | 27) AUD

Vanilla 6M | 6M Crv | 6M Bfly | Vanilla 3M | 3M Crv | 3M Bfly | 12M/1M | OIS | BFX | IMM 6M | IMM 3M

| 6M Euribor | | | | |
|---|---|---|---|---|
| 31) 1 Year | -0.059 | / | -0.053 | -0.001 |
| 32) 18 Month | -0.063 | / | -0.057 | -0.003 |
| 33) 2 Year | -0.050 | / | -0.046 | -0.002 |
| 34) 3 Year | 0.016 | / | 0.021 | +0.000 |
| 35) 4 Year | 0.128 | / | 0.132 | +0.000 |
| 36) 5 Year | 0.256 | / | 0.260 | -0.001 |
| 37) 6 Year | 0.393 | / | 0.398 | -0.003 |
| 38) 7 Year | 0.535 | / | 0.540 | -0.002 |
| 39) 8 Year | 0.675 | / | 0.680 | -0.004 |
| 40) 9 Year | 0.806 | / | 0.810 | -0.004 |
| 41) 10 Year | 0.920 | / | 0.924 | -0.006 |
| 42) 11 Year | 1.023 | / | 1.030 | -0.005 |
| 43) 12 Year | 1.115 | / | 1.121 | -0.004 |
| 44) 13 Year | 1.195 | / | 1.200 | -0.003 |
| 45) 14 Year | 1.262 | / | 1.268 | -0.002 |
| 46) 15 Year | 1.322 | / | 1.328 | -0.001 |
| 47) 16 Year | 1.371 | / | 1.377 | -0.001 |
| 48) 17 Year | 1.414 | / | 1.421 | +0.001 |

| 6M Euribor | | | | |
|---|---|---|---|---|
| 49) 18 Year | 1.449 | / | 1.456 | +0.001 |
| 50) 19 Year | 1.477 | / | 1.484 | +0.000 |
| 51) 20 Year | 1.500 | / | 1.506 | +0.003 |
| 52) 25 Year | 1.540 | / | 1.546 | +0.004 |
| 53) 30 Year | 1.545 | / | 1.551 | +0.005 |
| 54) 40 Year | 1.542 | / | 1.549 | +0.005 |
| 55) 50 Year | 1.496 | / | 1.503 | +0.005 |

| Other Markets | | | | |
|---|---|---|---|---|
| 100) EURO-SCHATZ | 10:41 | d | 111.500 | + 0.010 |
| 101) EURO-BOBL | 10:40 | d | 130.920 | + 0.060 |
| 102) EURO-BUND | 10:41 | d | 158.560 | + 0.300 |
| 103) EURO-BUXL | 10:41 | d | 152.920 | + 0.140 |

Figure 38: EUR IRS Quotes from Bloomberg IRS Trading Portal - Used with Permission.

20) EUR | 21) EUR MAC | 22) USD | 23) USD MAC | 24) USD IMM | 25) GBP | 26) CHF | 27) AUD

Semi 6M | Semi 6M Crv | Semi 6M Bfly | Semi 3M | Quarterly 3M | OIS

| GBP Semi-Annual vs 3M | | | | |
|---|---|---|---|---|
| 31) 18 Month | 0.767 | / | 0.786 | -0.004 |
| 32) 1 Year | 0.675 | / | 0.694 | -0.002 |
| 33) 2 Year | 0.868 | / | 0.879 | -0.005 |
| 34) 3 Year | 1.036 | / | 1.052 | -0.015 |
| 35) 4 Year | 1.181 | / | 1.197 | -0.018 |
| 36) 5 Year | 1.310 | / | 1.326 | -0.021 |
| 37) 6 Year | 1.426 | / | 1.441 | -0.021 |
| 38) 7 Year | 1.527 | / | 1.542 | -0.021 |
| 39) 8 Year | 1.616 | / | 1.630 | -0.022 |
| 40) 9 Year | 1.691 | / | 1.706 | -0.022 |
| 41) 10 Year | 1.755 | / | 1.770 | -0.021 |
| 42) 11 Year | 1.810 | / | 1.832 | -0.033 |
| 43) 12 Year | 1.859 | / | 1.873 | -0.022 |
| 44) 13 Year | 1.898 | / | 1.926 | -0.032 |
| 45) 14 Year | 1.932 | / | 1.960 | -0.031 |
| 46) 15 Year | 1.959 | / | 1.975 | -0.020 |
| 47) 16 Year | 1.979 | / | 2.007 | -0.030 |
| 48) 17 Year | 1.994 | / | 2.021 | -0.030 |

| GBP Semi-Annual vs 3M | | | | |
|---|---|---|---|---|
| 49) 18 Year | 2.004 | / | 2.031 | -0.030 |
| 50) 19 Year | 2.010 | / | 2.038 | -0.029 |
| 51) 20 Year | 2.014 | / | 2.030 | -0.019 |
| 52) 25 Year | 2.006 | / | 2.021 | -0.018 |
| 53) 30 Year | 1.990 | / | 2.006 | -0.018 |
| 54) 40 Year | 1.930 | / | 1.944 | -0.016 |
| 55) 50 Year | 1.886 | / | 1.901 | -0.017 |

| Other Markets | | | | |
|---|---|---|---|---|
| 100) EURO-SCHATZ | 10:42 | d | 111.500 | + 0.010 |
| 101) EURO-BOBL | 10:42 | d | 130.920 | + 0.060 |
| 102) EURO-BUND | 10:42 | d | 158.540 | + 0.280 |
| 103) EURO-BUXL | 10:42 | d | 152.880 | + 0.100 |
| 104) LONG GILT FT | 10:47 | d | 117.580 | + 0.340 |

Figure 39: GBP IRS Quotes from Bloomberg IRS Trading Portal - Used with Permission.



| | | | |
|---|---|---|---|
| 31) | 1 Year | 0.121 / 0.126 | +0.015 |
| 32) | 18 Month | 0.116 / 0.124 | +0.015 |
| 33) | 2 Year | 0.100 / 0.103 | +0.000 |
| 34) | 3 Year | 0.102 / 0.105 | -0.016 |
| 35) | 4 Year | 0.123 / 0.128 | -0.001 |
| 36) | 5 Year | 0.161 / 0.167 | +0.000 |
| 37) | 6 Year | 0.209 / 0.215 | +0.001 |
| 38) | 7 Year | 0.262 / 0.267 | -0.001 |
| 39) | 8 Year | 0.319 / 0.324 | +0.000 |
| 40) | 9 Year | 0.379 / 0.384 | -0.001 |
| 41) | 10 Year | 0.441 / 0.446 | -0.003 |
| 42) | 12 Year | 0.573 / 0.580 | -0.003 |
| 43) | 15 Year | 0.778 / 0.788 | -0.004 |
| 44) | 20 Year | 1.057 / 1.064 | +0.003 |
| 45) | 25 Year | 1.202 / 1.209 | +0.002 |
| 46) | 30 Year | 1.270 / 1.280 | -0.008 |
| 47) | 40 Year | 1.379 / 1.404 | -0.006 |

Figure 40: JPY IRS Quotes from Bloomberg IRS Trading Portal - Used with Permission.

# Appendix B: Excel Pricing Spreadsheet

This paper comes together with an Excel Swaps Pricer. The tool is not for live pricing, but for demonstration purposes. Kindly e-mail the author should you wish to receive a copy. A visual of the tool is displayed below.

**Refresh**

**Swap Trade Details**

| | |
|---|---|
| Payer/Receiver | RECEIVER |
| Currency | EUR |
| Notional, N | 1,000,000 |
| Fixed Rate, $r^{Fixed}$ | 1.0000% |
| Fixed Frequency | ANNUAL |
| Float Frequency | SEMI-ANNUAL |
| Libor Spread, s | 0.00 |
| Tenor, T | 5.00 |

**Swap Pricing**

| | |
|---|---|
| Swap PV | 31,873 |
| Fixed Leg PV | 50,348 |
| Float Leg PV | -18,475 |
| Par Rate | 0.3670% |

**Swap Risk**

| | |
|---|---|
| PV01 | 503 |
| Numerical DV01 | 512 |
| Analytical DV01 | 513 |

**Fixed Leg**

| Row | Accrual Start | Accrual End | Pay Date | $t_i$ | N | $r^{Fixed}$ | $\tau_i$ | $P(t_E, t_i)$ | $PV^{Fixed}$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 13-Jun-16 | 13-Jun-17 | 13-Jun-17 | 1.00 | 1,000,000 | 1.0000% | 1.00 | 1.004141 | 10,041 |
| 2 | 13-Jun-17 | 13-Jun-18 | 13-Jun-18 | 2.00 | 1,000,000 | 1.0000% | 1.00 | 1.006941 | 10,069 |
| 3 | 13-Jun-18 | 13-Jun-19 | 13-Jun-19 | 3.00 | 1,000,000 | 1.0000% | 1.00 | 1.008390 | 10,084 |
| 4 | 13-Jun-19 | 13-Jun-20 | 13-Jun-20 | 4.00 | 1,000,000 | 1.0000% | 1.00 | 1.008386 | 10,084 |
| 5 | 13-Jun-20 | 13-Jun-21 | 13-Jun-21 | 5.00 | 1,000,000 | 1.0000% | 1.00 | 1.006933 | 10,069 |
| 6 | | | | | | | | | |
| 7 | | | | | | | | | |
| 8 | | | | | | | | | |
| 9 | | | | | | | | | |
| 10 | | | | | | | | | |
| 11 | | | | | | | | | |
| 12 | | | | | | | | | |
| 13 | | | | | | | | | |
| 14 | | | | | | | | | |
| 15 | | | | | | | | | |
| 16 | | | | | | | | | |
| 17 | | | | | | | | | |
| 18 | | | | | | | | | |

Figure 41: Excel Swaps Pricer, Fixed Leg Calculations

**Float Leg**

| Row | Fixing Date | Accrual Start | Accrual End | Pay Date | $t_j$ | N | $l_{j-1}$ | s | $l_{j-1} + s$ | $\tau_j$ | $P(t_E, tj)$ | $PV^{Float}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 13-Jun-16 | 13-Jun-16 | 12-Dec-16 | 12-Dec-16 | 0.50 | 1,000,000 | 0.0487% | 0.00 | 0.0487% | 0.50 | 1.001938 | 244 |
| 2 | 12-Dec-16 | 12-Dec-16 | 13-Jun-17 | 13-Jun-17 | 1.00 | 1,000,000 | 0.1087% | 0.00 | 0.1087% | 0.50 | 1.004141 | 546 |
| 3 | 13-Jun-17 | 13-Jun-17 | 12-Dec-17 | 12-Dec-17 | 1.50 | 1,000,000 | 0.1887% | 0.00 | 0.1887% | 0.50 | 1.005506 | 949 |
| 4 | 12-Dec-17 | 12-Dec-17 | 13-Jun-18 | 13-Jun-18 | 2.00 | 1,000,000 | 0.2487% | 0.00 | 0.2487% | 0.50 | 1.006941 | 1,252 |
| 5 | 13-Jun-18 | 13-Jun-18 | 13-Dec-18 | 13-Dec-18 | 2.50 | 1,000,000 | 0.3287% | 0.00 | 0.3287% | 0.50 | 1.007934 | 1,657 |
| 6 | 13-Dec-18 | 13-Dec-18 | 13-Jun-19 | 13-Jun-19 | 3.00 | 1,000,000 | 0.4087% | 0.00 | 0.4087% | 0.50 | 1.008390 | 2,061 |
| 7 | 13-Jun-19 | 13-Jun-19 | 13-Dec-19 | 13-Dec-19 | 3.50 | 1,000,000 | 0.4687% | 0.00 | 0.4687% | 0.50 | 1.008609 | 2,364 |
| 8 | 13-Dec-19 | 13-Dec-19 | 13-Jun-20 | 13-Jun-20 | 4.00 | 1,000,000 | 0.5487% | 0.00 | 0.5487% | 0.50 | 1.008386 | 2,767 |
| 9 | 13-Jun-20 | 13-Jun-20 | 12-Dec-20 | 12-Dec-20 | 4.50 | 1,000,000 | 0.6287% | 0.00 | 0.6287% | 0.50 | 1.007929 | 3,169 |
| 10 | 12-Dec-20 | 12-Dec-20 | 13-Jun-21 | 13-Jun-21 | 5.00 | 1,000,000 | 0.6887% | 0.00 | 0.6887% | 0.50 | 1.006933 | 3,468 |

Figure 42: Excel Swaps Pricer, Floating Leg Calculations

# Appendix C: Pricing Formulae

Annuity

$$A = \sum_{i=1}^{n} \tau_i P(t_E, t_i)$$

Annuity Scaled by Notional

$$A_N = N \sum_{i=1}^{n} \tau_i P(t_E, t_i) \quad \text{when the notional is constant}$$

$$A_{N_i} = \sum_{i=1}^{n} N_i \tau_i P(t_E, t_i) \quad \text{when the notional is variable}$$

$$A = \sum_{i=1}^{n} \tau_i P(t_E, t_i) \quad \text{when the notional is unity}$$

Fixed Leg PV

$$PV^{\text{ Fixed Leg }} = r^{Fixed} \sum_{i=1}^{n} N_i \tau_i P(t_E, t_i)$$

Float Leg PV

$$PV^{\text{ Float Leg }} = \sum_{j=1}^{m} N_j (l_{j-1} + s) \tau_j P(t_E, t_j)$$

Swap PV

$$PV^{Swap} = \phi\left(\left(r^{Fixed} - p^{Market}\right) A_N^{Fixed} - s A_N^{Float}\right)$$

Swap PV01

$$PV01^{Swap} = \left(\phi A_{N_i}^{Fixed}\right) / 10,000$$

Swap DV01

$$DV01^{Swap} = PV01 + \phi\left(PV^{Swap\ No\ Spread} D_{Mod}^{Fixed} - PV^{Swap\ Spread} D_{Mod}^{Float}\right) / 10,000$$

Asset Swap Spread, Yield/Yield

$$\text{Yield-Yield Spread}\left(t, T\right) = \text{Bond Yield}\left(t, T\right) - \text{Swap Spread}\left(t, T\right)$$

Asset Swap Spread, Par/Par

$$s = \left(\frac{\left(r^{Fixed} - p^{Market}\right) A^{Fixed} + \left(\frac{100-B}{100}\right)}{A^{Float}}\right)$$

# Appendix D: Pricing Approximations

Annuity

$$A = T$$

Annuity Scaled by Notional

$$A_N = NT$$

Fixed Leg PV

$$PV^{Fixed} = r^{Fixed} NT$$

Float Leg PV

$$PV^{Float} = \left(p^{Market} + s\right) NT$$

Swap PV

$$PV^{Swap} = \phi \left(r^{Fixed} - p^{Market} - s\right) NT$$

Swap PV01

$$PV01^{Swap} = \frac{\phi NT}{10,000}$$

Swap DV01

$$DV01^{Swap} = \left(\frac{\phi \left(1 + \left(r^{Fixed} - p^{Market} + s\right) D^{Swap}_{Mod}\right) NT}{10,000}\right)$$

Asset Swap Spread, Yield/Yield

$$\text{Yield-Yield Spread}\left(t, T\right) = \text{Bond Yield}\left(t, T\right) - \text{Swap Spread}\left(t, T\right)$$

Asset Swap Spread, Par/Par

$$s = r^{Fixed} - p^{Market} + \textit{Par-Par Adjustment}$$

where

$$\textit{Par-Par Adjustment} = \left(\frac{100 - B}{100}\right) / T$$

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